
Preface

The field of inverse scattering theory has been a particularly active field in applied mathematics for the past 25 years. The aim of research in this field has been to not only detect but also to identify unknown objects through the use of acoustic, electromagnetic, or elastic waves. Although the success of such techniques as ultrasound and x-ray tomography in medical imaging has been truly spectacular, progress has lagged in other areas of application, which are forced to rely on different modalities using limited data in complex environments. Indeed, as pointed out in [88] concerning the problem of locating unexploded ordinance, “Target identification is the great unsolved problem. We detect almost everything, we identify nothing.”

Until a few years ago, essentially all existing algorithms for target identification were based on either a weak scattering approximation or on the use of nonlinear optimization techniques. A survey of the state of the art for acoustic and electromagnetic waves as of 1998 can be found in [54]. However, as the demands of imaging increased, it became clear that incorrect model assumptions inherent in weak scattering approximations imposed severe limitations on when reliable reconstructions were possible. On the other hand, it was also realized that for many practical applications nonlinear optimization techniques require a priori information that is in general not available. Hence, in recent years, alternative methods for imaging have been developed that avoid incorrect model assumptions but, as opposed to nonlinear optimization techniques, only seek limited information about the scattering object. Such methods come under the general title of *qualitative methods in inverse scattering theory*. Examples of such an approach are the linear sampling method, [54, 107], the factorization method [98, 107], the method of singular sources [138, 139], the probe method [91, 92], and the use of convex scattering supports [74, 116], all of which seek to determine an approximation to the shape of the scattering obstacle but in general provide only limited information about the material properties of the scatterer.

This book is designed to be an introduction to qualitative methods in inverse scattering theory, focusing on the basic ideas of the linear sampling

method and its close relative, the factorization method. The obvious question is: an introduction for whom? One of the problems in making these new ideas in inverse scattering theory available to the wider scientific and engineering community is that the research papers in this area make use of mathematics that may be beyond the training of a reader who is not a professional mathematician. This book represents an effort to overcome this problem and to write a monograph that is accessible to anyone having a mathematical background only in advanced calculus and linear algebra. In particular, the necessary material on functional analysis, Sobolev spaces, and the theory of ill-posed problems will be given in the first two chapters. Of course, to do this in a short book such as this one, some proofs will not be given, nor will all theorems be proven in complete generality. In particular, we will use the mapping and discontinuity properties of double- and single-layer potentials with densities in the Sobolev spaces $H^{1/2}(\partial D)$ and $H^{-1/2}(\partial D)$, respectively, but will not prove any of these results, referring for their proofs to the monographs [111] and [127]. We will furthermore restrict ourselves to a simple model problem, the scattering of time-harmonic electromagnetic waves by an infinite cylinder. This choice means that we can avoid the technical difficulties of three-dimensional inverse scattering theory for different modalities and instead restrict our attention to the simpler case of two-dimensional problems governed by the Helmholtz equation. For a glimpse of the problems arising in the three-dimensional “real world,” we refer the reader to [26].

Although, for the foregoing reasons we do not discuss the qualitative approach to the inverse scattering problem for modalities other than electromagnetic waves, the reader should not assume that such approaches do not exist! Indeed, having mastered the material in this book, the reader will be fully prepared to understand the literature on qualitative methods for inverse scattering problems arising in other areas of application, such as acoustics and elasticity. In particular, for qualitative methods in the inverse scattering problem for acoustic waves and underwater sound see [12, 133, 158, 159, 160], whereas for elasticity we refer the reader to [5, 37, 38, 73, 132, 135, 150].

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In closing, we note that this book is an updated and expanded version of an earlier book by the authors that originally appeared in the Springer Series on Interactions of Mechanics and Mathematics entitled *Qualitative Methods in Inverse Scattering Theory*.

A Qualitative Approach to Inverse Scattering Theory

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