

Preface to the Second Edition

More than 12 years have passed since the publication of the first edition of this book and the topic of coherent states (CS) is more flourishing than ever. The sheer abundance of new developments in the field made the appearance of a new edition both desirable and indeed necessary. The range of physical applications of CS has increased enormously in the past decade, from the traditional aspects of quantum physics (nuclear, condensed matter, optics) all the way to quantum gravity and quantum information theory. A good illustration of this situation is given by the recent special issue [38] of the *Journal of Physics A*, devoted to CS, which, in 37 articles, covers much of the wide panorama of the field.

On the other hand, CS have turned into a genuine domain of mathematics, reaching into applied group theory and harmonic analysis to the theory of quantization and its probabilistic formulation. The topic of quantization, using coherent states, was not dealt with in the first edition. We decided that the subject now fully merits due attention in a book such as ours. The other domain of focus in this book, that of wavelets, has continued its explosive growth. As before, our approach is to look upon CS and wavelets as mathematically related aspects of one theory. Wavelets on manifolds (spheres and other conics, for instance) and various generalizations (ridgelets, curvelets, shearlets) have been developed, prompted by the demands of signal and image processing. All these new aspects are covered, sometimes in considerable detail, in the present edition. As a consequence, our bibliography has grown considerably. Certain sections of the old book have been eliminated or rewritten, while others have been merged in an attempt to clarify, update, and streamline the presentation. On the other hand, there are other mathematical developments, such as CS on Hilbert C^* - and W^* -modules and CS on quaternionic Hilbert spaces, which are mentioned here but have not been discussed in any detail, for otherwise the book would have grown to ponderous lengths.

Additionally, we should also mention two new textbooks on CS, namely, those of Gazeau [Gaz09] (with emphasis on physical applications) and of Combes and Robert [Com12] (rather mathematically oriented).

In the course of the years past, all three of us have continued to teach and lecture about the various aspects of CS and wavelets. In doing so, we have had the pleasure

of discussing these topics with uncountably many colleagues and, of course, we benefited enormously from these discussions. We wish to thank them all for their fruitful suggestions. Particular thanks are due to Gitta Kutyniok and Daniela Roşca, who read and commented on Chaps. 14 and 15, respectively, and to Hervé Bergeron, who influenced heavily the contents of Chap. 11.

Altogether, the subject of CS continues to be a fascinating domain, both in physics and mathematics. As before we wanted to share this fascination with our readers. We made an effort to make the book useful to beginning students as well as to seasoned researchers.

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*Nitya kaaler utshab taba
Bishyer-i-dipaalika
Aami shudhu tar-i-mateer pradeep
Jaalao tahaar shikhaa¹*

– Tagore

Should authors feel compelled to justify the writing of yet another book? In an overpopulated world, must parents feel compelled to justify the bringing forth of yet another child? Perhaps not! But an act of creation is also an act of love and a love story can always be happily shared.

In writing this book, it has been the authors' feeling that in all the wealth of material on coherent states and wavelets, in which the literature on the subject abounds, there is lack of a discernable unifying mathematical perspective. The use of wavelets in research and technology has witnessed explosive growth in recent years, while the use of coherent states in numerous areas of theoretical and experimental physics has been an established trend since decades. Yet it is not at all uncommon to find practitioners in either one of the two disciplines, who are hardly aware of its links to the other. Currently, there are many books in the market which treat the subject of wavelets from a wide range of perspectives and with windows on one or several areas of a large spectrum of possible applications. On the theory of coherent states, likewise, there exist several excellent monographs, edited collections of papers, synthetic reviews or specialized articles. The emphasis in most of these works is usually on physical applications. In the more mathematical works, the focus is usually on specific properties—arising either from group theory, holomorphic function theory and, more recently, differential geometry. The point of view put forward in this book is that both the theory of wavelets and the theory of

¹Thine is an eternal celebration ...—A cosmic Festival of Lights! ... Therein I am a mere flicker of a wicker lamp ... O kindle its flame, (my Master!).

coherent states can be subsumed into certain broad functional analytic structures, namely, positive operator valued measures on a Hilbert space and reproducing kernel Hilbert spaces. The specific context in which these structures arise, to generate particular families of coherent states or wavelet transforms, could of course be very diverse, but typically they emanate either from the property of square integrability of certain unitary group representations on Hilbert spaces or from holomorphic structures associated to certain differential manifolds. In talking about square integrable representations, a broad generalization of the concept has been introduced here, moving from the well-known notion of square integrability with respect to the whole group to one based upon some of its homogeneous spaces. This generalization, while often implicit in the past, in physical discussions of coherent states (notably, in the works of Klauder, Barut and Girardello, or even in the case of the time honoured canonical coherent states, discovered by Schrödinger), had not been readily recognized in the mathematical literature until the work of Gilmore and Perelomov. In this book, this generalization is taken even further, with the result that the classes of coherent states that can be constructed and usefully employed extend to a vast array of physically pertinent groups. Similarly, it is generally known and recognized that wavelets are coherent states arising from the affine group of the real line. But using coherent states of other groups to generate higher dimensional wavelets, or alternative wavelet-like transforms, is not such a common preoccupation among practitioners of the trade (an exception being the recent book by Torr  sani). About a third of the present book is devoted to looking at wavelets from precisely this point of view, displaying thereby the richness of possibilities that exists in this domain. Considerable attention has also been paid in the book to the discretization problem, in particular with the discussion of τ -wavelets. The interplay between discrete and continuous wavelets is a rich aspect of the theory, which does not seem to have been exploited sufficiently in the past.

In presenting this unifying backdrop, for the understanding of a wide sweep of mathematical and physical structures, it is the authors' hope that the relationship between the two disciplines—of wavelets and coherent states—will have been made more transparent, aiding thereby the process of cross-fertilization as well. For graduate students or research workers, approaching the disciplines for the first time, such an overall perspective should also make the subject matter easier to assimilate with the book acting as a dovetailed introduction to both subjects—unfortunately, a frustrating incoherence blurs the existing literature on coherence! Besides being a primer for instruction, the book, of course, is also meant to be a source material for a wide range of very recent results, both in the theory of wavelets and of coherent states. The emphasis is decidedly on the mathematical aspect of the theory, although enough physical examples have been introduced, from time to time, to illustrate the material. While the book is aimed mainly at graduate students and entering research workers in physics and mathematical physics, it is nevertheless hoped that professional physicists and mathematicians would also find it interesting reading, being an area of mathematical physics in which the intermingling of theory and practice is most thoroughgoing. Prerequisites for an understanding of most of the material in the book is a familiarity with standard

Hilbert space operator techniques and group representation theory, such as every physicist would acknowledge from the days of graduate quantum mechanics and angular momentum. For the more specialized topics, an attempt has been made to make the treatment self-contained and indeed, a large part of the book is devoted to the development of the mathematical formalism.

If a book such as this can make any claims to originality at all, it can mainly be in the manner of its presentation. Beyond that, the authors believe that there is also a body of material presented here (for example in the use of POV measures or in dealing with the discretization problem), which has not appeared in book form before. An attempt has been made throughout to cite as many references to the original literature as were known to the authors—omissions should therefore be attributed to their collective ignorance and the authors would like to extend their unconditional apologies for any resulting oversight.

This book has grown out of many years of shared research interest and indeed, camaraderie, between the three authors. Almost all of the material presented here has been touched upon in courses, lectures and seminars, given to students and among colleagues at various institutions in Europe, America, Asia and Africa—notably in graduate courses and research workshops, given at different times by all three authors, in Louvain-la-Neuve, Montréal, Paris, Porto-Novo, Białystok, Dhaka, Fukuoka, Havana and Prague. One is tempted to say that the geographical diversity here rivals the mathematical menagerie!

To all their colleagues and students who have participated in these discussions, the authors would like to extend their heartfelt thanks. In particular, a few colleagues graciously volunteered to critically read parts of the manuscript and to offer numerous suggestions for improvement and clarity. Among them, one ought to specially mention J. Hilgert, G.G. Emch, S. De Bièvre and J. Renaud. In addition, the figures would not exist without the programming skills of A. Coron, L. Jacques and P. Vandergheynst (Louvain-la-Neuve), and we thank them all for their gracious help. During the writing of the book, the authors made numerous reciprocal visits to each others' institutions. To Concordia University, Montréal, the Université Catholique de Louvain, Louvain-la-Neuve and the Université Paris 7—Denis Diderot, Paris, the authors would like to express their appreciation for hospitality and collegiality. The editor from Springer-Verlag, Thomas von Foerster, deserves a special vote of thanks for his cooperation and for the exemplary patience he displayed, even as the event horizon for the completion of the manuscript kept receding further and further! It goes without saying, however, that all responsibility for errors, imperfections and residual or outright mistakes, is shared jointly by all three authors.

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