Chapter 2
Flow of Incompressible Newtonian Fluids in Pipes

Newtonians are fluids in which the relative slip of fluid elements past each other is proportional to the shear on the fluid, as shown in Fig. 2.1. All gases, liquid water, and liquids of simple molecules (ammonia, alcohol, benzene, oil, chloroform, butane, etc.) are Newtonians. Pastes, emulsions, biological fluids, polymers, suspensions of solids, and other mixtures are likely to be non-Newtonian.[1] This chapter deals with Newtonians.

When a fluid flows in a pipe, some of its mechanical energy is dissipated by friction. The ratio of this frictional loss to the kinetic energy of the flowing fluid is defined as the Fanning friction factor, f_F. Thus[2]

$$f_F = \left(\frac{\text{frictional drag force} \Big/ \text{area of pipe surface}}{\text{kinetic energy/m}^3 \text{ of fluid}} \right) = \frac{\tau_w}{\rho \frac{u^2}{2}} \quad [-] \tag{2.1}$$

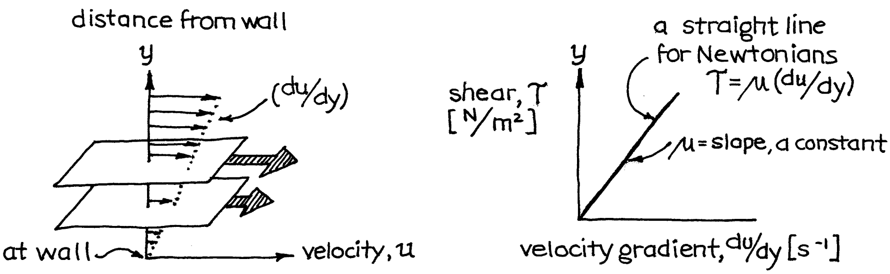

Fig. 2.1 Representation of a Newtonian fluid

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[1] Strictly speaking, we should use the terms *Newtonian fluids* and *non-Newtonian fluids*. However, there should not be much confusion or distress if we drop the word "fluid" and simply call these materials *Newtonians* and *non-Newtonians*.

[2] SI readers may ignore g_c in all equations, if they wish.

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Making a force balance about a section of pipe, as sketched in Fig. 2.2, relates the wall shear τ_w to the frictional loss ΣF (or the frictional pressure drop Δp_{fr}). In words then, for a length of pipe L,

$$\begin{pmatrix}\text{Force transmitted}\\ \text{to the walls}\end{pmatrix} = \begin{pmatrix}\text{frictional energy}\\ \text{loss by the fluid}\end{pmatrix}$$

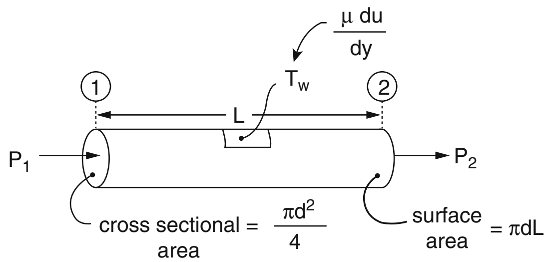


Fig. 2.2 A force balance on a section of pipe

or in symbols

$$(L\pi d)\tau_w = \frac{\pi d^2}{4}\Delta p_{\mathrm{fr}} = \frac{\pi d^2}{4}\rho\underbrace{\Sigma F}_{\mathrm{J/kg}} \qquad [\mathrm{N}]$$

Substituting in equation (2.1) and rearranging gives

$$\Sigma F = \frac{2f_{\mathrm{F}}Lu^2}{d} = gh_L \qquad \left[\frac{\mathrm{J}}{\mathrm{kg}} = \frac{\mathrm{m}^2}{\mathrm{s}^2}\right] \tag{2.2}$$

This frictional loss shows up as the lost head and is seen physically in the sketch of Fig. 2.3.

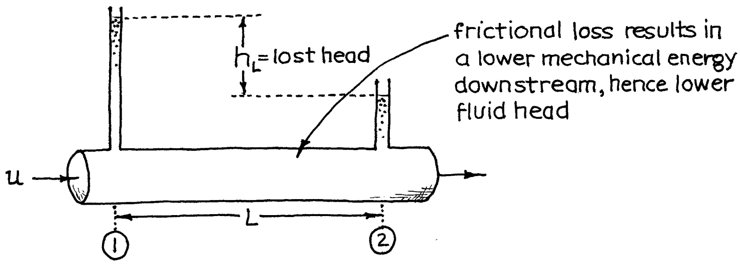


Fig. 2.3 Physical representation of the lost head

We would expect the friction factor to depend on the velocity of flow u, the fluid properties of density ρ and viscosity μ, the pipe size d, and its roughness, and so it does. Thus, we have

$$f_F = f\left[\begin{pmatrix}\text{Reynolds number :}\\ \text{a combination of } d,\ u,\ \mu,\ \rho\end{pmatrix},\ \begin{pmatrix}\text{pipe}\\ \text{roughness, } \in\end{pmatrix}\right]$$

where the Reynolds number is defined as

$$\mathrm{Re} = \frac{du\rho}{\mu}$$

Figures 2.4 and 2.5, prepared from both theory and experiment, represent this relationship for Newtonians in two alternative ways. Each figure is useful for

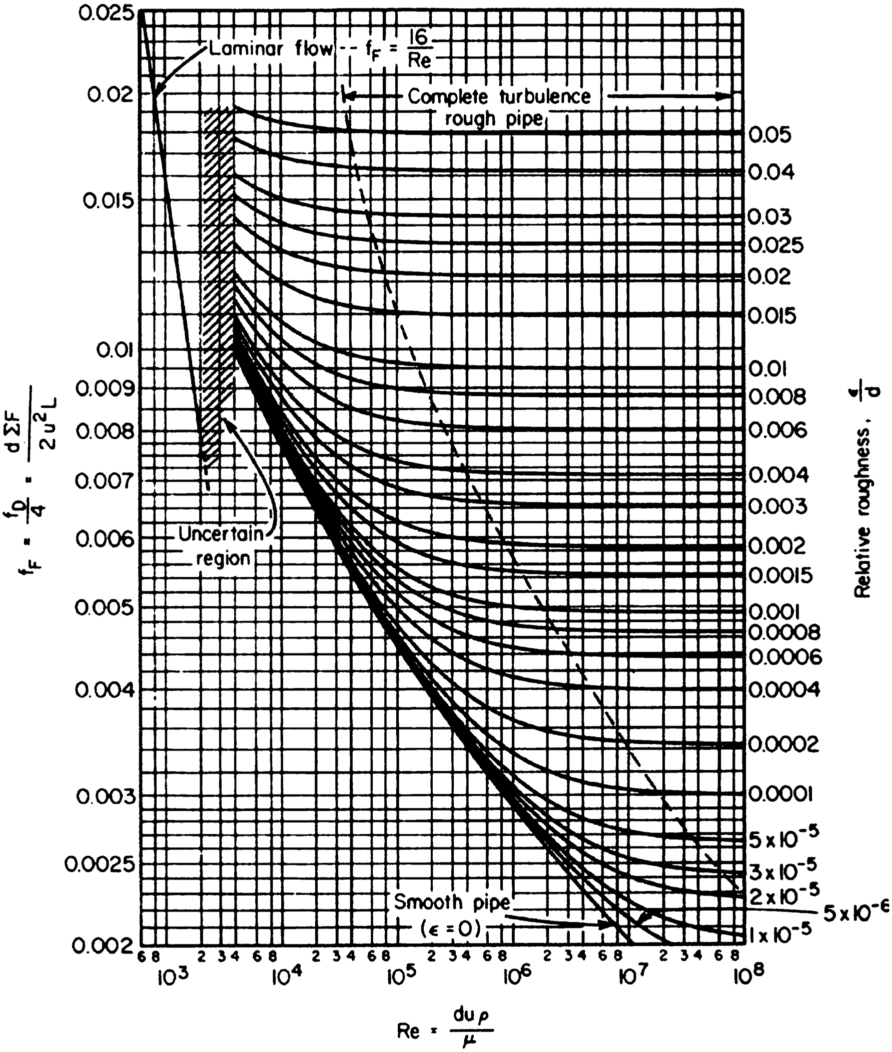


Fig. 2.4 This figure is useful for finding the pumping requirement or frictional loss when you are given the flow rate of fluid in a pipe (Adapted from Moody (1944)) or given d and u, find W_s or Δp

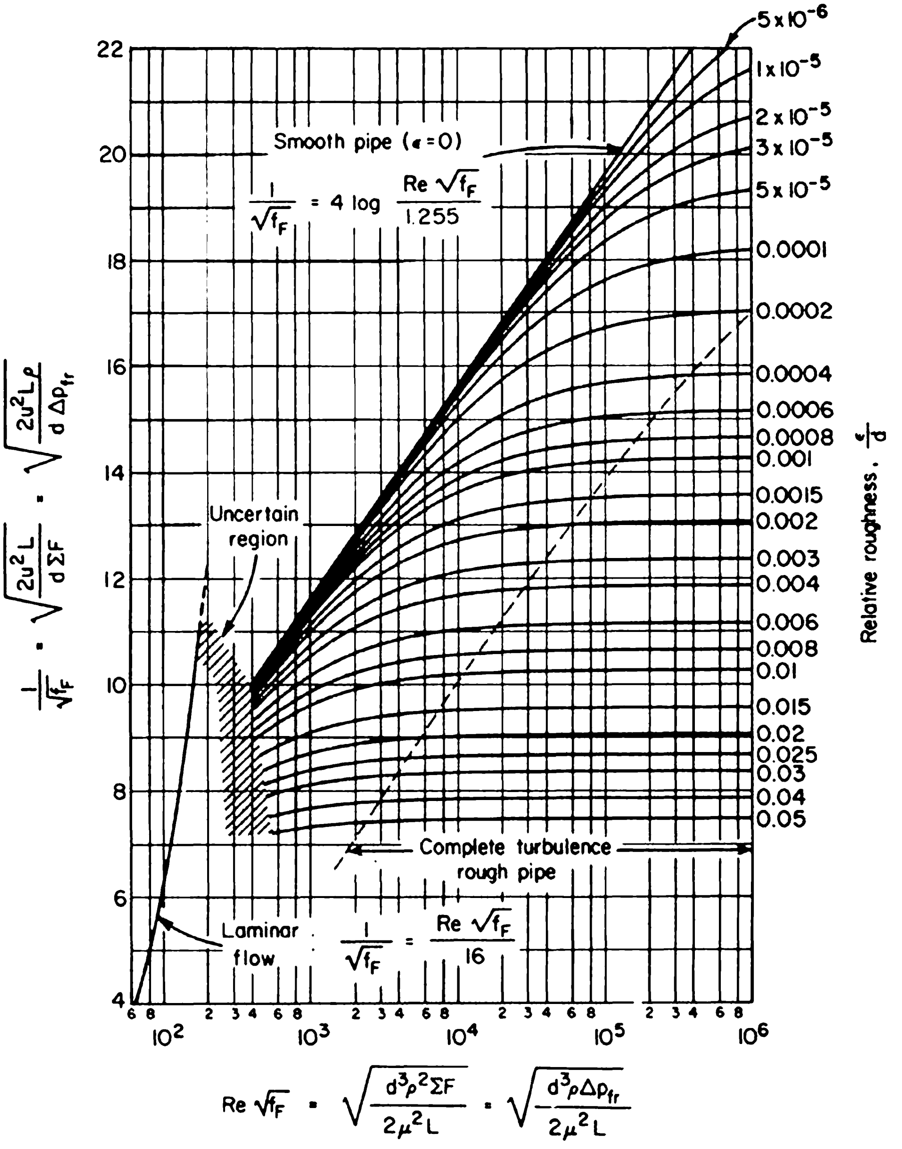


Fig. 2.5 This figure is useful for finding the flow rate when you are given the driving force for flow (gravitational head, pumping energy input, etc.) (Adapted from H. Rouse; see discussion after Moody (1944)) or given d and Δp or W_s, find u

certain purposes. The pipe roughness, needed in these charts, is given in Table 2.1 for various common pipe materials.

The mechanical energy balance for flow between points 1 and 2 in a pipe is then represented by equation (1.5); thus, referring to Fig. 2.6 we have

Table 2.1 Roughness of clean pipe[a]

Pipe material	ϵ, mm
Riveted steel	1–10
Concrete	0.3–3
Wood stave	0.2–1
Cast iron	0.26 (0.25)[b]
Galvanized iron	0.15 (0.15)[b]
Asphalted cast iron	0.12 (0.13)[b]
Commercial steel or wrought iron	0.046 (0.043)[b]
Drawn tubing	0.0015
Glass	0
Plastic (PVC, ABS, polyethylene)	0

[a]Adapted from Fischer and Porter Co., Hatboro, PA, catalogs section 98-A (1947)
[b]Values in parentheses from Colebrook (1939)

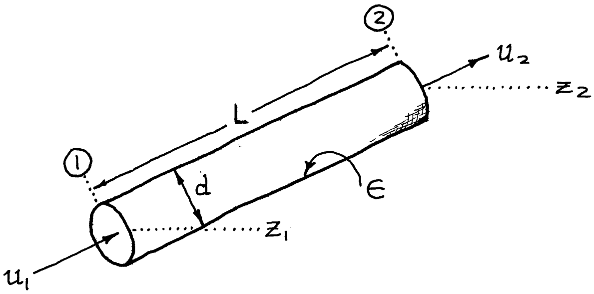


Fig. 2.6 Development of the mechanical energy balance for flow in pipes

$$\left.\begin{aligned} & g\Delta z + \Delta\left(\frac{u^2}{2}\right) + \int_1^2 \frac{dp}{\rho} + \overbrace{W_s}^{=0 \text{ for no pump or turbine in the line}} + \sum F = 0 \\ & \sum F = \frac{2 f_F L u^2}{d} \end{aligned}\right\} \quad \left[\frac{\mathrm{J}}{\mathrm{kg}}\right] \qquad (2.3)$$

where

or in terms of head of fluid

where

$$\left.\begin{aligned} &\Delta z + \Delta\left(\frac{u^2}{2g}\right) + \frac{1}{g}\int\frac{dp}{\rho} + \overbrace{\frac{1}{g}W_s}^{=0 \text{ for no pump or turbine in the line}} + h_L = 0 \\ &h_L = \frac{1}{g}\sum F = \frac{2 f_{\mathrm{F}} L u^2}{gd} \end{aligned}\right\} \quad [\mathrm{m}] \tag{2.4}$$

These equations are used with Figs. 2.4 and 2.5 to solve pipe flow problems.

2.1 Comments

1. *The Reynolds number Re* measures the importance of energy dissipation by viscous effects. Thus

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{du\rho}{\mu} = \frac{dG}{\mu} \quad [\text{-}]$$

When the flow is represented by a large Reynolds number, this means that viscous effects are relatively unimportant and contribute little to energy dissipation; a small Reynolds number means that viscous forces dominate and are the main mechanism for energy dissipation.

2. *Flow regimes* (see Fig. 2.7). Newtonians flowing in pipes exhibit two distinct types of flow, *laminar* (or streamline) when $\mathrm{Re} < 2{,}100$ and *turbulent* when $\mathrm{Re} > 4{,}000$. Between $\mathrm{Re} = 2{,}100$ and $\mathrm{Re} = 4{,}000$, we observe a transition regime with uncertain and sometimes fluctuating flow.

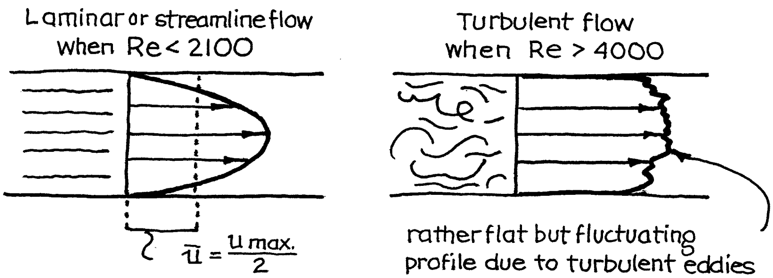


Fig. 2.7 The difference between laminar and turbulent flow in pipes

3. *For laminar flow* ($\mathrm{Re} < 2{,}100$), the friction factor and the frictional loss can be found either from Fig. 2.4 or 2.5 or from the following simple theoretical expressions derived by Poiseuille:

$$f_\mathrm{F} = \frac{16}{\mathrm{Re}} \quad \text{or} \quad \Sigma F = \frac{32u\mu L}{d^2\rho} \quad \text{or} \quad u = \frac{d^2\rho\Sigma F}{32\mu L} \tag{2.5}$$

4. *Two different friction factors* are in common use today:

   (i) f_F, the Fanning friction factor, defined in equation (2.1)
   (ii) f_D, the Darcy friction factor

   Chemical engineers favor f_F; most others prefer f_D. Don't confuse these two definitions; also, note that

$$f_\mathrm{D} = 4f_\mathrm{F} \quad \text{or} \quad f_\mathrm{F} = \frac{f_\mathrm{D}}{4}$$

   The simplest way to tell which one is being used (when no subscript is shown) is to look at the laminar flow regime in the f vs. Re chart. There

$$f_\mathrm{F} = \frac{16}{\mathrm{Re}} \quad \text{while} \quad f_\mathrm{D} = \frac{64}{\mathrm{Re}}$$

5. *In the turbulent regime* ($\mathrm{Re} > 4{,}000$), the friction factor and frictional loss are found from Fig. 2.4 or 2.5 or from the experimentally tested expressions of Nikuradse, which closely approximate the theoretical expressions of Nikuradse, Prandtl, and von Karman, as discussed in Schlichting (1979).

   For the range of flows from $\mathrm{Re} = 4{,}000$ to 10^8, these expressions were cleverly combined by Colebrook (1939) to give

$$\frac{1}{\sqrt{f_\mathrm{F}}} = -4\log\left(\frac{1}{3.7}\frac{\in}{d} + \frac{1.255}{\mathrm{Re}\sqrt{f_\mathrm{F}}}\right) \tag{2.6}$$

   In a form useful for calculating Re given the value of f_F

$$\frac{1}{\mathrm{Re}} = \frac{\sqrt{f_\mathrm{F}}}{1.255}\left[10^{-0.25/\sqrt{f_\mathrm{F}}} - \frac{1}{3.7}\frac{\in}{d}\right] \tag{2.7}$$

   and in a form useful for calculating f_F from Re Pavlov et al. (1981) give the very good approximation

$$\frac{1}{\sqrt{f_\mathrm{F}}} \cong -4\log\left[\frac{1}{3.7}\frac{\in}{d} + \left(\frac{6.81}{\mathrm{Re}}\right)^{0.9}\right] \tag{2.8}$$

The above expressions reduce to a number of special cases. Thus for *fully developed turbulence in rough pipes*, where f_F is independent of Re, equation (2.6) or (2.8) becomes

$$\frac{1}{\sqrt{f_F}} = 4\log\left(3.7\frac{d}{\in}\right) \tag{2.9}$$

For *smooth pipes* ($\epsilon/d = 0$) equation (2.8) simplifies to

$$\frac{1}{\sqrt{f_F}} = 3.6\log\frac{\mathrm{Re}}{6.81} \tag{2.10}$$

6. *Transition regime* (Re = 2,100 ~ 4,000). Here we have an uncertain situation where the flow may be turbulent, or laminar, or even fluctuating.

7. *Piping systems* have contractions, expansions, valves, elbows, and all sorts of fittings. Each has its own particular frictional loss. A convenient way to account for this is to put this loss in terms of an equivalent length of straight pipe. Thus, the equivalent length of a piping system as a whole is given by

$$\underset{\text{total}}{L_{\text{equiv}}} = \underset{\text{pipe}}{L_{\text{straight}}} + \sum_{\substack{\text{all fittings}\\ \text{contractions,}\\ \text{expansions, etc.}}} \left(L_{\text{equiv}}\right) \tag{2.11}$$

In turbulent flow the equivalent lengths of pipe fittings are independent of the Reynolds number, and Table 2.2 shows these values for various fittings. Unfortunately, in laminar flow the equivalent length varies strongly with the Reynolds number is distinctive for each fitting. Thus, simple generalizations such as Table 2.2 cannot be prepared for the laminar flow regime.

8. The kinetic and potential energy terms of flowing fluids. In solving flow problems and replacing values in the mechanical energy balance, we often find

- For liquids—the kinetic energy terms are negligible and can be ignored.
- For gases—the potential energy terms are negligible and can be ignored.

When in doubt, evaluate all the terms and then drop those which are small compared to the others.

In cases where the kinetic energy must be considered, the sketches of Fig. 2.8 show how to account for this effect. In general, then

Table 2.2 Equivalent pipe length for various pipe fittings (turbulent flow only)[a]

Pipe fitting	L_{equiv}/d
Globe valve, wide open	~300
Angle valve, wide open	~170
Gate valve, wide open	~7
3/4 open	~40
1/2 open	~200
1/4 open	~900
90° elbow, standard	30
long radius	20
45° elbow, standard	15
Tee, used as elbow, entering the stem	90
Tee, used as elbow, entering one of two side arms	60
Tee, straight through	20
180° close return bend	75
Ordinary entrance (pipe Hush with wall of vessel)	16
Borda entrance (pipe protruding into vessel)	30
Rounded entrance, union, coupling	Negligible
Sudden enlargement from d to D	
Laminar flow in d:	$\frac{\mathrm{Re}}{32}\left[1-\left(\frac{d^2}{D^2}\right)\right]^2$
Turbulent flow in d:	$\frac{1}{4f_{\mathrm{F,\,in}\,d}}\left[1-\left(\frac{d^2}{D^2}\right)\right]^2$
Sudden contraction from D to d; all conditions except high-speed gas How where $p_1/p_2 \geq 2$. For this see Chap. 3.	
Laminar How in d:	$\frac{\mathrm{Re}}{160}\left[1.25-\left(\frac{d^2}{D^2}\right)\right]$
Turbulent How in d:	$\frac{1}{10f_{\mathrm{F,\,in}\,d}}\left[1.25-\left(\frac{d^2}{D^2}\right)\right]$

[a]Adapted in part from Crane (1982) and from Perry (1950)

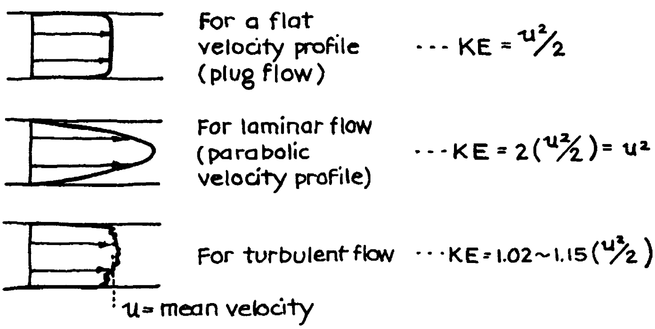


Fig. 2.8 An accounting of the kinetic energy term in the mechanical energy balance

$$\mathrm{KE} = \frac{u^2}{\alpha 2} \quad \text{where} \quad \alpha \begin{cases} = \dfrac{1}{2} \text{ for laminar flow} \\ = 1 \text{ for plug flow} \\ \rightarrow 1 \text{ for turbulent flow} \end{cases} \tag{2.12}$$

Since the kinetic energy of flowing fluids only contributes significantly at high velocities where flow is turbulent, α usually is close to 1. Only for gases can the flow be both laminar and at high velocity. This situation occurs only rarely.

9. *Evaluation of the* $\int(d\rho/\rho)$ *term* in the mechanical energy balance:

   - For liquids, $\rho \simeq$ constant so

$$\int_1^2 \frac{dp}{\rho} = \frac{1}{\rho}(p_2 - p_1) = \frac{\Delta p}{\rho} \tag{2.13}$$

   - For ideal gases with small density changes, we can use an average density and then treat the gas as an incompressible fluid; thus

$$\overline{\rho} = \frac{1}{\overline{v}} = \frac{\overline{p}(mw)}{R\overline{T}} \qquad \text{where} \qquad \begin{cases} \overline{p} = \dfrac{p_1 + p_2}{2} \\ \overline{T} = \dfrac{T_1 + T_2}{2} \end{cases} \tag{2.14}$$

   - For large fractional changes in pressure or density, and by this we mean when $p_1/p_2 > 2$ or $\rho_1/\rho_2 > 2$, we must use the treatment of Chap. 3.

10. *Aging of pipes*. The value of pipe roughness given in Table 2.1 is for clean pipe. With time, however, roughness may increase because of corrosion and scale deposition. Colebrook (1939) found that a simple linear expression can reasonably represent such a change

$$\epsilon_{\text{any time}} = \epsilon_{\text{time}=0} + \alpha t \tag{2.15}$$

    An increase in roughness will lower the flow rate for a given driving force or will increase the power requirement to maintain a given flow rate.

11. *Other shaped conduits*. In general for turbulent flow, one can approximate the frictional loss in other than circular-shaped conduits by representing the conduit by a circular pipe of equivalent diameter defined as

$$d_e = 4\begin{pmatrix}\text{hydraulic} \\ \text{radius}\end{pmatrix} = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right) \tag{2.16}$$

For certain shapes,

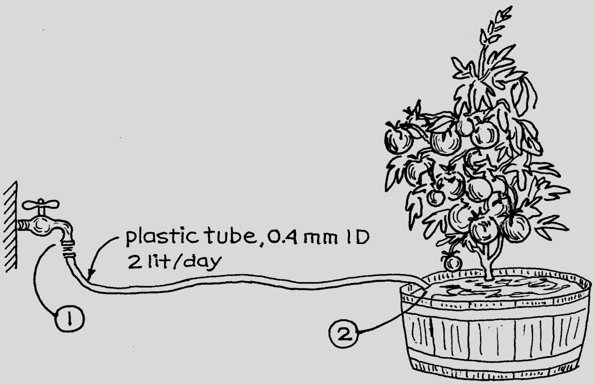
- Plain and eccentric annuli
- Pipes containing various shapes of internals, including finned tubes
- Parallel plates
- Rectangular, triangular, and trapezoidal conduits

—experiments have been made and frictional losses reported [see Knudsen and Katz (1958), Chaps. 4 and 7].

Example 2.1. Tomato Growing in Absentia

Every summer I carefully grow a giant tomato plant because I love the taste of its fresh-picked fruit. Since these plants need 2 lit of water each day of the growing season to produce these delectable and irresistible fruits, how do I grow my plant next summer when I will be away for 4 weeks with no way to water it?

One solution would be to connect a long plastic tube 0.4 mm i.d. to the faucet at my home where the water pressure is 100 kPa above atmospheric and lead it to the plant. Determine how long the tube would have to be to deliver 2 lit/day of water. Of course, everything is on the level.



Solution

Knowing the volumetric flow rate of water and the tube diameter will allow calculation of the Reynolds number for the flow in the plastic tube. Thus

$$u_2 = \frac{v}{A} = \frac{\left(2\frac{\text{lit}}{\text{day}}\right)\left(\frac{1\,\text{m}^3}{1{,}000\,\text{lit}}\right)\left(\frac{1\,\text{day}}{24\times 3{,}600\,\text{s}}\right)}{\frac{\pi}{4}(0.0004\,\text{m})^2} = 0.184\frac{\text{m}}{\text{s}}$$

(continued)

(continued)

and

$$\mathrm{Re}_2 = \frac{d_2 u_2 \rho}{\mu} = \frac{(0.0004)(0.184)(1{,}000)}{10^{-3}} = 73.7 \quad \text{thus, laminar flow}$$

After these preliminaries let us write the mechanical energy balance between points 1 and 2

$$\cancelto{=0}{g(z_2 - z_1)} + u_2^2 - u_1^2 + \frac{p_2 - p_1}{\rho} + \cancelto{=0}{W_s} + \frac{2 f_F L u^2}{d} = 0 \qquad \left[\frac{\mathrm{J}}{\mathrm{kg}}\right]$$

Note that "2" is absent from the denominator of the kinetic energy term. This is because the fluid is in laminar flow. Also, since the diameter of the faucet opening is large compared to that of the tube, we can reasonably assume that the velocity therein is negligible, or $u_1 = 0$. Thus, on replacing values we find

$$(0.184)^2 + \frac{101{,}325 - 201{,}325}{1{,}000} + \frac{2 f_F L (0.184)^2}{0.0004} = 0$$

or

$$0.034 - 100 + 169.6 f_F L = 0 \tag{i}$$

Then, either from an extrapolation of Fig. 2.4 or from equation (2.5), we find that

$$f_F = \frac{16}{\mathrm{Re}} = \frac{16}{73.7} = 0.2171 \tag{ii}$$

Combining (i) and (ii) then gives the length of tube needed, or

$$L = 2.715 \text{ m}$$

NOTE ON THE KE CONTRIBUTION: The numbers in equation (i) show that the kinetic energy contributes less than 0.04 % to the total energy loss. Thus, the kinetic energy term could very well have been ignored in this problem. This frequently is the case, especially when flow velocities are not great and when frictional losses are severe.

(continued)

(continued)

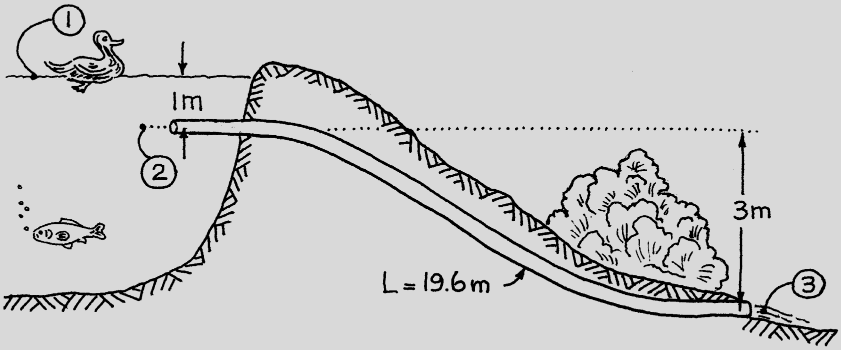
NOTE ON THE ENTRANCE LOSSES: In the above solution we ignored the entrance losses. Let us see if this is reasonable. From Table 2.2 we find that the extra length of tubing representing this loss is given by

$$\frac{L_{eq}}{d} = \frac{1.25(\mathrm{Re})}{160} = \frac{1.25(74)}{160} = 1.57, \quad \text{or} \quad L_{eq} = 0.2 \;\; \mathrm{mm}$$

This is quite negligible.

Example 2.2. Overflow Pipe for a Dam
Next summer I plan to dam Dope Creek to form a little lake. Building the dam is straightforward; however, I haven't yet figured out what size of galvanized pipe to use for the water overflow. The dry season is no problem; it is the wet season with its thunderstorms and flash floods that worries me. My personal meteorologist estimates that in the very worst conceivable situation, flow in Dope Creek can reach $\pi/2$ m^3/s.

If the pipe diameter is too small, the water level will rise too high (more than 1 m above the water intake) and will overflow and damage the dam. How large a pipe is needed to guarantee that the water level does not ever rise above this danger point? The equivalent length of this overflow pipe is 19.6 m, and its discharge is located 3 m below its intake.



Solution

We can choose to write the mechanical energy balance between points 1 and 3 or between points 2 and 3. The former pair seems simpler because then $\Delta p = 0$. Thus, between points 1 and 3, we have

$$g\Delta z + \cancel{\frac{\Delta u^2}{2}}^{=0} + \cancel{\frac{\Delta p}{\rho}}^{=0} + \cancel{W_s}^{=0} + \sum F = 0$$

In Example 2.1 we found that the kinetic energy contribution and pipe entrance losses were negligible, so let us start by assuming that they are negligible here as well (we will check this later), and with no pump or turbine in the line, the shaft work term disappears. On replacing values we find that

$$9.8(-4) + 0 + 0 + 0 + \frac{2f_F u_2^2 (19.6)}{d} = 0 \tag{i}$$

Next, relate the flow velocity and Reynolds number with the pipe diameter as follows:

$$u_2 = \frac{\dot{v}}{A} = \frac{\pi/2}{(\pi/4)d^2} = \frac{2}{d^2} \tag{ii}$$

$$\mathrm{Re} = \frac{du_2\rho}{\mu} = \frac{d\left(2/d^2\right)(1{,}000)}{10^{-3}} = \frac{2 \times 10^6}{d} \tag{iii}$$

Combining (i) and (ii) gives

$$f_F = d^5/4 \tag{iv}$$

Now as d changes, so does Re and f_F, and these in turn are related by Fig. 2.4. So let us solve for the pipe diameter by trial and error, as shown.

Guess d	Re [from (iii)]	ϵ/d [from Table 2.1]	f_F [from Fig. 2.4]	f_F [from (iv)]
0.1	2×10^7	0.0015	5.4×10^{-3}	0.0025×10^{-3}
0.4	5×10^6	0.00038	4.0×10^{-3}	2.6×10^{-3}
0.44	4.6×10^6	0.00034	3.9×10^{-3}	4.1×10^{-3} . . . close enough

Therefore, the pipe diameter needed is $d = 0.44$ m.

Finally, let us check to see if we are justified in assuming that the kinetic energy contribution and pipe entrance losses can be ignored. So let us evaluate these contributions and include them in equation (i)

(continued)

(continued)

KE contribution. From equations (2.12) and (ii)

$$\frac{\Delta u^2}{2} = \frac{u_2^2}{2} = \frac{2}{d^4}$$

Entrance loss. Referring to Table 2.2 and the sketch of the dam for Dope Creek above shows that the Borda entrance most closely represents the pipe; thus,

$$\frac{L_{eq}}{d} = 30 \quad \text{or} \quad L_{eq} = 30d$$

Including both these terms in (i) gives

$$9.8(-4) + \frac{2}{d^4} + 0 + 0 + \left[\frac{2f_F(4/d^4)(19.6 + 30d)}{d}\right] = 0 \qquad \text{(v)}$$

To solve this for the pipe diameter, we repeat the trial-and-error procedure, but with equation (v) in place of equation (iv). This gives the pipe diameter of

$$d = 0.56 \text{ m}$$

In which case equation (v) becomes

$$\underbrace{-39.2}_{\text{PE : 100\%}} + \underbrace{20.3}_{\text{KE : 52\%}} + 0 + 0 + [\underbrace{10.3}_{\text{Pipe : 26\%}} + \underbrace{8.8}_{\text{Entrance : 21\%}}] = 0$$

These numbers show that only 26 % of the potential energy is dissipated by pipe friction, not 100 %, and that the major transformation, over 50 %, is to kinetic energy of the flowing water.

NOTES: These two examples illustrate the general finding that for flow in rather long small-diameter pipes, the frictional resistance at the pipe walls dominates, while kinetic energy and entrance effects can be ignored. On the contrary, in short large-diameter pipes, kinetic energy and pipe entrance losses should not be ignored and can actually dominate.

Figures 2.4 and 2.5 allow us to solve flow problems without trial and error whenever frictional losses, flow rate, or pipe length are the unknowns. Unfortunately, when the pipe diameter is unknown, one needs to resort to trial-and-error procedures, as we have seen in this problem. Laminar flow is the exception; for then no trial and error is needed.

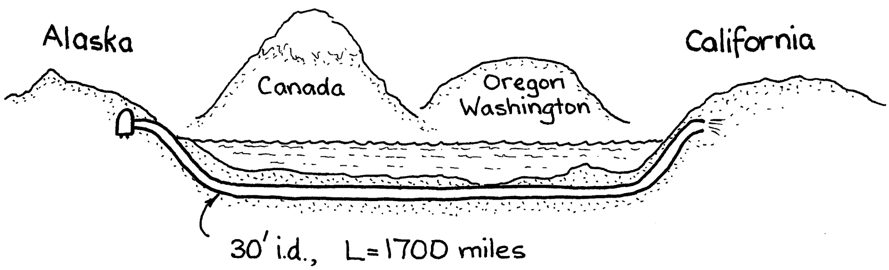
Problems on Incompressible Flow in Pipes

2.1. A 100 % efficient 1-kW pump–motor lifts water at 1.6 lit/s from a lake through 80 m of flexible hose into a tank 32 m up on a hill. A second pump with the same length of hose will be used to pump water from the lake at the same rate into a reservoir at lake level. What size of 100 %-efficient pump–motor is needed?

2.2. *California's savior.* "Big projects define a civilization. So why war, why not big projects," said Governor Walter Hickel of Alaska, referring to the following scheme.

California will run out of excess water by year 2000, Alaska has lots of excess, so why not build a suboceanic pipeline to pump freshwater from Alaska to California. It would be enormous, the cost would be very large (about \$150 billion), and it would take about 15 years to build, but it would supply about 10 % of the whole of California's needs.

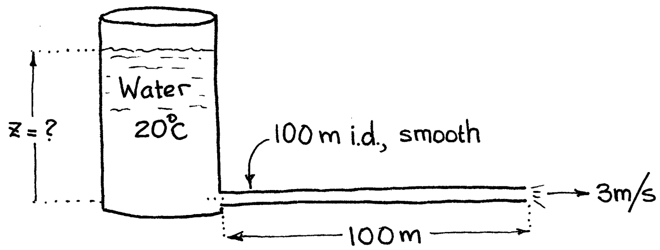
As proposed it would pump a trillion gallons of water per year through a 30-foot-diameter plastic pipe 1,700 miles long, buried in the ocean floor. What would be the pumping cost per year and per m^3 of water delivered if energy costs 3¢/kW · h?



2.3. *Acid pump.* 3 kg/s of 75 % H_2SO_4 ($\rho = 1{,}650$ kg/m^3, $\mu = 8.6 \times 10^{-3}$ kg/m s) is to be pumped from one open tank to another through 600 m (total equivalent length which includes bends, fittings, etc.) of 50-mm-i.d. pipe ($\epsilon = 0.046$ mm). The outlet of the first tank is 7 m below its surface, and the inlet of the second tank is 2 m below its surface and 13 m above the surface of the first tank. Find the power required for this operation if the pump–motor is 50 % efficient.

2.4. Water at 20 °C flows horizontally at 3 m/s from the base of a tank through 100 m of 100-mm-i.d. PVC plastic pipe. How high is the level of water in the tank?

(a) Solve ignoring kinetic energy and pipe entrance effects.
(b) Solve accounting for these effects.



2.5. Water at 20 °C is to flow out of a settling pond to a drain trench through 100-m equivalent length of 100-mm-i.d. galvanized pipe. The level of the pond is 10 m above the discharge end of the pipe. Find the flow rate of water in m^3/min.

2.6. Water at 20 °C flows from the base of a large storage tank through a smooth horizontal pipe (100 mm i.d., 1 km in length) at a velocity of 1 m/s. That is not fast enough. How much pipe must I saw off to get 2.5 times the velocity through the pipe? Ignore kinetic energy and entrance effects.

2.7. *The Sweat-hose.* We are testing a new type of soaker garden hose. It has porous walls through which water seeps. Calculate the seepage rate of this hose in liters/hour.

Data. The hose is 15 m long, 3 cm o.d., and 2 cm i.d. It is connected to a water faucet at one end, and it is sealed at the other end. It has 100 pores/cm^2 based on the outside of the hose surface. Each pore is tubular, 0.5 cm long and 10 μm in diameter. The water pressure at the faucet feeding the hose is 100 kPa above atmospheric pressure.

2.8. *The incredible osmotic pump.* Unbelievable though it may seem, if you could sink a pipe about 10 km down into the ocean capped with an ideal semipermeable membrane at its bottom, *fresh*water would pass through the membrane, rise up the pipe, and burble out above the surface of the ocean . . . all by itself and with no expenditure of energy! However, there are not many places in the world where one could go that far down into the ocean.

A somewhat more practical alternative for getting freshwater from the ocean requires sinking the pipe only about 250 m down. Freshwater would pass through the membrane into the pipe and stop at the 231-m depth. Then all you need to do is pump this freshwater to the surface.

What would be the pumping cost/m^3 of water to pump 1 lit/s of 20 °C freshwater up 240 m to an inland storage tank through a 50.8-mm-i.d. pipe 300 m in equivalent length?

2.9. *Taming the Mekong.* The giant Mekong River runs from the Himalayas through Southeast Asia, and the Mekong Development Project proposed that 35 dams be built along the river to tap its vast hydroelectric potential. One of these dams, the Pa Mong, is to stand 100 m high, be 25 % efficient in overall conversion to electricity, and have an annual output of 20 billion (US) kW h of electrical energy. Preliminary design by engineer Kumnith Ping suggests using 25 water intakes, each leading to a turbine located 100 m below the upstream reservoir through 200 m of concrete pipe. The guaranteed total flow rate of water to the turbines would be 14,800 m^3/s. Find the size of pipes needed. [See *National Geographic* **134**, 737 (1968).]

2.10. The *Alaska Pipeline*. Oil is pumped right across Alaska from Prudhoe Bay to Valdez in a 1.22-m-i.d. pipe, 1,270 km long at pressures as high as 8 MPa. The crude is at 50 °C, and the flow rate of the line is 2.2 m^3/s. Calculate:

(a) The theoretical number of pumping stations needed
(b) The kW rating of the pump–motor set

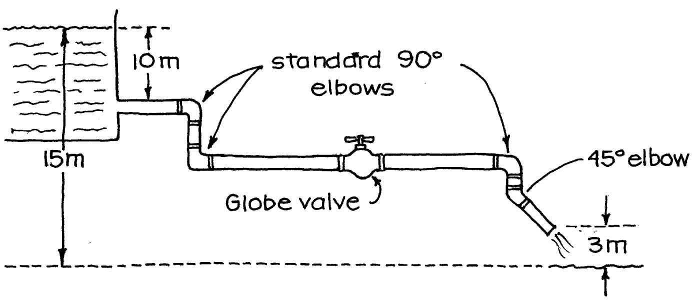
NOTE: This data is from *National Geographic* **150**, 684 (1976). In addition, estimate for crude oil at this temperature: $\rho = 910\ \mathrm{kg/m^3}$, $\mu = 6 \times 10^{-3}\ \mathrm{kg/m\ s}$.

Geothermal energy. Northwest Natural Gas Co. is doing exploratory work for a possible $50,000,000 geothermal development to supply Portland, Oregon, with hot water from Mt. Hood. Wells would be drilled on the slopes of Mt. Hood, 760 m above Portland to obtain 74 °C hot water at 1 atm. This would then be piped to Portland at 1.6 m^3/s in a 1.1-m-i.d. pipe 70 km in length.

2.11. Calculate the required size of motor and the pumping cost assuming 50 % efficiency for the pump–motor and 2¢/kW h for electricity, or else, if no pump is needed, find the water pressure in the pipeline at Portland.

2.12. We would like the water pressure at Portland to be 700 kPa, and this can be obtained without pumps by choosing the proper pipe size. What size would do the job? [See *Corvallis Gazette-Times* (September 28, 1977).]

2.13. Water at 10 °C is to flow out of a large tank through a 30-gauge 8-in. commercial steel (0.205-m-i.d.) piping system with valve open, as shown below. What length of pipe could be used while still maintaining a flow rate of 0.2 m^3/s?



2.14. In the above problem replace the globe valve with a gate valve. What maximum length can now be used?

2.15. *Swiss ingenuity.* The village of Schaffzell high in the Alps operates its own modest hydroelectric plant which produces electricity continuously whether it is used or not. When not needed the 240 kW of electricity runs a motor-turbine at 75 % efficiency which pumps 5 °C water at 0.1 m^3/s through an equivalent length of 780 m of pipe to a little lake located 153 m uphill.

When extra electricity is needed, the flow is reversed, and water runs downhill at the same flow rate, 0.1 m^3/s, from the little lake through the turbine to generate the needed electricity, again at 75 % efficiency. How much power can be generated with this downflow from the little lake?

2.16. What size of pipe was used by the village of Schaffzell in their system that pumps water up to the little lake (see previous problem)?

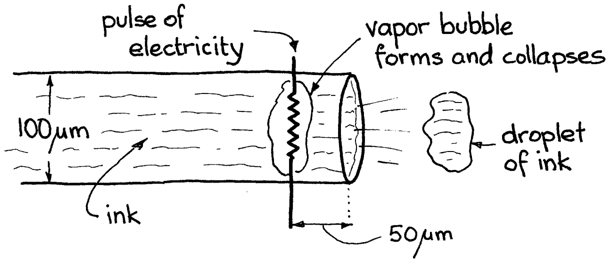
2.17. *Drip irrigation* is a means of getting water directly to growing plants with very little waste. One method uses a large-diameter polyethylene "mother" tube (10–15 mm i.d.) from which lead many small-diameter polyethylene feeder tubes, called drippers, which go directly to the individual plants. What length of 0.5-mm tubing will give a flow rate of 20 °C water of 6 lit/day if the pressure is 2 bar in the mother tube and 1 bar in the surroundings?

2.18. *More on drip irrigation.* Referring to the above problem, what diameter and length of dripper should we use if we want a flow rate of 4 lit/h to each plant and if the desired dripper length is to be somewhere between 0.5 and 1.5 m?
Data: The temperature is 20 °C; mother tube pressure = 200 kPa; ambient pressure = 100 kPa; drippers are manufactured in three diameters: 0.5 mm, 1.0 mm, and 1.5 mm.

2.19. *Jet printers.* The heart of jet printers for computers is a cluster of very thin ink-filled tubes, each about the thickness of a human hair or 100 μm. These spit out tiny droplets of ink to form desired characters on paper. About 4,000 drops/s are produced by each tube, and each drop has a mass of 15 μg and is ejected in about 3 μs.

These drops are ejected from the tube by passing pulses of electricity, about 15 μ**J**, through a heater located very close to the end of the tube, or 50 μm. This heat vaporizes a bit of ink which raises the pressure high enough to force a drop out of the tube. The vapor then condenses, and the process is repeated rapidly enough to generate these 4,000 drops/s.

Estimate the pressure in these vapor bubbles, and ignore possible surface tension effects. Accounting for surface tension effects would only change the answer by about 13 %.

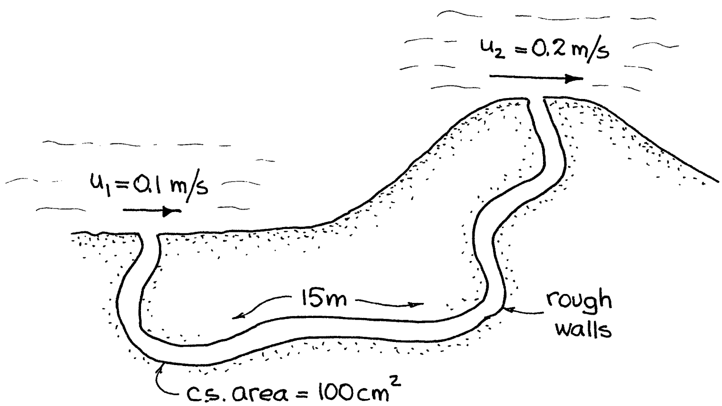


2.20. *Air-conditioned homes for little animals.* Prairie dogs, cute squirrel-like creatures minus bushy tails, live in large colonies in underground burrows connected by tunnels. A curious feature of these tunnels is that some of their exits are at ground level while others are in carefully built up mounds of earth. Is there a purpose to this?

Some speculate that these mounds which dot the colony act as lookout posts while their tunnels act as emergency getaways. You see, hawks love prairie dogs, but not vice versa. But do they need that many lookout stations, seeing that just about every second tunnel exit is in a raised mound?

Vogel and Bretz (*Science* **175**, 210 (1972)) propose that these mounds are part of the scheme for ventilating the burrows. Since air flows more slowly past a ground level exit than past one higher up, they feel that this should cause air to flow through the tunnel. Let us see if this explanation makes sense by considering the idealized situation sketched below and with values as shown. For this:

(a) Find the direction and velocity of air flow through the tunnel.
(b) Find the mean replacement time for the air in the tunnel.



2.21. *Oil pipeline*. A steel pipe 0.5 m i.d. is to carry 18,000 m^3/day of oil from an oil field to a refinery located 1,000 km away. The difference in elevation of the two ends of the line is negligible.

(a) Calculate the power required to overcome friction in the pipeline.
(b) Since the maximum allowable pressure in any section of the line is 4 MPa (about 40 atm), it will be necessary to have pumping stations at suitable intervals along the pipeline. What is the smallest number of pumping stations required?

Data: At the temperature involved the oil has a viscosity of 0.05 kg/m s and a density of 870 kg/m^3.

2.22. Williams Brothers–CMPS Engineers are presently designing a pipeline to transport 40 m^3/h of concentrated sulfuric acid from Mt. Isa to Phosphate Hill, both in Queensland, Australia. At the head of the pipeline will be one pumping station, the line will be constructed of soft iron because this material is only very slowly attacked by the concentrated acid (at most 6 mm in the estimated 20-year life of the line), and the line is to discharge into a storage tank.

(a) Determine the highest pressure expected in the pipeline. Where should this be?
(b) Determine the pressure at the pipeline discharge; note that the pressure should not go below atmospheric anywhere in the line.
(c) Calculate the pumping power requirement for this pipeline for a 33 % pump–motor efficiency. This low value is to account for corrosion of the pump, low winter temperatures, etc.

Data

- The pipe to be used is API standard seamless 6″ pipe; 219-mm o.d. and 9.52-mm wall thickness.
- The length of the pipe is 142 km.
- Elevations above sea level are as follows:

  360 m at Mt. Isa
  265 m at Phosphate Hill
  450 m at Thunder Ridge, the highest point in the line and 50 km from Mt. Isa

- Properties of 98 weight % H_2SO_4 at 15 °C:

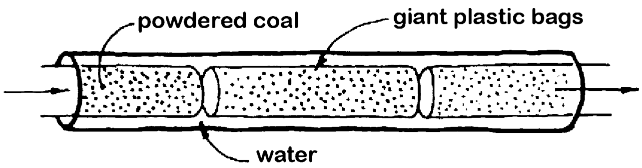
$\rho = 1{,}800$ kg/m^3, $\mu = 29 \times 10^{-3}$ kg/m-s.

2.23. Air at rest, at 20 °C and 100 kPa, is forced by means of a fan through a horizontal galvanized flue 1 m in diameter and 10 m long at a velocity of 10 m/s. What size motor should be used if the motor is 90 % efficient and the fan with its entrance resistance is 20 % efficient?

2.24. *Teaser.* Consider Problem 2.25 on the horizontal flue. If the flue were vertical and flow were upward, would you get a different answer? Calculate it please. Did you find that the size of motor needed changes from 3 to 8 kW? If so, then what would happen for downflow in the flue? Would you be generating 2 kW of useful work for free? Try to resolve the dilemma of a vertical flue.

2.25. *The Aquatrain.* Low-sulfur coal, highly prized by electric utilities because it needs no pretreatment for sulfur removal, is mined in Colorado. Los Angeles needs this coal, and the slurry pipeline is one way to transport this coal from mine to city. Some claim that this method of transportation is cheaper than by railroad. However, there is one overwhelming drawback to this method—the use of freshwater:

- Freshwater must be used in the slurry; otherwise salts will remain with the powdered coal to cause corrosion problems when the coal is burned.
- On arrival at Los Angeles the freshwater will be contaminated and not usable further.
- Freshwater is scarce in the Southwest United States and too valuable to be used this way.



W.R. Grace and Co. has an alternative proposal called the Aquatrain: The powdered coal is placed in giant cylindrical plastic bags 5 m long and 0.75 m in diameter, and the bags are then pumped along with water in a 0.91-m-i.d. pipeline all the way from their mine near Axial, Colorado (elevation = 2,000 m), to Los Angeles (at sea level), 2,000 km away. It is reported that 15 million tons of coal can be transported per year this way.

This method has another big advantage. At Glenwood Springs some very salty water enters the Colorado River, significantly raising the saline content of the river. By using this saline water in the pipeline instead of fresh river water, one can keep away 250,000 t/yr of salt from the river, thereby reducing the load on the desalination plants presently being built downstream on the Colorado River.

Suppose the pipeline operates 360 days in the year and that the plastic bags are carried down the river in neutral buoyancy and are in end-to-end contact along the whole length of the pipeline. For a preliminary rough estimate:

(a) Calculate the mean velocity of coal in the pipeline.
(b) Estimate the volumetric flow rate of water needed to transport this coal assuming first laminar flow, then turbulent flow.
(c) Determine whether the flow of water is laminar or turbulent.

(d) Find the theoretical pumping cost per ton to transport the coal assuming a straight rundown to Los Angeles in a commercial steel pipe and 2.5¢/kW h for electrical energy.

Information for this problem is from *Christian Science Monitor* (June 4, 1982).

2.26. Water at 20 °C flows horizontally at 3 m/s from the base of a tank through a Borda entrance and 100 m of 100-m-i.d. PVC pipe. How high is the level of the water in the tank?

(a) Solve ignoring kinetic energy effects and pope entrance effects.
(b) Solve accounting for these effects.

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