Preface

Overview

The theory of partially ordered set is dynamically evolving, as demonstrated by the number of publications in the mathematical journals. Also the rather special binary relation used for evaluating data matrices with respect to possible rankings is rapidly developing. The special order relation used is the reason why often Hasse Diagram Technique (HDT) is referred to instead of partial order technique or partial order ranking. This restriction on the possible variety of binary relations fulfilling the axioms of partial order is immediately related to its use in the evaluation of data matrices, namely the component-wise order or, as it is also often called, product order. The restriction to the component-wise order has a severe consequence: The application of partial order becomes now a discipline in multivariate statistics too.

Because of the pretty quick development of HDT, the international workshops about HDT become an important platform to exchange ideas not only in theoretical questions but also within the field of applications.

The book "Multi-indicator Systems and Modelling in Partial Order" contains the newest theoretical concepts as well as new applications or even applications, where standard multivariate statistics fail. Some of the presentations have their counterpart in the book; however, there are many contributions, which are completely new in the field of applied partial order.

Why do we use the term "modelling" in the title? It turns out that complex processes or complex properties may simply be understood, if the ranking that is induced can be analyzed in terms of multi-indicator systems. In the field of chemistry this kind of analysis has a long-lasting tradition; however, the techniques can be applied by far more generally. Even the outcome of ranking indices, as calculated from traditional decision support systems (such as PROMETHEE or ELECTRE), may be of interest in terms of modelling.

Partial order applied to multi-indicator systems gets an additional quality because the relations, which may be drawn as arrows or edges in a Hasse diagram, do not only tell us the order between any two objects, but also that a data pattern is

behind this finding. Hence, Chap. 3 makes this statement clear. Incomparability in a general Hasse diagram is an inherent feature. Thus, where a data vector is in the background, the incomparability as well as the comparability can get different degrees. This aspect becomes also clear in Chap. 18 of this book, where chain and antichain analysis are specifically based on the multivariate character, which is behind the posetic analysis of matrices suitable for evaluation. Another statement was already several times mentioned: partial order theory provides analytical tools to understand results arising from multicriteria techniques. The composite indicator as the most simple and most transparent variant is studied in several chapters of this book.

In sum the book informs about recent developments in theory and in applications of partial order under the special relation as used in HDT.

The Book Chapters

It is clear that with so many different topics, theoretical and applied, one cannot easily give a logical sequence. Thus, a not necessarily innovative concept is followed to present first the chapters, which are theoretical, then chapters with applicational character and finally chapters where software aspects are considered.

The book takes care for the different aspects attributed to partial order theory. It is organized in the following sections:

(a) Theory
(b) Partial Order as Analysis Tool of Composite Indicators
(c) New Trends in Partial Order
(d) Applications
(e) Software Aspects

Hence, the first section provides an introduction to partial order theory in a general sense, however aiming at evaluation. It starts with a chapter about basics of evaluation where not only the inherent character of evaluation, namely order theory is regarded, but also a unifying concept is represented taking into account that evaluations, even based on a single dimension, can be unsharp. Whereas the first chapter clarifies the interaction between order, fuzziness and evaluation, the second chapter asks whether a given data matrix is suitable for evaluation or not. Obviously, in full generality this is not the case. Only after a series of additional inputs, for example giving the columns of a data matrix (quantifying the attributes) an orientation, a data matrix can be considered as evaluation matrix. Even if partial order is an analytical tool by which results of conventional multicriteria decision systems can be discussed, as shown in the last part of the second chapter, the most typical striking fact is the existence of incomparability. The third chapter shows how the dual character "incomparable"—yes or no—can give a quantification. How incomparable is one object with another, which is a nice example as to how far modelling within partial order can be perfomed. A multi-indicator system demands for such a modelling step.

By incomparability the graph of order relations gets a structure: Whereas a linear order would lead to a Hasse diagram consisting of a single chain, the appearance of incomparability leads to branching points, isolated points, in brief, to all the diversity directed acyclic graphs can have. Then it is a natural question, as to how far different partially ordered sets (posets) and their graphs can be compared. In the fourth chapter, the dissimilarity of posets is modelled by an embedding into a lattice and from that new measures are derived. By this contribution the methods of quantifying distances among posets get an additional sharp instrument. When we discuss similarity or dissimilarity among posets, then it is pretty natural to characterize posets by new measures, which themselves could be a basis for quantification of dissimiarity. With the concept of complexity as a poset-characterizing quantity, the first step is done. Similar to the lattice concept of Chap. 4, another lattice concept, that of Young diagrams, is the vehicle to derive a measure of complexity.

The second section is more specifically contributing to the application of partial order as an analytical tool in the decision process. A chapter about comparative knowledge discovery shows how far stakeholders and decision makers may get support by the instruments partial order is offering. When an evaluation matrix is at hand, then often weights are considered as additional and subjective information. One may take the other way round and see data-based weights, i.e. weights derived from the evaluation matrix as inherent posetic information about the importance of the indicators, which are the columns of the evaluation matrix. In the same direction aims the next chapter. Here, weights as needed to construct composite indicators, widely used in the multicriteria-decision scene, are seen as objects of a modelling process: Which weight system allows the closest coincidence between linear (weak) orders as a result of partial order theory and of composite indicators?

The next section has the title “New Trends in Partial Order”.

> One trend is directed toward partial order itself and its development beyond the graphical representation by Hasse diagrams, another is complementary: What does all the theoretical innovation help, when the applicational sciences do not use the new theoretical findings.

In that sense this section contributes to new developments within partial order theory; especially it shows how the concept of Hasse diagrams, which is limited to only a few objects, can be replaced by more general—data-mining-suitable—concepts. Although the concept is not verbally mentioned in that chapter, but practically it shows how one can use posetic coordinates instead of the Hasse diagram. The other chapter discusses the potential use of partial order concepts especially in socioeconomics. Poverty is a topic of general relevance. Although it has so many facets, the one-dimensional scale is still most often intrinsically assumed. Instead, poverty as many other concepts should be seen as multi-indicator system and hence a suitable object of partial order studies.

The following two sections are devoted to applications and sotware aspects. Both application and software are mutually stimulating. Hence, the separation into more applicational and more software-oriented sections is somewhat arbitrary. Many of the applicational chapters suggest or describe new theoretical developments besides the specific applicational field. So the first chapter in this section (Chap. 10) offers

an heuristic solution, for what often can be seen as a conundrum in Hasse diagram technique, namely the isolated vertices. Although the fact of isolatedness indicates a data pattern of specific interest, they are not facilitating a practical evaluation. In this chapter an idea is offered to remedy this problem. The heuristic idea is now—with some modifications—realized by a module of PyHasse. The application field of this chapter is taken from technical chemistry and the risk of accidents attributed to single chemicals. The following chapter combines concepts of Geographical Information Systems and Geostatistics with partial order theory and shows how chains in poset can be helpful in the interpretation of monitoring results. Here the data matrix is almost rectangular, i.e. there are many indicators and relatively few objects. This is a situation which the authors of the next chapter face too. Their question is, how to prioritize waste disposal sites for a remediation? There are many criteria that are taken into account. The authors show how a concept, already published, namely the Hierarchical Partial Order Ranking, can be applied to systematically reduce the criteria in order to arrive at a handsome priority list. In another chapter (Chap. 13) it is shown how indicators in project management can be evaluated by tools offered by partial order theory: As to how far the single indicators are responsible for the structure of the Hasse diagram and hence for the position of the objects. So, skilled processes to study sensitivity of indicators are applied (global sensitivity analysis). Furthermore, the role of the "Local Partial Order Model" was illustrated with exciting results.

When a trend should be extracted from the more applicational section, then it seems as if more and more partial order is applied together with other typical multivariate concepts, so for example partial order and geostatistics. Hence, the next chapter combines partial order with neural networks, where the neural networds are applied as a preprocessing tool to condense the data matrix into a form which is managable within conventional partial order graphical displays, i.e. with Hasse diagrams. The applicational field is taken from monitoring of sediments.

The section, "software aspects", could equally well be a part of the former one. However, in the chapters in this section software aspects play a slightly more central role.

Section "software aspects" starts with the usage of the software **R**. Here it is shown how macros can be written, which facilitate the programming work. In the same direction aims Chap. 16. The applicational field is the evaluation of geographic units with respect to the landscape inventory and applies cluster analysis besides the already detailed described posetic coordinates. The next two chapters apply the software package PyHasse. In Chap. 17, a typical modelling aspect is on the focus, namely as to how far the proximity of one poset can be used to explain causally the results of the other poset: The next chapter deals with the topic of Failed Nations and shows in a pretty systematic way, how different tools of PyHasse can be applied. The main focus is on the role the different indicators have. As the two chapters (17 and 18) widely apply modules of the software PyHasse, it may be a good idea to conclude the book with a short description of PyHasse itself (Chap. 19).

With this chapter the book finds its end. It spanned a spectrum from theoretical concepts to applicational ones and finally to software aspects. What is the summarizing statement we can draw and what are the trends aiming into the future?

What are the trends in the future?

Starting with environmental chemistry, the domain of applicability was steadily increasing, so it is hoped that new fields of applications are opened; perhaps the chapter "Partial orders in socio-economics: an applicative challenge for poset theorists or a cultural challenge for social scientists" is initiating new interests.

Almost all theoretical investigations, explained here, are the germ for further developments, for example the question as to how far partial order can be helpful for stakeholders seems to direct towards algebraic topology, the concepts of dissimilarity and complexity may get a unifying theoretical framework and the modelling by proximity of posets may get a safe background by suitable methods of statistical test theories. We could elongate this list easily; here only some few examples are mentioned. However there is still a main deficit in partial order theory: Incomparabilities are used as in explorative statistics: They inform us about pretty specific data configurations. This trend was inforced by publications about separability and dominance; nevertheless, the practioner, i.e. the decision maker, is not happy with that. What still and most urgently is needed is a transparent procedure to perform the trade-off among contradicting indicator values. It is to be hoped that in the future we can add corresponding concepts into our tool box.

Finally what about software? Theoretical development without its practical applicability by means of software is in the long run not helpful. Indeed software is developing and the interested reader may find either R-software practicable for her/his needs or other packages such as DART, WHASSE, PRORANK or PyHasse. The development of software is (or should be) teamwork. Nevertheless, most developments of PyHasse are on the shoulders of one single scientist. A necessary condition to get a broader personal basis is the visibility of the software. Some first steps in this direction, i.e. representation of the posetic software in the Internet, are already started. For the future, it is hoped that these web-based steps can be intensified.

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