Chapter 2
The IQ Imbalance Model

Abstract Both analog In-phase and Quadrature (IQ) modulator and demodulator may introduce IQ imbalance. If it is left uncompensated, the system performance will be impaired. In this chapter, we first derive the IQ imbalance model without any other impairments. Then, we consider the IQ imbalance model in presence of frequency offset and phase noise. After that, we consider the IQ imbalance model in multiple antenna systems, firstly space-time or space-frequency encoded systems, then spatial multiplexing systems. At the end, we show the signal-to-interference ratio (SIR) degradation due to the IQ imbalance.

2.1 Introduction

In-phase and quadrature (IQ) modulator and demodulator are commonly used in wireless communication systems [1]. The IQ modulator at transmitter transforms the complex baseband signals to passband centered at carrier frequency, which is suitable for wireless transmission, while the IQ demodulator transforms the passband signals to complex baseband, which is suitable for using signal processing techniques to recover the transmitted signals. Ideal IQ modulator and demodulator provide two orthogonal channels for the real and imaginary parts of a complex signal. In reality, the IQ modulator and demodulator are not ideal, especially for wireless transceiver designed with direct conversion radio frequency (RF) architectures [2]. This introduces interference between the two orthogonal channels. If it is left uncompensated, the system may not have sufficient performance to support high order modulation schemes, and thus may not be able to support high data rate. To compensate IQ imbalance, we first need to build an IQ imbalance model. In this chapter, we start with general frequency dependent (FD) IQ imbalance model without other impairments. Then we consider the the IQ imbalance model when there are frequency offset and phase noise. Considering multiple antennas, we show the IQ imbalance model for space-time/space-frequency encoded systems

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as well as the IQ imbalance model for spatial multiplexing systems. This chapter prepares for the discussion of estimation and compensation in the next two chapters.

2.2 The IQ Imbalance Model Without Other Impairments

In this section, we derive the IQ imbalance model without other impairments. This IQ imbalance model can be found in almost all literatures about IQ imbalance estimation and compensation, see for example [3–15] and the references therein. We unify these models in this chapter. We start from general FD time domain IQ imbalance model, then the frequency independent (FI) one is just a special case. We then look at the FD IQ model in the frequency domain assuming that OFDM is used.

2.2.1 The Time Domain Transmitter-Side IQ Imbalance Model

Figure 2.1 shows the IQ modulation with amplitude and phase imbalances. In the system, there are two sources that may cause IQ imbalance. One is the phase and amplitude mismatch in the clocks used by the I and Q branches. This mismatch is constant for different frequency components in the transmitted signal, and is called FI IQ imbalance. The other is caused by the discrepancy in the filters used by the I and Q branches. This mismatch is different for different frequency components in the transmitted signal, and is called FD IQ imbalance.

At the transmitter, we assume that the clocks used by the I and Q branches have amplitude mismatch ε_t and phase mismatch θ_t, the discrete time impulse response of the analog filters in the I and Q branches are $g_t^I(n)$ and $g_t^Q(n)$, respectively, and the discrete time signals to be transmitted in the I and Q branches are $s^I(n)$ and $s^Q(n)$, respectively. Then, the modulated passband signal $x_p(n)$ equals

$$\begin{aligned} x_p(n) &= \left[s^I(n) \otimes g_t^I(n)\right](1-\varepsilon_t)\cos(\omega_c n - \theta_t) \\ &\quad - \left[s^Q(n) \otimes g_t^Q(n)\right](1+\varepsilon_t)\sin(\omega_c n + \theta_t) \\ &= x^I(n)\cos(\omega_c n) - x^Q(n)\sin(\omega_c n), \end{aligned}$$

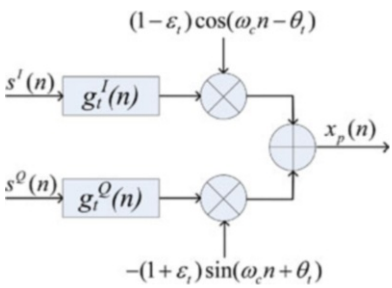


Fig. 2.1 The transmitter IQ imbalance signal model

where ω_c is the carrier frequency, $\otimes$ means convolution, and $x^I(n)$ and $x^Q(n)$ are defined respectively as

$$\begin{aligned} x^I(n) \triangleq & \left[s^I(n) \otimes g_t^I(n)\right](1-\varepsilon_t)\cos\theta_t \\ & -\left[s^Q(n) \otimes g_t^Q(n)\right](1+\varepsilon_t)\sin\theta_t, \end{aligned} \tag{2.1}$$

$$\begin{aligned} x^Q(n) \triangleq & -\left[s^I(n) \otimes g_t^I(n)\right](1-\varepsilon_t)\sin\theta_t \\ & +\left[s^Q(n) \otimes g_t^Q(n)\right](1+\varepsilon_t)\cos\theta_t. \end{aligned} \tag{2.2}$$

Writing the complex baseband signal to be transmitted as

$$s(n) \triangleq s^I(n) + \mathbf{j}\, s^Q(n),$$

and the TX IQ imbalance distorted complex baseband signal as

$$x'(n) \triangleq x^I(n) + \mathbf{j}\, x^Q(n),$$

based on (2.1) and (2.2), we have

$$x'(n) = \mu_t(n) \otimes s(n) + \nu_t(n) \otimes s^*(n), \tag{2.3}$$

where

$$\mu_t(n) \triangleq \left(\frac{\alpha_t - \beta_t}{2}\right) g_t^I(n) + \left(\frac{\alpha_t + \beta_t}{2}\right) g_t^Q(n), \tag{2.4}$$

$$\nu_t(n) \triangleq \left(\frac{\alpha_t - \beta_t}{2}\right) g_t^I(n) - \left(\frac{\alpha_t + \beta_t}{2}\right) g_t^Q(n), \tag{2.5}$$

and α_t and β_t are defined as

$$\alpha_t \triangleq \cos\theta_t + \mathbf{j}\,\varepsilon_t \sin\theta_t, \tag{2.6}$$

$$\beta_t \triangleq \varepsilon_t \cos\theta_t + \mathbf{j}\,\sin\theta_t, \tag{2.7}$$

respectively.

If there is no FD IQ imbalance, i.e., assume that $g_t(n) = g_t^I(n) = g_t^Q(n)$, then we have

$$\begin{aligned} \mu_t(n) &= \alpha_t g_t(n) \\ \nu_t(n) &= -\beta_t g_t(n). \end{aligned}$$

Substituting them into (2.3), we have

$$x'(n) = \left[\alpha_t s(n) - \beta_t s^*(n)\right] \otimes g_t(n). \tag{2.8}$$

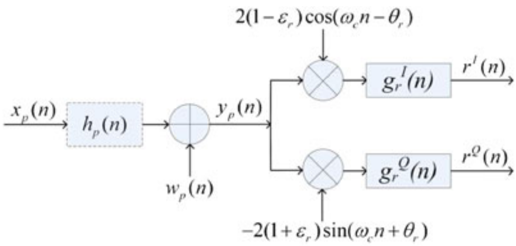


Fig. 2.2 The receiver IQ imbalance signal model

2.2.2 *The Time Domain Receiver-Side IQ Imbalance Model*

Figure 2.2 shows the IQ demodulation with amplitude and phase imbalances. Also, in the system, we assume that both FI IQ imbalance caused by mixer and FD IQ imbalance caused by filters are present.

At the receiver, the received passband signal can be written as

$$y_p(n) = x_p(n) \otimes h_p(n) + w_p(n), \tag{2.9}$$

where $h_p(n)$ is the discrete time passband channel response and $w_p(n)$ is the passband additive noise. If we denote $y^I(n)$ and $y^Q(n)$ as the output of ideal IQ demodulator at the I and Q branches, respectively, then, $y_p(n)$ can be written as

$$y_p(n) = y^I(n)\cos(\omega_c n) - y^Q(n)\sin(\omega_c n), \tag{2.10}$$

and its equivalent complex baseband signal can be written as

$$\begin{aligned} y'(n) &\triangleq y^I(n) + \mathbf{j}\, y^Q(n) \\ &= x'(n) \otimes h'(n) + w'(n), \end{aligned} \tag{2.11}$$

where $h'(n)$ is the equivalent complex baseband channel response, and $w'(n)$ is the equivalent complex noise with two-sided power spectrum density σ^2.

Based on the RX IQ demodulation model in Fig. 2.2, with amplitude mismatch ε_r and phase mismatch θ_r at RX mixer and analog filters with discrete time impulse response $g_r^I(n)$ and $g_r^Q(n)$ at I and Q branches, respectively, we have that the demodulated output at I and Q branches equals to

$$r^I(n) = \text{LPF}\left\{\left[2y_p(n)(1-\varepsilon_r)\cos(\omega_c n - \theta_r)\right] \otimes g_r^I(n)\right\}, \tag{2.12}$$

$$r^Q(n) = \text{LPF}\left\{\left[-2y_p(n)(1+\varepsilon_r)\sin(\omega_c n + \theta_r)\right] \otimes g_r^Q(n)\right\}, \tag{2.13}$$

respectively, where the operation LPF$\{\cdot\}$ removes the frequency content at $2\omega_c$. Substituting (2.10) into (2.12) and (2.13), expanding (2.12) and (2.13), and removing the high frequency signals at $2\omega_c$, we have

$$
\begin{aligned}
r^I(n) &= \left[y^I(n) \otimes g_r^I(n)\right](1-\varepsilon_r)\cos\theta_r \\
&\quad - \left[y^Q(n) \otimes g_r^I(n)\right](1-\varepsilon_r)\sin\theta_r \qquad (2.14)\\
r^Q(n) &= -\left[y^I(n) \otimes g_r^Q(n)\right](1+\varepsilon_r)\sin\theta_r \\
&\quad + \left[y^Q(t) \otimes g_r^Q(n)\right](1+\varepsilon_r)\cos\theta_r. \qquad (2.15)
\end{aligned}
$$

Writing the RX IQ imbalance distorted complex baseband signal as

$$
r(n) \triangleq r^I(n) + \mathbf{j}\, r^Q(n),
$$

we have that $r(n)$ equals to

$$
r(n) = \mu_r(n) \otimes y'(n) + \nu_r(n) \otimes (y'(n))^*, \qquad (2.16)
$$

where $\mu_r(n)$ and $\nu_r(n)$ are

$$
\mu_r(n) \triangleq \left(\frac{\alpha_r - \beta_r^*}{2}\right) g_r^I(n) + \left(\frac{\alpha_r + \beta_r^*}{2}\right) g_r^Q(n), \qquad (2.17)
$$

$$
\nu_r(n) \triangleq \left(\frac{\alpha_r^* - \beta_r}{2}\right) g_r^I(n) - \left(\frac{\alpha_r^* + \beta_r}{2}\right) g_r^Q(n), \qquad (2.18)
$$

and α_r and β_r are defined as

$$
\alpha_r \triangleq \cos\theta_r - \mathbf{j}\,\varepsilon_r \sin\theta_r, \qquad (2.19)
$$

$$
\beta_r \triangleq \varepsilon_r \cos\theta_r + \mathbf{j}\, \sin\theta_r. \qquad (2.20)
$$

If there is no FD IQ imbalance, i.e., assume $g_r(n) = g_r^I(n) = g_r^Q(n)$, we have

$$
\begin{aligned}
\mu_r(n) &= \alpha_r g_r(n) \\
\nu_r(n) &= -\beta_r g_r(n).
\end{aligned}
$$

Substitute them into (2.16) gives

$$
r(n) = \left[\alpha_r y'(n) - \beta_r (y'(n))^*\right] \otimes g_r(n). \qquad (2.21)
$$

2.2.3 *The Time Domain Transmitter and Receiver IQ Imbalance Model*

Now, we assume that in the system, IQ imbalance exists in both transmitter and receiver. Substituting (2.3) into (2.11), we have

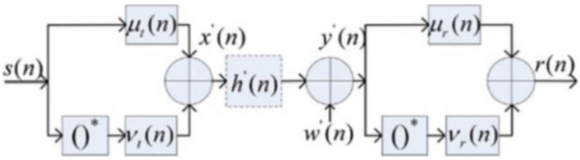


Fig. 2.3 Transmitter and receiver IQ imbalance complex signal model

$$y'(n) = [\mu_t(n) \otimes s(n) + \nu_t(n) \otimes s^*(n)] \otimes h'(n) + w'(n), \tag{2.22}$$

and further substituting (2.22) into (2.16), we have

$$\begin{aligned} r(n) = &\left[\mu_r(n) \otimes \mu_t(n) \otimes h'(n) + \nu_r(t) \otimes \nu_t^*(n) \otimes (h'(n))^*\right] \otimes s(n) \\ &+ \left[\mu_r(n) \otimes \nu_t(n) \otimes h'(n) + \nu_t(t) \otimes \mu_t^*(n) \otimes (h'(n))^*\right] \otimes s^*(n) \\ &+ \left[\mu_r(n) \otimes w'(n) + \nu_r(n) \otimes (w'(n))^*\right]. \end{aligned} \tag{2.23}$$

Figure 2.3 shows the time domain model which includes both transmitter and receiver IQ imbalances.

To simplify the signal model, we can absorb the filtering operations that are common to both I and Q branches in the transmitter and receiver to the channel. This will make the design of the estimation and compensation schemes easier. To do so, we can write $y'(n)$ in (2.22) as

$$\begin{aligned} y'(n) &= [\mu_t(n) \otimes s(n) + \nu_t(n) \otimes s^*(n)] \otimes h'(n) + w'(n) \\ &= [s(n) + \underbrace{\nu_t(n) \otimes \overline{\mu}_t(n)}_{\triangleq \xi_t(n)} \otimes s^*(n)] \otimes \underbrace{\mu_t(n) \otimes h'(n)}_{\triangleq h''(n)} + w'(n), \end{aligned}$$

where $\overline{\mu}_t(n)$ satisfies

$$\overline{\mu}_t(n) \otimes \mu_t(n) = \delta(t)$$

and $\delta(n)$ is the delta function, which equals

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}.$$

If we denote

$$x(n) \triangleq s(n) + \xi_t(n) \otimes s^*(n), \tag{2.24}$$

we can write $y'(n)$ as

$$y'(n) = x(n) \otimes h''(n) + w'(n). \tag{2.25}$$

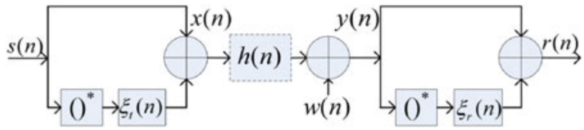


Fig. 2.4 IQ imbalance equivalent complex signal model

If we define

$$y(n) \triangleq \mu_r(n) \otimes y'(n), \tag{2.26}$$

and substitute (2.25) into (2.26), we have

$$\begin{aligned} y(n) &= \mu_r(n) \otimes (h''(n) \otimes x(n) + w'(n)) \\ &= \underbrace{\mu_r(n) \otimes h'(n) \otimes \mu_t(n)}_{\triangleq h(n)} \otimes x(n) + \underbrace{\mu_r(n) \otimes w'(n)}_{\triangleq w(n)} \\ &= h(n) \otimes x(n) + w(n). \end{aligned} \tag{2.27}$$

Then, the received signal $r(n)$ in Eq. (2.16) becomes

$$r(n) = y(n) + \underbrace{\nu_r(n) \otimes \overline{\mu}_r^*(n)}_{\triangleq \xi_r(n)} \otimes y^*(n) \tag{2.28}$$

$$= y(n) + \xi_r(n) \otimes y^*(n), \tag{2.29}$$

where $\overline{\mu}_r^*(n)$ satisfies that

$$\overline{\mu}_r^*(n) \otimes \mu_r^*(n) = \delta(n).$$

Figure 2.4 shows the equivalent complex signal model from the transmitted signal $s(n)$ to the received signal $r(n)$, where $x(n)$ is related to $s(n)$ based on (2.24), $y(n)$ is related to $x(n)$ based on (2.27), and $r(n)$ is related to $y(n)$ based on (2.29).

Substitute (2.24) into (2.27), and then substitute (2.27) into (2.29), we can get the relation between the transmit signal $s(n)$ and the received signal $r(n)$, which is

$$\begin{aligned} r(n) = {} & h(n) \otimes s(n) + h(n) \otimes \xi_t(n) \otimes s^*(n) \\ & + \xi_r(n) \otimes h^*(n) \otimes s^*(n) + \xi_r(n) \otimes h^*(n) \otimes \xi_t^*(n) \otimes s(n) \\ & + w(n) + \xi_r(n) \otimes w^*(n). \end{aligned} \tag{2.30}$$

2.2.4 The Frequency Domain Transmitter and Receiver IQ Imbalance Model

We assume that an OFDM system with DFT/IDFT (Discrete Fourier Transform/Inverse Discrete Fourier Transform) size of N is used, where N is usually a power of 2 and thus an even number. Assume that $\xi_t(n)$, $h(n)$ and $\xi_r(n)$ have length L_t, L_h and L_r, respectively. Assume that the cyclic prefix length $L_{cp} > L_t + L_h + L_r - 2$. After CP removal, define the N sampled value of $x(n)$ and $s(n)$ in one OFDM symbol respectively as

$$\bar{\mathbf{x}} = [x(0), x(1), \cdots, x(N-1)]^T,$$

$$\bar{\mathbf{s}} = [s(0), s(1), \cdots, s(N-1)]^T.$$

Then, based on (2.24), we have

$$\bar{\mathbf{x}} = \bar{\mathbf{s}} + \Lambda_t \bar{\mathbf{s}}^*, \tag{2.31}$$

where, $\bar{\mathbf{s}}^*$ is the component-wise conjugate of $\mathbf{s}$, and Λ_t is an $N \times N$ circulant matrix with the first column equals $\left[\xi_t(0), \xi_t(1), \cdots, \xi_t(L_t - 1), \mathbf{0}_{1\times(N-L_t)}\right]^T$, where $\mathbf{0}_{1\times(N-L_t)}$ is a size $N - L_t$ row vector with all zero elements.

Decompose Λ_t as

$$\Lambda_t = \mathbf{F}^H \Xi_t \mathbf{F}, \tag{2.32}$$

where

$$\Xi_t \triangleq \text{diag}\left\{\xi_{t,0}, \xi_{t,1}, \cdots, \xi_{t,N-1}\right\},$$

$\xi_{t,k} \triangleq \sum_{n=0}^{N-1} \xi_t(n) e^{-j\frac{2\pi nk}{N}}$, and $\mathbf{F}$ is an $N \times N$ DFT matrix.

Substitute (2.32) into (2.31), and multiply both sizes of (2.31) by $\mathbf{F}$, we have

$$\mathbf{x} = \mathbf{s} + \Xi_t \mathbf{s}^{\#}, \tag{2.33}$$

where $\mathbf{x}$, $\mathbf{s}$ and $\mathbf{s}^{\#}$ are defined respectively as

$$\begin{aligned}
\mathbf{x} &= [X_0, X_1, \cdots, X_{N-1}]^T, \\
\mathbf{s} &= [S_0, S_1, \cdots, S_{N-1}]^T, \\
\mathbf{s}^{\#} &= [S_0^*, S_{N-1}^*, S_{N-2}^*, \cdots, S_1^*]^T,
\end{aligned}$$

and X_k and S_k are defined respectively as $X_k = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi nk}{N}}$ and $S_k = \sum_{n=0}^{N-1} s(n)e^{-j\frac{2\pi nk}{N}}$. Expanding (2.33) gives us

$$X_k = S_k + \xi_{t,k} S_{N-k}^*,$$
$$X_{N-k} = S_{N-k} + \xi_{t,N-k} S_k^*,$$

for $k = 1, 2, \cdots, N-1$ or in matrix format

$$\begin{bmatrix} X_k \\ X_{N-k}^* \end{bmatrix} = \begin{bmatrix} 1 & \xi_{t,k} \\ \xi_{t,N-k}^* & 1 \end{bmatrix} \begin{bmatrix} S_k \\ S_{N-k}^* \end{bmatrix}. \tag{2.34}$$

Similarly, based on (2.27), after removing CP, and multiplying size $N \times N$ DFT matrix $\mathbf{F}$ on both sides of (2.27), we have

$$\begin{bmatrix} Y_k \\ Y_{N-k}^* \end{bmatrix} = \begin{bmatrix} H_k & 0 \\ 0 & H_{N-k}^* \end{bmatrix} \begin{bmatrix} X_k \\ X_{N-k}^* \end{bmatrix} + \begin{bmatrix} W_k \\ W_{N-k}^* \end{bmatrix}, \tag{2.35}$$

where $Y_k = \sum_{n=0}^{N-1} y(n)e^{-j\frac{2\pi nk}{N}}$, $H_k = \sum_{n=0}^{L_h-1} h(n)e^{-j\frac{2\pi nk}{N}}$, and $W_k = \sum_{n=0}^{N-1} w(n)e^{-j\frac{2\pi nk}{N}}$. Based on (2.27), W_k can be further written as

$$W_k = \mu_{r,k} W_k', \tag{2.36}$$

where $\mu_{r,k}$ and W_k equal to $\mu_{r,k} = \sum_{n=0}^{N-1} \mu_r(n)e^{-j\frac{2\pi nk}{N}}$ and $W_k' = \sum_{n=0}^{N-1} w'(n)e^{-j\frac{2\pi kn}{N}}$, respectively.

Also, following the same procedure, based on (2.29), we have

$$\begin{bmatrix} R_k \\ R_{N-k}^* \end{bmatrix} = \begin{bmatrix} 1 & \xi_{r,k} \\ \xi_{r,N-k}^* & 1 \end{bmatrix} \begin{bmatrix} Y_k \\ Y_{N-k}^* \end{bmatrix} \tag{2.37}$$

$$= \mathbf{H}_k \begin{bmatrix} S_k \\ S_{N-k}^* \end{bmatrix} + \mathbf{v}_k, \tag{2.38}$$

where $R_k = \sum_{n=0}^{N-1} r(n)e^{-j\frac{2\pi kn}{N}}$ and $\xi_{r,k} = \sum_{n=0}^{L_r-1} \xi_r(n)e^{-j\frac{2\pi kn}{N}}$. $\mathbf{H}_k$ is the equivalent channel matrix that combines channel response, transmitter and receive IQ imbalances for subcarrier pair k and $N-k$, and can be written as

$$\begin{aligned} \mathbf{H}_k &= \begin{bmatrix} 1 & \xi_{r,k} \\ \xi_{r,N-k}^* & 1 \end{bmatrix} \begin{bmatrix} H_k & 0 \\ 0 & H_{N-k}^* \end{bmatrix} \begin{bmatrix} 1 & \xi_{t,k} \\ \xi_{t,N-k}^* & 1 \end{bmatrix} \\ &= \begin{bmatrix} H_k + \xi_{r,k}\xi_{t,N-k}^* H_{N-k}^* & \xi_{t,k} H_k + \xi_{r,k} H_{N-k}^* \\ \xi_{r,N-k}^* H_k + \xi_{t,N-k}^* H_{N-k}^* & \xi_{r,N-k}^* \xi_{t,k} H_k + H_{N-k}^* \end{bmatrix}. \end{aligned} \tag{2.39}$$

$\mathbf{v}_k$ is the colored noise defined as

$$\mathbf{v}_k = \begin{bmatrix} V_k \\ V_{N-k}^* \end{bmatrix} = \begin{bmatrix} 1 & \xi_{r,k} \\ \xi_{r,N-k}^* & 1 \end{bmatrix} \begin{bmatrix} W_k \\ W_{N-k}^* \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \xi_{r,k} \\ \xi_{r,N-k}^* & 1 \end{bmatrix} \begin{bmatrix} \mu_{r,N-k} & 0 \\ 0 & \mu_{r,k}^* \end{bmatrix} \begin{bmatrix} W_k' \\ (W_{N-k}')^* \end{bmatrix}$$
$$= \begin{bmatrix} \mu_{r,N-k} & \nu_{r,k} \\ \nu_{r,N-k}^* & \mu_{r,k}^* \end{bmatrix} \begin{bmatrix} W_k' \\ (W_{N-k}')^* \end{bmatrix}, \tag{2.40}$$

where $\nu_{r,k} = \sum_{n=0}^{N-1} \nu_r(n) e^{-j\frac{2\pi kn}{N}}$. In (2.40), we used the definition $\xi_r(n) \triangleq \nu_r(n) \otimes \overline{\mu}_r^*(n)$ and the fact that $\xi_{r,k} = \nu_{r,k}/\mu_{r,k}^*$. From (2.40) we can see that the covariance of the noise depends on the individual values of $\nu_{r,k}$ and $\mu_{r,k}$, not just their ratio.

2.3 The IQ Imbalance Model with Frequency Offset

In practical wireless communication system, the frequency of local oscillator at the transmitter and receiver may not be the same, this introduces frequency offset. In presence of frequency offset, the IQ imbalance model is different from that derived in Sect. 2.2. There are also many literatures that discussed IQ imbalance with frequency offset, see for example [15–22] and the references therein. In this section, we first look at the IQ imbalance model in presence of frequency offset in the time domain, and then look at it in the frequency domain.

Assume that the carrier frequency of the receiver is offset by Δf with respect to the carrier frequency of the transmitter, let us denote $\Delta\omega \triangleq 2\pi \Delta f T$, where T is the sample period. Then, at the receiver, the frequency offset causes the baseband equivalent signal before the IQ demodulation rotated by an angle $e^{j\Delta\omega n}$, i.e., instead of $y'(n)$ defined in (2.11) is received, $e^{j\Delta\omega n} y'(n)$ is received. After IQ demodulation, based on (2.16), the baseband equivalent IQ imbalance distorted signal $r(n) = r^I(n) + \mathbf{j}\, r^Q(n)$ equals

$$r(n) = \mu_r(n) \otimes \left[e^{j\Delta\omega n} y'(n)\right] + \nu_r(n) \otimes \left[e^{j\Delta\omega n} y'(n)\right]^*$$
$$= e^{j\Delta\omega n} \left[\left(e^{-j\Delta\omega n} \mu_r(n)\right) \otimes y'(n)\right]$$
$$+ e^{-j\Delta\omega n} \left[\left(e^{j\Delta\omega n} \nu_r(n)\right) \otimes (y'(n))^*\right], \tag{2.41}$$

where $\mu_r(n)$ and $\nu_r(n)$ are defined as Eqs. (2.19) and (2.20), respectively.

From (2.41) we can see that when RX IQ exists, we cannot multiply $e^{-j\Delta\omega n}$ to $r(n)$ to compensate the frequency offset, since there are $e^{-j\Delta\omega n}$ component in $r(n)$, which cannot be removed by multiplying $e^{-j\Delta\omega n}$. So, at the receiver, before compensating frequency offset, the RX IQ imbalance must be compensated.

When both transmitter and receiver IQ imbalances exist, the baseband equivalent received signal before IQ demodulation equals (2.22), which is

$$y'(n) = [\mu_t(n) \otimes s(n) + \nu_t(n) \otimes s^*(n)] \otimes h'(n) + w'(n). \tag{2.42}$$

Substituting it into (2.41) gives

$$\begin{aligned} r(n) = e^{j\Delta\omega n}&\{\mu_t(n) \otimes h'(n) \otimes [e^{-j\Delta\omega n}\mu_r(n)] \otimes s(n)\} \\ &+ e^{j\Delta\omega n}\{\nu_t(n) \otimes h'(n) \otimes [e^{-j\Delta\omega n}\mu_r(n)] \otimes s^*(n)\} \\ &+ e^{-j\Delta\omega n}\{\mu_t^*(n) \otimes (h'(n))^* \otimes [e^{j\Delta\omega n}\nu_r(n)] \otimes s^*(n)\} \\ &+ e^{-j\Delta\omega n}\{\nu_t^*(n) \otimes (h'(n))^* \otimes [e^{j\Delta\omega n}\nu_r(n)] \otimes s(n)\} \\ &+ q(n), \end{aligned} \tag{2.43}$$

where $q(n)$ is the transformed noise, which equals

$$q(n) = \mu_r(n) \otimes \left[e^{j\Delta\omega n} w'(n)\right] + \nu_r(n) \otimes \left[e^{j\Delta\omega n} w'(n)\right]^*. \tag{2.44}$$

From (2.43) we can see how the frequency offset is added to the received signal. If there is no frequency offset, i.e., $\Delta\omega = 0$, then (2.43) is equivalent to (2.23). If there is no transmitter and receiver IQ imbalance, we have $\mu_t(n) = \mu_r(n) = \delta(n)$ and $\nu_t(n) = \nu_r(n) = 0$. Substituting them into (2.43), we have $r(n) = e^{j\Delta\omega n}[h'(n) \otimes s(n) + w'(n)]$, which is a standard received signal model with frequency offset.

Also, to simplify the notations, we can absorb the filtering operations that are common to the I and Q branches to the channel. In this case, the received signal before IQ demodulator can still be represented as

$$y'(n) = x(n) \otimes h''(n) + w'(n), \tag{2.45}$$

where

$$\begin{aligned} x(n) &= s(n) + \xi_t(n) \otimes s^*(n) \\ h''(n) &= \mu_t(n) \otimes h'(n) \end{aligned} \tag{2.46}$$

the same as that in Sect. 2.2.3. If we define

$$y(n) = \mu_r(n) \otimes \left[e^{j\Delta\omega n} y'(n)\right], \tag{2.47}$$

then $r(n)$ in (2.41) can be represented as

$$r(n) = y(n) + \xi_r(n) \otimes y^*(n), \tag{2.48}$$

which is the same as Eq. (2.29) in Sect. 2.2.3.

Compared with the IQ model without frequency offset, the difference here is in $y(n)$, which is different from that in Eq. (2.26) in Sect. 2.2.3. Substituting $y'(n) = x(n) \otimes h''(n) + w'(n)$ into the new $y(n)$ in (2.47), we have

$$\begin{aligned} y(n) &= e^{j\Delta\omega n}\left[\left(e^{-j\Delta\omega n}\mu_r(n)\right) \otimes y'(n)\right] \\ &= e^{j\Delta\omega n}\left\{x(n) \otimes \underbrace{\mu_t(n) \otimes h'(n) \otimes [e^{-j\Delta\omega n}\mu_r(n)]}_{\triangleq h(n)} + \underbrace{w'(n) \otimes [e^{-j\Delta\omega n}\mu_r(n)]}_{\triangleq w(n)}\right\} \\ &= e^{j\Delta\omega n}\{x(n) \otimes h(n) + w(n)\}. \end{aligned} \tag{2.49}$$

Compared with the $y(n)$ defined in (2.26), we can find that the frequency offset affects the equivalent channel $h(n)$ as well as the equivalent noise $w(n)$.

Now, we consider the IQ imbalance in the frequency domain with frequency offset. We assume that the channel impulse response keeps constant during one OFDM symbol period, and look at the impact of frequency offset on the IQ imbalance model. We assume that size N FFT/IFFT is used and the CP is greater than the length of the equivalent channel. After removing CP, we write the $y(n)$ in one OFDM symbol as

$$\overline{\mathbf{y}} = [y(0), y(1), \cdots, y(N-1)]^T$$

and it can be represented as

$$\overline{\mathbf{y}} = \overline{\Lambda}\overline{\mathbf{H}}\overline{\mathbf{x}} + \overline{\mathbf{w}}, \tag{2.50}$$

where

$$\overline{\Lambda} \triangleq \mathrm{diag}\left\{1, e^{j\Delta\omega}, \cdots, e^{j(N-1)\Delta\omega}\right\},$$

$\overline{\mathbf{H}}$ is a circulant matrix with the first column equals $[h(0), h(1), \cdots, h(N-1)]^T$, $\overline{\mathbf{x}}$ and $\overline{\mathbf{w}}$ are defined respectively as

$$\overline{\mathbf{x}} = [x(0), x(1), \cdots, x(N-1)]^T \tag{2.51}$$

$$\overline{\mathbf{w}} = [w(0), w(1), \cdots, w(N-1)]^T. \tag{2.52}$$

The matrix $\overline{\mathbf{H}}$ can be decomposed as $\mathbf{F}^H\mathbf{H}\mathbf{F}$, where $\mathbf{F}$ is a size $N \times N$ DFT matrix,

$$\mathbf{H} = \mathrm{diag}\left\{H_0, H_1, \cdots, H_{N-1}\right\}, \tag{2.53}$$

and $H_k = \sum_{n=0}^{N-1} h(n)e^{-j\frac{2\pi kn}{N}}$. Substituting it into (2.50) and multiplying both sides of (2.50) by $\mathbf{F}$, we have

$$\mathbf{y} = \mathbf{F}\overline{\mathbf{y}} = \Lambda \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{2.54}$$

$$= [Y_0, Y_1, \cdots, Y_{N-1}]^T, \tag{2.55}$$

where Λ is a circulant matrix which equals

$$\Lambda = \mathbf{F}\overline{\Lambda}\mathbf{F}^H \tag{2.56}$$

and $\mathbf{x}$ and $\mathbf{w}$ are equal to

$$\mathbf{x} = \mathbf{F}\overline{\mathbf{x}} = [X_0, X_1, \cdots, X_{N-1}]^T \tag{2.57}$$

$$\mathbf{w} = \mathbf{F}\overline{\mathbf{w}} = [W_0, W_1, \cdots, W_{N-1}]^T. \tag{2.58}$$

In (2.54), $\mathbf{x}$ still equals that in (2.33), however, due to the circulant matrix Λ, vector $\mathbf{y}$ cannot be decomposed as that in (2.35). By multiplying the circulant matrix Λ caused by the frequency offset, ICI is introduced.

2.4 The IQ Imbalance Model with Phase Noise

Ideally the carrier frequency is a pure sine wave with frequency f_c or with normalized digital frequency $\omega_c = 2\pi f_c T$, where T is the sample period. In reality, the carrier frequency may suffer from phase noise, i.e., the generated carrier frequency equals $\sin(\omega_c n + \phi_n)$ where ϕ_n is a random process. For the discussion of IQ imbalance when there is phase noise, please see [23, 24].

In free running oscillators, it has been found that the phase error ϕ_n becomes asymptotically a Brownian motion (Wiener-Levy process) as the time index $n \rightarrow \infty$. The phase noise process is given by $\phi_n = \phi_{n-1} + \varphi_n$, where the phase noise innovations φ_n are modelled as i.i.d. Gaussian random variables with 0 mean and variance σ_φ^2. Assuming perfect synchronization at the beginning of each frame, $\varphi_0 = 0$. The variance of the phase noise process increases linearly with time index n, making it non-stationary. However, the phase noise process itself is stationary and displays the so called Lorenzian spectrum. The variance of the phase noise innovations is given by

$$\sigma_\varphi^2 = 2\pi\beta T_s/N = 2\pi\beta/R \tag{2.59}$$

where T_s is the duration of an OFDM symbol, N is the number of sub-carriers, R is the symbol rate as $R \triangleq \frac{N}{T_s}$, and β is the 3 dB bandwidth of the Lorenzian spectrum for the phase noise.

Compared with the case with frequency offset, we can see that in case of phase noise, the received signal now is equal to $e^{j\phi_n}y'(n)$, where $y'(n)$ is the received baseband equivalent signal before IQ demodulation. As shown in Fig. 2.3, $y'(n)$ equals

$$
\begin{aligned}
y'(n) &= x'(n) \otimes h'(n) + w'(n) \\
&= x(n) \otimes \mu_t(n) \otimes h'(n) + w'(n),
\end{aligned} \tag{2.60}
$$

where $x(n) = s(n) + \xi_t(n) \otimes s^*(n)$.

Using approaches similar as deriving the model for the case with frequency offset, we have that after RX IQ imbalance, the received distorted signal equals

$$
\begin{aligned}
r(t) &= \mu_r(n) \otimes \left[e^{j\varphi_n} y'(n)\right] + \nu_r(n) \otimes \left[e^{j\varphi_n} y'(n)\right]^* \\
&= y(n) + \xi_r(n) \otimes y^*(n),
\end{aligned} \tag{2.61}
$$

where $y(n)$ equals to

$$
\begin{aligned}
y(n) &= \mu_r(n) \otimes \left[e^{j\varphi_n} y'(n)\right] \\
&= e^{j\varphi_n} \left[\left(e^{-j\varphi_n} \mu_r(n)\right) \otimes y'(n)\right].
\end{aligned} \tag{2.62}
$$

Substituting (2.60) into (2.62) gives

$$
\begin{aligned}
y(n) &= e^{j\varphi_n} \left\{ x(n) \otimes \underbrace{\mu_t(n) \otimes h'(n) \otimes [e^{-j\varphi_n} \mu_r(n)]}_{\triangleq h(n)} + \underbrace{w'(n) \otimes [e^{-j\varphi_n} \mu_r(n)]}_{\triangleq w(n)} \right\} \\
&= e^{j\varphi_n} \{x(n) \otimes h(n) + w(n)\}.
\end{aligned} \tag{2.63}
$$

In frequency domain, the received signal can be written similar as Eq. (2.54), which is

$$
\mathbf{y} = \Lambda \mathbf{H} \mathbf{x} + \mathbf{w}, \tag{2.64}
$$

where $\mathbf{H}$, $\mathbf{x}$ and $\mathbf{w}$ are defined the same as (2.53), (2.57) and (2.58), respectively. The difference is the definition of Λ, which now is equal to

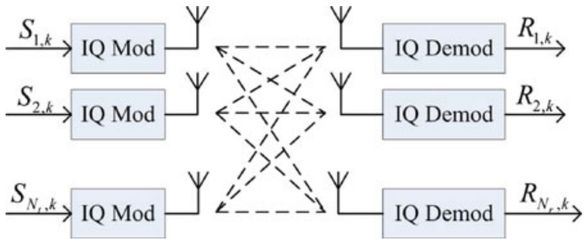
$$
\Lambda = \mathbf{F} \mathrm{diag} \left\{ e^{j\varphi_0}, e^{j\varphi_1}, \cdots, e^{j\varphi_{N-1}} \right\} \mathbf{F}^H. \tag{2.65}
$$

2.5 The IQ Imbalance Model with Spatial Multiplexing

Spatial multiplexing or BLAST (Bell-Labs Layered Space-Time) [32] is a way to achieve high capacity in rich scattering wireless channels [32, 33]. Figure 2.5 shows the spatial multiplexing with N_t transmit and N_r receive antennas. It is also called multiple-input multiple-output (MIMO) system. For the IQ imbalance with MIMO, see for example [29, 34–36] and the references therein.

For clarity while without loss of generality, as in previous sections, we assume that an OFDM system with size N FFT/IFFT is used. We consider the transmit and

Fig. 2.5 The IQ imbalance model with MIMO spatial multiplexing



receive symbols at the k-th subcarrier. If there are no TX and RX IQ imbalances, the transmit and receive signals equal to

$$\mathbf{s}_k = [S_{1,k}, S_{2,k}, \cdots, S_{N_t,k}]^T,$$
$$\mathbf{y}_k = [Y_{1,k}, Y_{2,k}, \cdots, Y_{N_r,k}]^T.$$

They have the relation

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{s}_k + \mathbf{w}_k, \tag{2.66}$$

where $\mathbf{H}_k$ is a $N_r \times N_t$ matrix, whose i-th row and j-th column $H_{i,j,k}$ is the channel coefficient between the i-th receive and j-th transmit antennas at subcarrier k. $\mathbf{w}_k$ is the $N_r \times 1$ noise vector.

If TX IQ imbalance exists, subcarrier k and $(N-k)$ interfere with each other according to Eq. (2.34). Denote the TX IQ imbalance corrupted signal as

$$\mathbf{x}_k = [X_{1,k}, X_{2,k}, \cdots, X_{N_t,k}]^T.$$

Then, after TX IQ imbalance, the signal is

$$\begin{bmatrix} \mathbf{x}_k \\ \mathbf{x}_{N-k}^* \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{N_t} & \Xi_{t,k} \\ \Xi_{t,N-k}^* & \mathbf{I}_{N_t} \end{bmatrix} \begin{bmatrix} \mathbf{s}_k \\ \mathbf{s}_{N-k}^* \end{bmatrix}, \tag{2.67}$$

where $\Xi_{t,k}$ equals

$$\Xi_{t,k} = \mathrm{diag}\{\xi_{t,1,k}, \xi_{t,2,k}, \cdots, \xi_{t,N_t,k}\},$$

and $\xi_{t,n,k}$ means the IQ imbalance at the k-subcarrier of the n-th TX antenna branch.

Correspondingly, the received signal equals

$$\begin{bmatrix} \mathbf{y}_k \\ \mathbf{y}_{N-k}^* \end{bmatrix} = \begin{bmatrix} \mathbf{H}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{N-k}^* \end{bmatrix} \begin{bmatrix} \mathbf{I}_{N_t} & \Xi_{t,k} \\ \Xi_{t,N-k}^* & \mathbf{I}_{N_t} \end{bmatrix} \begin{bmatrix} \mathbf{s}_k \\ \mathbf{s}_{N-k}^* \end{bmatrix} + \begin{bmatrix} \mathbf{w}_k \\ \mathbf{w}_{N-k}^* \end{bmatrix}. \tag{2.68}$$

If RX IQ imbalance exists, the received corrupted signal equals

$$\mathbf{r}_k = [R_{1,k}, R_{2,k}, \cdots, R_{N_r,k}]^T.$$

Then, we have that

$$\begin{aligned}\begin{bmatrix}\mathbf{r}_k \\ \mathbf{r}^*_{N-k}\end{bmatrix} &= \begin{bmatrix}\mathbf{I}_{N_r} & \Xi_{r,k} \\ \Xi^*_{r,N-k} & \mathbf{I}_{N_r}\end{bmatrix}\begin{bmatrix}\mathbf{y}_k \\ \mathbf{y}^*_{N-k}\end{bmatrix} \\ &= \hat{\mathbf{H}}_{k,N-k}\begin{bmatrix}\mathbf{s}_k \\ \mathbf{s}^*_{N-k}\end{bmatrix} + \mathbf{v}_{k,N-k},\end{aligned} \tag{2.69}$$

where $\Xi_{r,k}$ equals

$$\Xi_{r,k} = \mathrm{diag}\{\xi_{r,1,k}, \xi_{r,2,k}, \cdots, \xi_{r,N_r,k}\},$$

and $\xi_{r,m,k}$ means the IQ imbalance at the k-subcarrier of the m-th RX antenna branch. $\hat{\mathbf{H}}_{k,N-k}$ equals

$$\hat{\mathbf{H}}_{k,N-k} = \begin{bmatrix}\mathbf{H}_k + \Xi_{r,k}\mathbf{H}^*_{N-k}\Xi^*_{t,N-k} & \mathbf{H}_k\Xi_{t,k} + \Xi_{r,k}\mathbf{H}^*_{N-k} \\ \Xi^*_{r,N-k}\mathbf{H}_k + \mathbf{H}^*_{N-k}\Xi^*_{t,N-k} & \Xi^*_{r,N-k}\mathbf{H}_k\Xi_{t,k} + \mathbf{H}^*_{N-k}\end{bmatrix}, \tag{2.70}$$

and $\mathbf{v}_{k,N-k}$ equals

$$\mathbf{v}_{k,N-k} = \begin{bmatrix}\mathbf{I}_{N_r} & \Xi_{r,k} \\ \Xi^*_{r,N-k} & \mathbf{I}_{N_r}\end{bmatrix}\begin{bmatrix}\mathbf{w}_k \\ \mathbf{w}^*_{N-k}\end{bmatrix}. \tag{2.71}$$

From above IQ imbalance model for spatial multiplexing systems, we can see that due to the interference between the k-th and $(N-k)$-th subcarriers, original $N_r \times N_t$ MIMO system is changed to a $2N_r \times 2N_t$ system.

2.6 The IQ Imbalance Model with Space-Time/Space-Frequency Code

Space-Time Code is a method to collect transmit spatial diversity when there are multiple antennas in transmitter [25–28]. The best known space-time code is the Alamouti's scheme [25], which works for two transmit antennas. Figure 2.6 shows

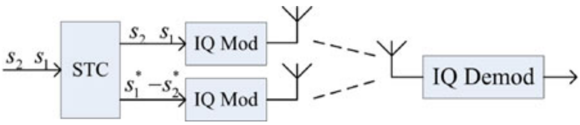


Fig. 2.6 The IQ imbalance model with space-time code/space-frequency code

the Alamouti's scheme. The impact of IQ imbalance on the space-time code is discussed in [29–31]. In this section, we follow the steps used in [30] to derive the IQ imbalance model when Alamouti's scheme is used. We assume that there are two transmit antennas and one receive antenna. For clarity, we first assume that the IQ imbalance is frequency independent, and the channel is one tap flat fading channel, then we extend the model to the frequency dependent IQ imbalance case with space-frequency code using OFDM.

For space-time code, it is assumed that the channel is constant in consecutive M time domain symbols, where $M > N_t$ and N_t is the number of transmit antennas. As described in Fig. 2.6, assume that the symbols in two consecutive time slots, i.e., s_1 and s_2, are encoded by the Alamouti's space-time code. First assume that there is no IQ imbalance at both transmitter and receiver, then after encoding, the symbols transmitted by the two transmit antennas equal to

$$\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}, \tag{2.72}$$

where row is the spatial domain and column is the time domain. Assuming that the channel coefficients are the same for the two time slots but are different for the two transmit antennas, denote them as h_1 and h_2 for the first and second transmit antennas, respectively. At the receiver, the received signals in two time slots equal to

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{2.73}$$

To do decoding, the decoupled received signal for detecting s_1 and s_2 equal to

$$\hat{y}_1 \triangleq h_1^* y_1 + h_2 y_2^* = (|h_1|^2 + |h_2|^2)\, s_1 + \hat{w}_1 \tag{2.74}$$

$$\hat{y}_2 \triangleq h_2^* y_1 - h_1 y_2^* = (|h_1|^2 + |h_2|^2)\, s_2 + \hat{w}_2, \tag{2.75}$$

where $\hat{w}_1$ and $\hat{w}_2$ are equivalent transformed noise.

If TX IQ imbalance exists, the transmitted space-time code does not equal to (2.72), but equals to the following

$$\begin{bmatrix} s_1 + \xi_{t,1} s_1^* & s_2 + \xi_{t,2} s_2^* \\ -s_2^* - \xi_{t,1} s_2 & s_1^* + \xi_{t,2} s_1 \end{bmatrix}, \tag{2.76}$$

where $\xi_{t,1}$ and $\xi_{t,2}$ are TX IQ imbalances for the first and second transmit antennas, respectively.[1]

At the receiver, if there is no IQ imbalance, the received signals now are equal to

[1]With a slight abuse of notations, here we use $\xi_{t,1}$ to denote the IQ imbalance for the first TX antennas, instead of the IQ imbalance at the first subcarrier, as is used in previous sections.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_1 + \xi_{t,1}s_1^* & s_2 + \xi_{t,2}s_2^* \\ -s_2^* - \xi_{t,1}s_2 & s_1^* + \xi_{t,2}s_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{2.77}$$

If IQ imbalance exists at the receiver, assume that it is ξ_r, then the received signals at the first and second times slots are

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} y_1 + \xi_r y_1^* \\ y_2 + \xi_r y_2^* \end{bmatrix}. \tag{2.78}$$

Substituting (2.77) into (2.78) gives us

$$\begin{aligned} r_1 &= as_1 + bs_2 + cs_1^* + ds_2^* + v_1, \\ r_2 &= ds_1 - cs_2 + bs_1^* - as_2^* + v_2, \end{aligned} \tag{2.79}$$

where a,b,c, and d are defined as

$$a = h_1 + \xi_r \xi_{t,1}^* h_1^*, \tag{2.80}$$

$$b = h_2 + \xi_r \xi_{t,2}^* h_2^*, \tag{2.81}$$

$$c = \xi_{t,1} h_1 + \xi_r h_1^*, \tag{2.82}$$

$$d = \xi_{t,2} h_2 + \xi_r h_2^*, \tag{2.83}$$

and v_1 are v_2 are noise terms, which equals

$$v_1 = w_1 + \xi_r w_1^*, \tag{2.84}$$

$$v_2 = w_2 + \xi_r w_2^*. \tag{2.85}$$

If OFDM is used, the frequency domain Alamouti's scheme encodes symbols over two contiguous subcarriers, instead of two contiguous time slots, and it is assumed that the channel coefficients are the same for these two contiguous subcarriers. The frequency domain Alamouti's scheme is also called space-frequency code. In this case, the encoded matrix equals

$$\begin{bmatrix} S_k & S_{k+1} \\ -S_{k+1}^* & S_k^* \end{bmatrix}. \tag{2.86}$$

IF TX IQ imbalance exists, we need to consider the space-frequency encoding at subcarrier $N-k$ and $N-k-1$, which equals

$$\begin{bmatrix} S_{N-k} & S_{N-k-1} \\ -S_{N-k-1}^* & S_{N-k}^* \end{bmatrix}. \tag{2.87}$$

According to the TX IQ imbalance interference model (2.34), assuming that IQ imbalances in consecutive subcarriers are equal, and the IQ imbalances for different transmit antennas are different, i.e. $\xi_{t,1,k} = \xi_{t,1,k+1}$ and $\xi_{t,2,k} = \xi_{t,2,k+1}$, but $\xi_{t,1,k} \neq \xi_{t,2,k}$, then the actual transmitted signal from the two transmit antennas in subcarrier k,$k+1$,$N-k$, and $N-k-1$ equal to

$$\begin{bmatrix} X_{1,k} \\ X_{1,k+1} \\ X_{1,N-k} \\ X_{1,N-k-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \xi_{t,1,k} & 0 \\ 0 & 1 & 0 & \xi_{t,1,k} \\ \xi^*_{t,1,N-k} & 0 & 1 & 0 \\ 0 & \xi^*_{t,1,N-k} & 0 & 1 \end{bmatrix} \begin{bmatrix} S_k \\ -S^*_{k+1} \\ S^*_{N-k} \\ -S_{N-k-1} \end{bmatrix} \tag{2.88}$$

$$\begin{bmatrix} X_{2,k} \\ X_{2,k+1} \\ X_{2,N-k} \\ X_{2,N-k-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \xi_{t,2,k} & 0 \\ 0 & 1 & 0 & \xi_{t,2,k} \\ \xi^*_{t,2,N-k} & 0 & 1 & 0 \\ 0 & \xi^*_{t,2,N-k} & 0 & 1 \end{bmatrix} \begin{bmatrix} S_{k+1} \\ S^*_k \\ S^*_{N-k-1} \\ S_{N-k} \end{bmatrix}. \tag{2.89}$$

At the receiver, the received signals in the four subcarriers are

$$\begin{bmatrix} Y_k \\ Y_{k+1} \end{bmatrix} = \begin{bmatrix} X_{1,k} & X_{2,k} \\ X_{1,k+1} & X_{2,k+1} \end{bmatrix} \begin{bmatrix} H_{1,k} \\ H_{2,k} \end{bmatrix} + \begin{bmatrix} W_k \\ W_{k+1} \end{bmatrix} \tag{2.90}$$

$$\begin{bmatrix} Y_{N-k} \\ Y_{N-k-1} \end{bmatrix} = \begin{bmatrix} X_{1,N-k} & X_{2,N-k} \\ X_{1,N-k-1} & X_{2,N-k-1} \end{bmatrix} \begin{bmatrix} H_{1,N-k} \\ H_{2,N-k} \end{bmatrix} + \begin{bmatrix} W_{N-k} \\ W_{N-k-1} \end{bmatrix}. \tag{2.91}$$

When there is IQ imbalances at the receiver, we also assume that $\xi_{r,k} = \xi_{r,k+1}$ and $\xi_{r,N-k} = \xi_{r,N-k-1}$, then, based on (2.37), the received signal corrupted by the RX IQ imbalance equals

$$\begin{bmatrix} R_k \\ R_{k+1} \\ R^*_{N-k} \\ R^*_{N-k-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \xi_{r,k} & 0 \\ 0 & 1 & 0 & \xi_{r,k} \\ \xi^*_{r,N-k} & 0 & 1 & 0 \\ 0 & \xi^*_{r,N-k} & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_k \\ Y_{k+1} \\ Y^*_{N-k} \\ Y^*_{N-k-1} \end{bmatrix}. \tag{2.92}$$

Reorganizing these equations, and defining

$$\mathbf{r}_k \triangleq [R_k, R_{k+1}, R^*_{N-k}, R^*_{N-k-1}]^T,$$

$$\mathbf{s}_k \triangleq [S_k, S_{k+1}, S^*_{N-k}, S^*_{N-k-1}]^T,$$

$$\mathbf{v}_k \triangleq [V_k, V_{k+1}, V^*_{N-k}, V^*_{N-k-1}]^T,$$

we have

$$\mathbf{r}_k = \mathbf{A}_k \mathbf{s}_k + \mathbf{B}_k \mathbf{s}^*_k + \mathbf{v}_k, \tag{2.93}$$

where $\mathbf{A}_k$ and $\mathbf{B}_k$ are defined as

$$\mathbf{A}_k = \begin{bmatrix} H_{1,k} & H_{1,k}\xi_{t,1,k} & H_{2,k} & H_{2,k}\xi_{t,2,k} \\ \xi_{r,k}^* H_{1,k} & \xi_{r,N-k}^* H_{1,k}\xi_{t,1,k} & \xi_{r,N-k}^* H_{2,k} & \xi_{r,N-k}^* H_{2,k}\xi_{t,2,k} \\ \xi_{r,k} H_{2,N-k}^* \xi_{t,2,N-k} & \xi_{r,k} H_{2,N-k}^* & -\xi_{r,k} H_{1,N-k}^* \xi_{t,1,N-k} & -\xi_{r,k} H_{1,N-k}^* \\ H_{2,N-k}^* \xi_{t,2,N-k} & H_{2,N-k}^* & -H_{1,N-k}^* \xi_{t,1,N-k} & -H_{1,N-k}^* \end{bmatrix}$$

$$\mathbf{B}_k = \begin{bmatrix} \xi_{r,k} H_{1,N-k}^* \xi_{t,1,N-k} & \xi_{r,k} H_{1,N-k}^* & \xi_{r,k} H_{2,N-k}^* \xi_{t,2,N-k} & \xi_{r,k} H_{2,N-k}^* \\ H_{1,N-k}^* \xi_{t,1,N-k} & H_{1,N-k}^* & H_{2,N-k}^* \xi_{t,2,N-k} & H_{2,N-k}^* \\ H_{2,k} & H_{2,k}\xi_{t,2,k} & -H_{1,k} & -H_{1,k}\xi_{t,1,k} \\ \xi_{r,N-k}^* H_{2,k} & \xi_{r,N-k}^* H_{2,k}\xi_{t,2,k} & -\xi_{r,N-k}^* H_{1,k} & -\xi_{r,N-k}^* H_{1,k}\xi_{t,1,k} \end{bmatrix},$$

and $\mathbf{v}_k$ is the colored noise, which equals

$$\mathbf{v}_k = \begin{bmatrix} 1 & 0 & \xi_{r,k} & 0 \\ 0 & 1 & 0 & \xi_{r,k} \\ \xi_{r,N-k}^* & 0 & 1 & 0 \\ 0 & \xi_{r,N-k}^* & 0 & 1 \end{bmatrix} \begin{bmatrix} W_k \\ W_{k+1} \\ W_{N-k}^* \\ W_{N-k-1}^* \end{bmatrix}. \tag{2.94}$$

If there is no TX and RX IQ imbalances, i.e., $\xi_{t,1,k} = \xi_{t,2,k} = \xi_{t,1,N-k} = \xi_{t,2,N-k} = 0$ and $\xi_{r,k} = \xi_{r,N-k} = 0$, then we have decoupled $[R_k, R_{k+1}]$ and $[R_{N-k}, R_{N-k-1}]$. By further processing, we can get decoupled S_k, S_{k+1}, S_{N-k}, and S_{N-k-1}. However, when TX and RX IQ imbalances exist, we lose these good properties.

2.7 The Impact of IQ Imbalance Model on Performance

In this section, we discuss the impact of IQ imbalance on the system performance. We mainly look at the signal-to-interference ratio (SIR) degradation caused by IQ imbalance.

From Eq. (2.34), we can see that when there is IQ imbalance, the actual transmitted signal at subcarrier k, i.e., X_k, not only includes the original signal S_k, but also includes the interference from its image subcarrier, i.e., S_{N-k}.

If there is only transmitter IQ imbalance, it is not difficult to see from Eq. (2.34) that the SIR due to transmitter IQ imbalance is

$$\mathrm{SIR}_{t,k} = \left| \frac{1}{\xi_{t,k}} \right|^2, \tag{2.95}$$

for $k = 0, 1, \cdots, N-1$. Assume that only FI IQ imbalance exists, then the SIR is the same for all subcarriers and $\xi_{t,k} = -\frac{\beta_t}{\alpha_t}$. Figure 2.7 shows how $\mathrm{SIR}_{t,k}$ changes with the transmitter IQ imbalance θ_t and ε_t when there is only FI IQ imbalance.

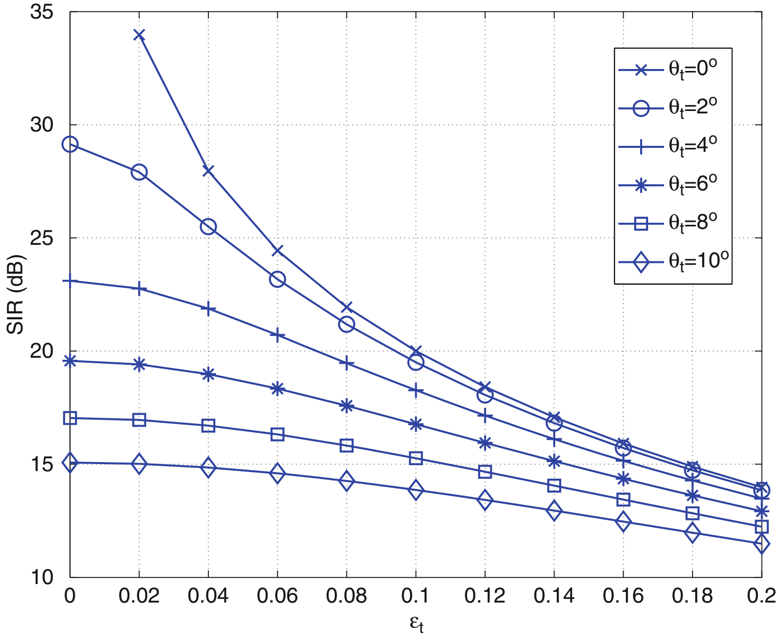


Fig. 2.7 The impact of transmitter only IQ imbalance on the SIR of the transmitted signal

Similarly, based on Eq. (2.37), we can see that when there is only receiver IQ imbalance, the SIR due to receiver IQ imbalance is

$$\mathrm{SIR}_{r,k} = \left|\frac{1}{\xi_{r,k}}\right|^2. \tag{2.96}$$

Also, if only FI IQ imbalance exists, the SIR is the same for all subcarriers and $\xi_{r,k} = -\frac{\beta_r}{\alpha_r^*}$. Figure 2.8 shows how $\mathrm{SIR}_{r,k}$ changes with the transmitter IQ imbalance θ_r and ε_r.

If both transmitter and receiver IQ imbalances exist, then the SIR can be calculated from Eq. (2.38), which is

$$\mathrm{SIR}_k = \frac{|H_k + \xi_{r,k}\xi_{t,N-k}^* H_{N-k}^*|^2}{|\xi_{t,k} H_k + \xi_{r,k} H_{N-k}^*|^2}. \tag{2.97}$$

Let us ignore the impact of channel, i.e., assume that $H_k = 1$ for $k = 0, 1, \cdots, N-1$, then, the SIR equals

$$\mathrm{SIR}_k = \frac{|1 + \xi_{r,k}\xi_{t,N-k}^*|^2}{|\xi_{t,k} + \xi_{r,k}|^2}. \tag{2.98}$$

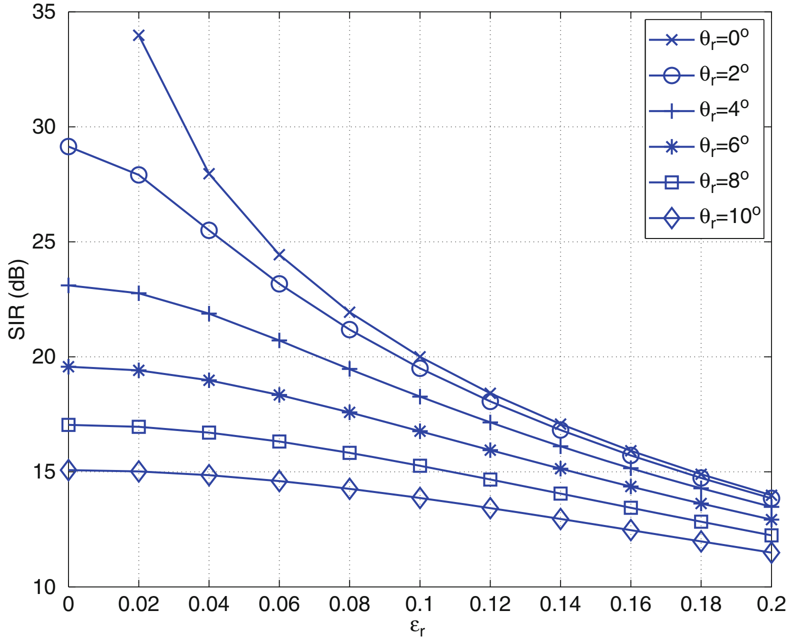


Fig. 2.8 The impact of receiver only IQ imbalance on the SIR of the received signal

If only FI IQ imbalance exists, then $\xi_{t,k} = -\frac{\beta_t}{\alpha_t}$ and $\xi_{r,k} = -\frac{\beta_r}{\alpha_r}$ for $k = 0, 1, \cdots, N-1$. In this case, the SIR equals

$$\mathrm{SIR}_k = \frac{|\alpha_r \alpha_t + \beta_r \beta_t|^2}{|\alpha_r \beta_t + \alpha_t \beta_r|^2}. \tag{2.99}$$

Figure 2.9 shows the SIR under the assumption that $\theta_t = \theta_r$ and $\epsilon_t = \epsilon_r$. Compared with the case of TX and RX only IQ imbalance, i.e., Figs. 2.7 and 2.8, we can see that there are about 6 dB degradation at the same value of phase and amplitude mismatch.

From Figs. 2.7 and 2.8 we can see that where there is only TX or RX IQ imbalance, in order to achieve 25 dB SIR, the required amplitude mismatch is about 0.04 and phase mismatch is about 2°, which is a quite stringent requirements. If both TX and RX IQ imbalances exist, at the same amplitude and phase mismatch at TX and RX, there is further 6 dB SIR degradation. This calls for the investigation of estimation and compensation of IQ imbalances.

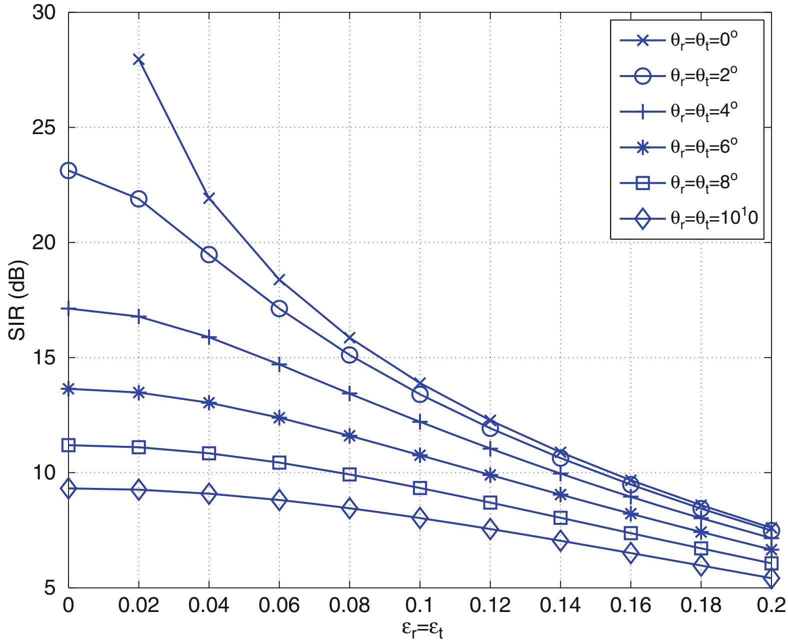


Fig. 2.9 The impact of transmitter and receiver IQ imbalance on the SIR of the received signal

2.8 Conclusions

In this chapter, we built the IQ imbalance model. We started from the general frequency dependent time domain model, and then looked at the frequency domain model. We then combined the IQ imbalance with frequency offset and phase noise. After that we discussed the IQ imbalance in the multiple antenna systems, including both spatial multiplex systems and space-time/space-frequency encoded systems. At the end, we looked at the SIR degradation due to IQ imbalance. This chapter built the foundation for the discussion of estimation and compensation of IQ imbalances in the next two chapters.

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