

# Preface

## Why Quirky?

The purpose of the “Quirky” series is to help develop an accurate physical, conceptual, geometric, and pictorial understanding of important physics topics. We focus on areas that do not seem to be covered well in most texts. The Quirky series attempts to clarify those neglected concepts, and others that seem likely to be challenging and unexpected (quirky?). The Quirky books are intended for serious students of physics; they are not “popularizations” or oversimplifications.

Physics includes math, and we are not shy about it, but we also do not hide behind it.

Without a conceptual understanding, math is gibberish.

We seek to be accurate, but not pedantic. When mathematical or physical words have precise meanings, we adhere to those meanings. Words are the tools of communication; it is impossible to make fine points with dull tools.

## Who Is It For?

This work is one of the several aimed at graduate and advanced-undergraduate physics students, engineers, scientists, and anyone else who wants a serious understanding of Quantum Mechanics. The material ranges from fairly elementary (though often neglected) to graduate level. Go to <http://physics.ucsd.edu/~emichels> for the latest versions of the Quirky Series, and for contact information. We are looking for feedback, so please let us know what you think.

## How to Use This Book

This book is an informal, topical review. We strive to be accurate, but not tedious.

This work is not a text book.

There are plenty of those, and they cover most of the topics quite well. This work is meant to be used *with* a standard text, to help emphasize those things that are most confusing for new students. When standard presentations do not make sense, come here. In short, our goal is to provide the foundation that enables future learning from other sources.

If you have some background in quantum mechanics, then most of the sections stand alone. The larger sections start by naming the prerequisites needed to understand that section. This work is deliberately somewhat redundant, to make sections more independent, and because learning requires repetition.

You should read all of Chap. 1, Basic Wave Mechanics Concepts, to familiarize yourself with the notation and contents. After the first two chapters, this book is meant to be read in any order that suits you. Each section stands largely alone, though the sections are ordered logically. You may read it from beginning to end, or skip around to whatever topic is most interesting. The “Desultory” chapter is a diverse set of short topics, each meant for quick reading.

We must necessarily sometimes include forward references to material which has not yet been covered in this book. If they are unfamiliar, most such references may be ignored without loss of continuity.

If you don’t understand something, read it again *once*, then keep reading. *Don’t get stuck on one thing*. Often, the following discussion will clarify things.

## Scope What This Text Covers

This text covers most of the unusual or challenging concepts in a first-year graduate course in nonrelativistic Quantum Mechanics (QM). Much of it is suitable for undergraduate QM, as well, because it provides a conceptual foundation for all of QM. We expect that you are taking or have taken such a QM course, and have a good text book. This text supplements those other sources.

## What This Text Doesn't Cover

This text is neither a QM course in itself, nor a review of such a course. We do not cover all basic QM concepts; only those that are unusual or especially challenging (quirky?). There is almost no relativistic QM here.

## What You Already Know

This text assumes you understand basic integral and differential calculus, partial differential equations, have seen complex numbers, and have some familiarity with probability. You must have a working knowledge of basic physics: mass, force, momentum, energy, etc. Further, it assumes you have a QM text for the bulk of your studies, and are using *Quirky Quantum Concepts* to supplement it. You must have been introduced to the idea of particles as waves, and photons as particles of light. Beyond that, different sections require different levels of preparation; some are much more advanced than others. Each section lists any particular prerequisites at the beginning. Some sections require some familiarity with classical Lagrangian and Hamiltonian mechanics, including canonical momentum.

## Notation

Important points are highlighted in solid-border boxes.

Common misconceptions are sometimes written in dashed-line boxes.

**References:** As is common, we include references to published works in square brackets, where the abbreviations in the brackets are defined in the “References” section of this document. Where page numbers are given, they may be augmented by “t”, “m”, or “b”, for “top”, “middle”, and “bottom” of the page.

**Unit Vectors:** We use  $\mathbf{e}_x$ ,  $\mathbf{e}_y$ , etc. for unit spatial vectors. In other disciplines, these are more likely written as  $\hat{x}$ ,  $\hat{y}$ , etc., but we reserve “hats” to indicate quantum operators.

Keywords are listed in **bold** near their definitions. All keywords also appear in the glossary.

We use the following symbols regularly:

Symbol	Name	Common scientific meanings
$\nabla$	Del	Gradient of a scalar field, divergence or curl of a vector field
$\equiv$	Identity	(1) is defined as; (2) identically (always) equal to
$\forall$	For all	for all
$\in$	Element	Is an element of (sometimes written as epsilon, $\epsilon$ )
$\approx$	Approximately equals	Approximately equals
$\sim$	Tilde	Scales like; is proportional to in some limit. For example, $1/(r+1) \sim 1/r$ for large $r$ because $\lim_{r \rightarrow \infty} \frac{1}{r+1} = \frac{1}{r}$ . Note that we want to preserve the scaling property, so we don't take such a limit all the way to $r \rightarrow \infty$ (limit of zero), which would hide any $r$ dependence
$!!$	Double factorial	$n!! \equiv n(n-2)(n-4) \dots$ (2 or 1)

**Integrals:** In many cases, we present a general integral, whose exact limits depend on what problem or coordinate system the integral is applied to. To provide a general formula, independent of such particulars, we give the limits of integration as a single “ $\infty$ ”, meaning integrate over the entire domain appropriate to the problem:

$$\int_{\infty} f(x) \, dx \equiv \text{integrate over entire domain relevant to the given problem.}$$

**Open and Closed Intervals:** An **open interval** between  $c$  and  $d$  is written “ $(c, d)$ ”, and means the range of numbers from  $c$  to  $d$  *exclusive* of  $c$  and  $d$ . A **closed interval** between  $c$  and  $d$  is written “ $[c, d]$ ”, and means the range of numbers from  $c$  to  $d$  *including*  $c$  and  $d$ . A half-open interval “ $[c, d)$ ” has the expected meaning of  $c$  to  $d$  including  $c$  but not  $d$ , and “ $(c, d]$ ” means  $c$  to  $d$  excluding  $c$  but including  $d$ .

**Operators:** I write most operators with a “hat” over them, e.g.  $\hat{x}$ . Rarely, the hat notation is cumbersome, so I sometimes use the subscript  $_{op}$  to denote quantum operators, as in [12]. Thus the symbol  $x$  is a real variable,  $\hat{x}$  is the position operator, and  $(p^2)_{op}$  is the operator for  $p^2$ .

**Conjugates and Adjoints:** We use “\*” for complex conjugation, and “†” for adjoint:  $z^* \equiv$  complex conjugate of the number ‘ $z$ ’,  $\hat{a}^\dagger \equiv$  adjoint operator of  $\hat{a}$ .

[Note that some math texts use a bar for conjugate:  $\bar{a} \equiv$  complex conjugate of ‘ $a$ ’, and a “\*” for adjoint. This is confusing to physicists, but c’est la vie.]

[Interesting paragraphs that may be skipped are “asides,” shown in square brackets, smaller font, and narrowed margins.]

[Short asides may be also be written in-line in square brackets.]

**Vector Variables:** In some cases, to emphasize that a variable is a vector, it is written in bold; e.g.,  $V(\mathbf{r})$  is a scalar function of the vector,  $\mathbf{r}$ .  $\mathbf{E}(\mathbf{r})$  is a vector function of the vector,  $\mathbf{r}$ . We write a zero vector as  $\mathbf{0}_v$  (this is different than the number zero).

**Matrices:** Matrices are in bold, **B**. A particular element of a single matrix may be specified with subscripts, e.g.  $B_{ij}$ . A particular element of a matrix expression uses brackets, e.g.  $[\mathbf{AB}]_{ij} \equiv$  the  $ij^{\text{th}}$  element of the matrix product  $\mathbf{AB}$ .

**Tensor Products:** Sometimes, we write a tensor product explicitly with the  $\otimes$  symbol.

In-line derivatives sometimes use the notation  $d/dx$  and  $\partial/\partial x$ . There is not always a clear mathematical distinction between  $d/dx$  and  $\partial/\partial x$ . When the function arguments are independent, they are both the same thing. I use  $d/dx$  when a function is clearly a total derivative, and  $\partial/\partial x$  when it is clearly a partial derivative. However, in some cases, it's not clear what arguments a function has, and it is not important. In that case, I tend to use  $\partial/\partial x$  for generality, but do not worry about it.

Also, for the record, derivatives *are* fractions, despite what you might have been told in calculus. They are a special case of fraction: the limiting case of fractions of differentially small changes. But they are still fractions, with all the rights and privileges thereof. All of physics treats them like fractions, multiplies and divides them like fractions, etc., because they *are* fractions.

## Greek Letters

The Greek alphabet is probably the next-best well known alphabet (after our Latin alphabet). But Greek letters are often a stumbling block for readers unfamiliar with them. So here are all the letters, their pronunciations, and some common meanings from all over physics. Note that every section defines its own meanings for letters, so look for those definitions.

The Greek alphabet has 24 letters, and each has both upper-case (capital) and lower-case forms. Not all can be used as scientific symbols, though, because some look identical to Latin letters. When both upper- and lower-case are useable, the lower-case form is listed first. Lower case Greek variables are italicized, but by convention, upper case Greek letters are not. Do not worry if you do not understand all the common meanings; we will define as we go everything you need to know for this book.

Letter	Name (pronunciation)	Common scientific meanings
$\alpha$	Alpha (al'fuh)	Coefficient of linear thermal expansion. (Capital: A, not used.)
$\beta$	Beta (bae'tuh)	Velocity as a fraction of the speed of light ( $\beta \equiv v/c$ ). (Capital: B, not used)
$\gamma$	Gamma (gam'uh)	The relativistic factor $(1 - \beta^2)^{-1/2}$ , aka time-dilation/length-contraction factor
$\Gamma$	Capital gamma	Christoffel symbols (General Relativity); generalized factorial function
$\delta$	Delta (del'tuh)	The Dirac delta (impulse) function; the Kronecker delta; an inexact differential (calculus)

Letter	Name (pronunciation)	Common scientific meanings
$\partial$	Old-style delta	Partial derivative (calculus)
$\Delta$	Capital delta	A small change
$\varepsilon$	Epsilon (ep'si-lon)	A small value. (Capital: E, not used.)
$\zeta$	Zeta (zae'tuh)	Damping ratio. (Capital: Z, not used.)
$\eta$	Eta (ae'tuh)	Efficiency; flat-space metric tensor. (Capital: H, not used.)
$\theta$	Theta (thae'tuh)	Angle
$\Theta$	Capital theta	Not commonly used. Sometimes angle
$\iota$	Iota (ie-o'tuh)	Not commonly used. (Capital: I, not used.)
$\kappa$	Kappa (kap'uh)	Decay constant. (Capital: K, not used.)
$\lambda$	Lambda (lam'duh)	Wavelength
$\Lambda$	Capital lambda	Cosmological constant.
$\mu$	Mu (mew)	Micro ( $10^{-6}$ ); reduced mass. (Capital: M, not used.)
$\nu$	Nu (noo)	Frequency. Not to be confused with an italic $\nu$ : $\nu$ vs. $\nu$ . (Capital: N, not used.)
$\xi$	Xi (zie, sometimes ksee)	Dimensionless distance measure.
$\Xi$	Capital xi	Not commonly used
$o$	Omicron (oe'mi-kron)	Not used. (Capital: O, not used.)
$\pi$	Pi (pie)	Ratio of a circle's circumference to its diameter, $\approx 3.14159\dots$
$\Pi$	Capital pi	Product (multiplication)
$\rho$	Rho (roe)	Mass density; charge density; correlation coefficient. (Capital: P, not used.)
$\sigma$	Sigma (sig'muh)	Standard deviation; surface charge density
$\Sigma$	Capital sigma	Sum (addition)
$\tau$	Tau (rhyme: cow, or sometimes saw)	Time; torque. (Capital: T, not used.)
$\upsilon$	Upsilon (oops'i-lon)	Not commonly used. (Capital: Y, not used.)
$\phi$	Phi (fee or fie)	Angle
$\varphi$	Old-style phi	Angle
$\Phi$	Capital phi	Electric potential; general potential.
$\chi$	Chi (kie)	Degrees of freedom. (Capital: X, not used.)
$\psi$	Psi (sie)	Wave-function amplitude
$\Psi$	Capital psi	Not commonly used
$\omega$	Omega (oe-mae'guh)	Angular velocity; angular frequency
$\Omega$	Capital omega	Angle; solid angle; ohm (unit of electrical resistance)

<http://www.springer.com/978-1-4614-9304-4>

Quirky Quantum Concepts

Physical, Conceptual, Geometric, and Pictorial Physics  
that Didn't Fit in Your Textbook

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2014, XIX, 361 p. 72 illus., Softcover

ISBN: 978-1-4614-9304-4