Chapter 2
Performance Measurement Using Data Envelopment Analysis (DEA)

2.1 DEA in Health Care

The 1980s brought many challenges to hospitals as they attempted to improve the efficiency of health care delivery through the fixed pricing mechanism of Diagnostic Related Groupings (DRGs). In the 1990s, the federal government extended the fixed pricing mechanism to physicians' services through Resource-Based Relative Value Scale (RBRVS). Enacted more recently, the Hospital Value-Based Purchasing (HVPB) program demands higher performance and quality of care and reduces payments for those providers that cannot achieve certain performance levels. Although these pricing mechanisms attempted to influence the utilization and quality of services by controlling the amount paid to hospitals and professionals, effective cost control must also be accompanied by a greater understanding of variation in physician practice behavior and development of treatment protocols for various diseases.

Although the origins of efficiency or benchmarking trace back to Farrell's (1957) study, theoretical development of the Data Envelopment Analysis (DEA) approach was started in 1978 by Charnes et al., who produced a measure of efficiency for decision making units (DMU). DEA is a nonparametric linear programming based technique that develops an efficiency frontier by optimizing the weighted output/input ratio of each provider, subject to the condition that this ratio can equal, but never exceed, unity for any other provider in the data set (Charnes et al. 1978). In health care, the first application of DEA dates to 1983, in the work of Nunamaker and Lewin, who measured routine nursing service efficiency. Since then DEA has been used widely in the assessment of hospital technical efficiency in the United States as well as around the world at different levels of decision making units. For example, Sherman (1984) was first in using DEA to evaluate overall hospital efficiency.

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2.2 Efficiency and Effectiveness Models

In order to understand the nature of the models that will be shown throughout the book, expanding on the definitions of the efficiency and effectiveness measures presented in Chap. 1 is in order. This will help not only in understanding the models developed here, but will also be useful for the curious reader examining other research in this area.

2.2.1 *Efficiency Measures*

As shown in formula (1.1), basic efficiency is a ratio of output over input. To improve efficiency one has to either increase the outputs or decrease the inputs. If both outputs and inputs increase, the rate of increase for outputs should be greater than the rate of increase for inputs. Conversely, if both outputs and inputs are decreasing, the rate of decrease for outputs should be lower than the rate of decrease for inputs. Another way to achieve higher efficiency is to introduce technological changes, or to reengineer service processes—lean management—which in turn may reduce inputs or increase the ability to produce more outputs (Ozcan 2009; pp. 121–123 and 222–224).

DEA models can generate new alternatives to improve performance compared to other techniques. Linear programming is the backbone methodology that is based on optimization platform. Hence, what differentiates the DEA from other methods is that it identifies the optimal ways of performance rather than the averages. In today's world, no health care institution can afford to be an average performer in a competitive health market.

Identification of optimal performance leads to benchmarking in a normative way. Using DEA, health care managers can not only identify top performers, but also discover the alternative ways to spur their health care organizations into becoming one of the best performers.

Since the seminal work of Charnes et al. (1978), DEA has been subject to countless research publications, conferences, dissertations, and applications within both the non-profit and for-profit sectors. Until now, the use of DEA within health care has been limited to conference sessions and research publications. Thus health care managers have not adopted DEA as a standard tool for benchmarking and decision-making. Part of this is due to its complicated formulation and to the failure of DEA specialists to adequately bridge the theory–practice gap. The aim of this book is to present DEA from a practical perspective, leaving the black box of sophisticated formulations in the background, so that health care managers, or those who are not trained in mathematical fields, can use Excel spreadsheet-based software, which they are familiar with, to analyze the performance of their organizations. The practical approach shown in this book will not only ease the fears of managers towards a new technique, but will also enable them to understand the pitfalls of the performed evaluations so they will feel confident in presenting, validating, and making decisions based on DEA results.

DEA is a comparative approach for identifying performance or its components by considering multiple resources that are used to achieve outputs or outcomes in health care organizations. These evaluations can be conducted not only at the organization level, but also in sub-units, such as departmental comparisons, where many areas of improvement in savings of particular input resources or strategies to augment the outputs can be identified.

In summary, DEA can help health care managers to:

1) assess their organization's relative performance,
2) identify top performance in the health care market, and
3) identify ways to improve their performance, if their organization is not one of the top performing organizations.

2.2.2 *Efficiency Evaluations Using DEA*

As described in Chap. 1, one of the major components of performance is efficiency. Efficiency is defined as the ratio of output(s) to input(s). Efficiency calculated by DEA is relative to these health organizations analyzed in a particular evaluation. The efficiency score for best performing (benchmark) health organizations in this evaluation would only represent the set of organizations considered in the analysis. Health care organizations identified as top performers in 1 year may not achieve this status if evaluations are repeated in subsequent years. Additionally, if more health organizations are included in another evaluation, their status may change since the relative performance will consider the newcomers. Although DEA can clearly identify improvement strategies for those non-top-performing health care organizations, further improvement of top performers depends on other factors, such as new technologies and other changes in the health service production process.

Efficiency attainment of health care organizations may also be the result of various factors, such as the price of the inputs or scope of the production process (scale) and other factors. Thus, it is prudent to understand types and components of efficiency in more depth. Major efficiency concepts can be described as technical, scale, price and allocative efficiency.

2.2.2.1 Technical Efficiency

Consider Hospital A treating brain tumors using the Gamma Knife technology. Hospital A can provide 80 procedures per month with 120 h of neurosurgeon time. Last month Hospital A produced 60 procedures while neurosurgeons were on the premises for 120 h. As shown in Table 2.1, the best achievable efficiency score for Hospital A is 0.667 (80/120), while due to their output of 60 procedures, their current efficiency score is 0.5 (60/120). We assess that Hospital A is operating at

Table 2.1 Technical efficiency

Hospital	Treatment capacity per month	Neurosurgeon time in hours	Current treatments per month	Best achievable efficiency	Efficiency
A	80	120	60	0.667	0.500

Table 2.2 Technical and scale efficiency

Hospital	Treatment capacity per month	Neurosurgeon time in hours	Current treatments per month	Best achievable efficiency	Efficiency	Scale efficiency
A	80	120	60	0.667	0.500	–
B	30	180	30	0.167	0.167	0.333

75 % (0.75 = 0.5/0.667) efficiency. This is called technical efficiency. In order for Hospital A to become technically efficient, it would have to increase its current output by 20 procedures per month.

2.2.2.2 Scale Efficiency

Also consider Hospital B, which does not have the Gamma Knife. Hence neurosurgeons at Hospital B remove tumors using the standard surgical technique (i.e., resection); to conduct 30 procedures a month, a neurosurgeon spends 180 h. The efficiency score of Hospital B is 0.167 (30/180). Compared to what Hospital A could ideally provide, Hospital B is at 25 % efficiency (0.25 = 0.167/0.667) in utilizing the neurosurgeon's time. If we consider only what Hospital A was able to achieve, Hospital B is operating at 33.3 % (0.333 = 0.167/0.5) relative efficiency in this comparison. If Hospital B used similar technology as Hospital A, then it could have produced 90 additional procedures given 180 h of neurosurgeon time; it would need to produce an additional 60 treatments to achieve the same efficiency level as Hospital A. The total difference between Hospital B's efficiency score and Hospital A's best achievable efficiency score is 0.5 (0.667 − 0.167). The difference between Hospital B's efficiency score and Hospital A's current efficiency score is 0.333 (0.5 − 0.167). Thus, we make the following observations (Table 2.2):

1) Hospital B is technically inefficient, illustrated by the component 0.167.
2) Hospital B is also scale inefficient, illustrated by the difference of 0.333.

The scale inefficiency can only be overcome by adopting the new technology or new service production processes. By contrast, the technical inefficiency is a managerial problem, where more outputs are required for a given level of resources.

We should also add that even though Hospital A produced 80 procedures a month, we cannot say that Hospital A is absolutely efficient unless it is compared to other hospitals with similar technology. However, at this point we know that differences in technology can create economies of scale in the health service production process. Using various DEA methods, health care managers can calculate both technical and scale efficiencies.

2.2.2.3 Price Efficiency

Efficiency evaluations can be assessed using price or cost information for inputs and/or outputs. For example, if the charge for the Gamma Knife procedure is $18,000 and for traditional surgery is $35,000, the resulting efficiency for Hospital A and Hospital B would be as follows:

Efficiency (A) = (60*18,000)/120 = $9,000.00
Efficiency (B) = (30*35,000)/180 = $5,833.33

Assuming that a neurosurgeon's time is reimbursed at the same rate for either traditional surgery or Gamma Knife procedures, Hospital A appears more efficient than Hospital B; however, the difference in this case is due to price of the output. If Hospital B used 120 h to produce half as many procedures (30) as Hospital A, its price efficiency score would have been $8,750, which clearly indicates the effect of the output price. If health care managers use the cost information in inputs or charge/revenue values for outputs, DEA can provide useful information for those inefficient health care organizations about potential reductions in input costs and needed revenue/charges for their outputs. In health care, although charges/revenues are generally negotiated with third-party payers, these evaluations would provide valuable information to health care managers while providing a basis for their negotiations.

2.2.2.4 Allocative Efficiency

When more than one input (and/or output) is part of health services delivery, health care managers are interested in the appropriate mix of the inputs to serve patients so the organization can achieve efficiency. Let us consider three group practices—A, B and C—where two types of professionals, physicians (P) and nurse practitioners (NP), provide health services. Furthermore assume that a physician's time costs $100/h, whereas a nurse practitioner's time costs $60/h. Let us suppose that Group Practice A employs three physicians and one nurse practitioner, that Group Practice B employs two physicians and two nurse practitioners, and that Group Practice C employs three physicians and three nurse practitioners. Assume that all group practices produce 500 equivalent patient visits during a week. Further assume that the practices are open for 8 h a day for 5 days a week (40 h). Input prices for the group practices are:

Inputs for Group Practice A = [(3*100) + (1*60)]*40 = $14,400
Inputs for Group Practice B = [(2*100) + (2*60)]*40 = $12,800
Inputs for Group Practice C = [(3*100) + (3*60)]*40 = $19,200

Table 2.3 Allocative efficiency

Group practice	Physicians ($100/h)	Nurse practitioners ($60/h)	Input: prices ($)	Output: visits	Efficiency	Allocative efficiency
A	3	1	14,400	500	$28.80	0.889
B	2	2	12,800	500	$25.60	1.000
C	3	3	19,200	500	$38.40	0.667

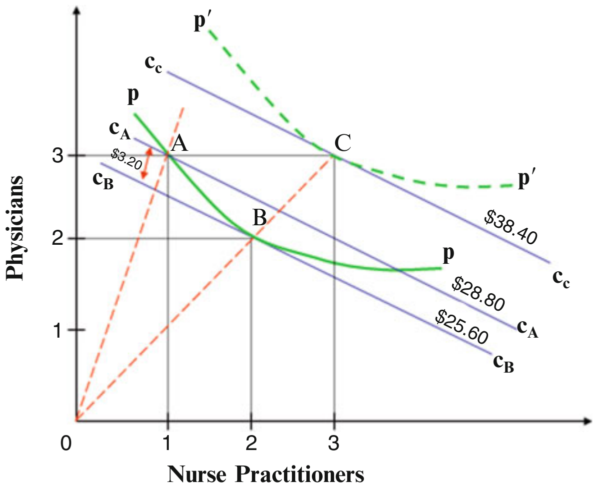


Fig. 2.1 Allocative efficiency

Since the output is the same, evaluating the input mix for these three group practices per visit yields the following ratios:

Group Practice A = 14,400/500 = $28.80
Group Practice B = 12,800/500 = $25.60
Group Practice C = 19,200/500 = $38.40

Table 2.3 summarizes these calculations as follows:

We can also illustrate these three group practices graphically on a production possibilities curves [pp] and [p′p′] shown in Fig. 2.1. Group Practices A and B lie on production possibilities curve [pp]. Because Group Practice C operates with a higher number of physicians and nurse practitioners when compared to Practices A and B, the production possibilities curve [p′p′] is in a higher position. Furthermore, the cost per case is shown using cost lines c_A ($28.80), c_B ($25.60), and c_C ($38.40), where Group Practice B is producing the services for $3.20 less per case compared to Group Practice A, as shown by cost line c_A. Furthermore, Group Practice B is producing the services for $12.80 less per case when compared to Group Practice C.

Comparing these costs, one can conclude that Group Practice A is 88.9 % (25.60/28.80) efficient compared to Group Practice B. Similarly, Group Practice C is

66.7 % (25.60/38.40) efficient compared to Group Practice B. In addition, Group Practice C is not only allocatively inefficient, but it is also technically inefficient, since it operates on a less efficient production possibilities curve [p′p′]. This example illustrates the concept of allocative efficiency, where various combinations (mixes) of inputs and their prices will yield different efficiencies.

We should also note that the contribution to outputs from each input might be different. In this example, while physicians can provide a full spectrum of services to the patients, nurse practitioners may be able to provide only a percentage, say, 70 %, due to their limited scope of practice. This raises the concern of whether using physicians and nurse practitioners as equal professions (e.g., substitutes for each other) in efficiency calculations is appropriate, or if a weighting scheme should be imposed to correctly assess the nurse practitioners' contributions to the total output. These weights are not readily available in most instances; however, DEA can estimate these weights in comparative evaluations.

2.2.3 Effectiveness Measures

Effectiveness in health care measured by outcomes or quality is of prime importance to many constituencies including patients, clinicians, administrators, and policy makers. Measuring the outcomes and quality is more problematic than efficiency measures. While inputs and outputs of the processes are relatively known to health care managers, multiple perspectives on outcomes and quality introduce additional practical difficulties in measurement. Although most hospitals report their inputs and outputs, until recently most outcome measures and quality measures, aside from mortality and morbidity statistics, were not reported on a systematic basis. The current quality reports from hospitals will be discussed in Chap. 7, and appropriate models will be developed to evaluate performance using both efficiency and effectiveness components.

2.3 Data Envelopment Analysis (DEA)

DEA essentially forms a frontier using the efficient organizations. To illustrate the conceptualization of the DEA frontier, consider the performance ratios of the first five hospitals from the example in Chap. 1. Here we consider two inputs, nursing hours and medical supplies, by dividing them by inpatient admissions; thus we obtain standardized usage of each input per inpatient admission.

As we observed before, H1 and H4 are efficient providers with their respective mix of use on these two inputs. We also know that H3 was an efficient provider from other dimensions of the performance. Graphically, as shown in Fig. 2.2 below, we can draw lines connecting these three efficient providers. As can be observed, there are two more hospitals, H5 and H8, that fall on the boundaries drawn by these

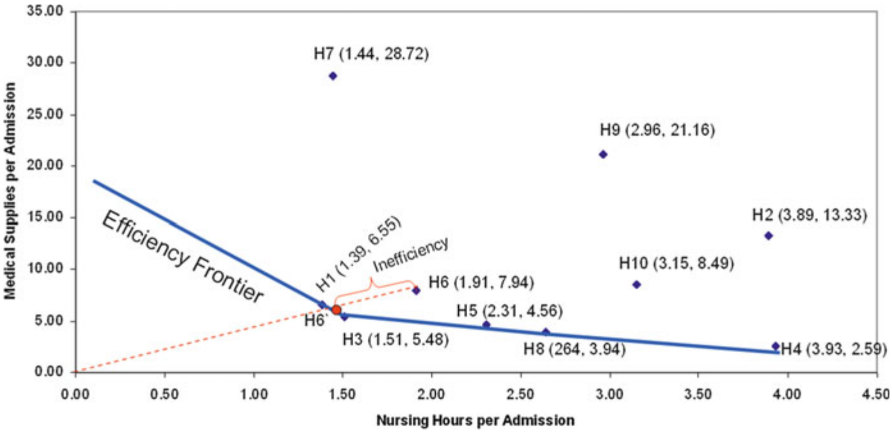


Fig. 2.2 Efficiency frontier

Table 2.4 Hospital performance ratios (bold numbers indicate best performance)

Provider ID	Nursing hours/ inpatient admissions	Medical supplies/ inpatient admissions
H1	**1.39**	6.55
H2	3.89	13.33
H3	1.51	5.48
H4	3.93	**2.59**
H5	2.31	4.56
H6	1.91	7.94
H7	1.44	28.72
H8	2.64	3.94
H9	2.96	21.16
H10	3.15	8.49

lines between H1 and H4. Hence, these lines connecting H1, H3, H4, H5, and H8 represent the efficiency frontier for this example; they are among the benchmark hospitals because these hospitals have the lowest combinations of the inputs when both ratios are taken into account (Table 2.4).

If we go back to the logic used to create the Table 1.4, where standardized efficiency ratios were calculated, we can observe that H1 and H4 received a standardized efficiency score of 1, and the other hospitals' standardized efficiency scores were somewhere between 0 and less than 1 from one dimension of the performance. Here also in DEA, the efficient hospitals will receive a score of 1 and those that are not on the efficiency frontier line will be less than 1 but greater than 0. Although we cannot explain why H5 and H8 are on the frontier line based on the graphic (due to its two dimensions), it suffices to say that they also have the lowest combinations of the inputs when both ratios are taken into account. Later when we employ all inputs and outputs into the model, we will demonstrate with DEA why H5 and H8 receive a score of 1 and are efficient.

Hospital 6 compared to H1 and H3 is considered inefficient using these input combinations. The amount of inefficiency can be understood by examining the dashed line from the origin to H6. In this dashed line, the amount of inefficiency exists from the point it crosses the efficiency frontier to H6. So, for H6 to become efficient, it must reduce usage of both inputs proportionately to reach point H6′. This is the normative power of DEA, where it can suggest how much improvement by each inefficient hospital is needed in each dimension of the resources.

2.4 Model Orientation

As in ratio analysis, when we calculate efficiency output over input and place emphasis on reduction of inputs to improve efficiency, in DEA analysis this is called *input orientation*. Input orientation assumes health care managers have more control over the inputs than over the number of patients arriving either for outpatient visit or admissions. Figure 2.2 is an example of an input-oriented model, where H2 must reduce its inputs to achieve efficiency.

However, the reverse argument can be made: that the health care managers, through marketing, referrals, or by other means (such as reputation on quality of services), can attract patients to their facilities. This means they can augment their outputs given their capacity of inputs to increase their organization's efficiency. Output augmentation to achieve efficiency in DEA is called *output orientation*. Output orientation will be further discussed in Sect. 2.11 below.

Various DEA models have been developed to use either the input or output orientation, and these models emphasize proportional reduction of excessive inputs (input slacks) or proportional augmentation of lacking outputs (output slacks). However, there are also models where health care managers can place emphasis on both output augmentation and input reduction at the same time by improving output slacks and decreasing input slacks. These slack-based models are also called the additive model or non-oriented models in DEA literature and software.

2.5 Basic Frontier Models

This book will consider various models that would be needed by health care managers. In this chapter, the basic frontier models will be presented. The subsequent chapters will introduce the extensions to these basic models for those specific management needs in evaluation of health care organizational performance.

There are various types of DEA models that may be used depending on the conditions of the problem on hand. Types of DEA models concerning a situation can be identified based on scale and orientation of the model. If one can assume that

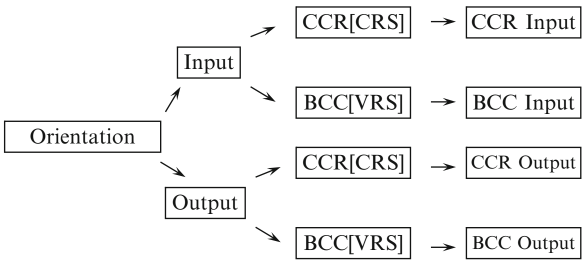


Fig. 2.3 DEA model classifications—basic envelopment models

economies of scale do not change as size of the service facility increases, then constant returns to scale (CRS) type DEA models are an appropriate choice.

The initial basic frontier model was developed by Charnes et al. (1978), known as the CCR model, using the initials of the developers' last names, but now is widely known as the *constant returns to scale* (CRS) model. The other basic frontier model, known as BCC (Banker, Charnes and Cooper), followed CCR as the *variable returns to scale* (VRS) model, though in this model one cannot assume that economies of scale do not change as size of the service facility increases. Figure 2.3 shows the basic DEA models based on returns to scale and model orientation. These models will be referred as "Basic Envelopment Models."

2.6 Decision Making Unit (DMU)

Organizations subject to evaluation in the DEA literature are called Decision Making Units (DMUs). For example, the hospitals, nursing homes, group practices, and other facilities that are evaluated for performance using DEA are considered as DMUs by many popular DEA software programs.

2.7 Constant Returns to Scale CCR [CRS] Model

The essence of the constant returns to scale model is the maximization of the ratio of weighted multiple outputs to weighted multiple inputs. Any health care organization compared to others should have an efficiency score of 1 or less, with either 0 or positive weights assigned to the inputs and outputs.

Here, the calculation of DEA efficiency scores are briefly explained using mathematical notations (adapted from Cooper et al. 2007, p. 23). The efficiency scores (θ_o) for a group of peer DMUs ($j = 1 \ldots n$) are computed for the selected outputs (y_{rj}, $r = 1, \ldots, s$) and inputs (x_{ij}, $i = 1, \ldots, m$) using the following fractional programming formula:

$$Maximize\ \ \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \tag{2.1}$$

$$subject\ \ to\ \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \tag{2.2}$$

$$u_r, v_i \geq 0 \quad for\ \ all\ \ r\ \ and\ \ i.$$

In this formulation, the weights for the outputs and inputs, respectively, are u_r and v_i, and "o" denotes a focal DMU (i.e., each hospital, in turn, becomes a focal hospital when its efficiency score is being computed relative to others). Note that the input and output values, as well as all weights, are assumed by the formulation to be greater than zero. The weights u_r and v_i for each DMU are determined entirely from the output and input data of all DMUs in the peer group of data. Therefore, the weights used for each DMU are those that maximize the focal DMU's efficiency score. In order to solve the fractional program described above, it needs to be converted to a linear programming formulation for easier solution.

Since the focus of this book is not on the mathematical aspects of DEA, an interested reader is referred to the appendix at the end of this chapter for more detail on how the above equations are algebraically converted to a linear programming formulation. Other DEA books listed in the references may also be consulted for an in-depth exposure.

In summary, the DEA identifies a group of optimally performing hospitals that are defined as efficient and assigns them a score of one. These efficient hospitals are then used to create an "efficiency frontier" or "data envelope" against which all other hospitals are compared. In sum, hospitals that require relatively more weighted inputs to produce weighted outputs or, alternatively, produce less weighted output per weighted inputs than do hospitals on the efficiency frontier, are considered technically inefficient. They are given efficiency scores of strictly less than 1, but greater than zero.

Although DEA is a powerful optimization technique to assess the performance of each hospital, it has certain limitations which need to be addressed. When one has to deal with significantly large numbers of inputs and outputs in the service production process and a small number of organizations are under evaluation, discriminatory power of the DEA will be limited. However, the analyst could overcome this limitation by only including those factors (input and output) which provide the essential components of the service production process, thus not distorting the outcome of the DEA results. This is generally done by eliminating one of pair of factors that are strongly positively correlated with each other.

Cooper et al. (2007, p. 116) suggest that in order to have adequate numbers of degrees of freedom (adequate discriminatory power for the DEA model), the "*n*" (number of DMUs) should exceed the number of inputs (*m*) and outputs (*s*) by several times. More specifically, they suggest a rule of thumb formula that "n" should be greater than $max\{m*s, 3*(m+s)\}$.

2.8 Example for Input-Oriented CCR [CRS] DEA Model

Consider again the sample data presented in Chap. 1 with ten hospitals, two inputs, and two outputs. Table 2.5 depicts the inputs and outputs according to formulation discussions presented above. As one can observe, peer hospitals ($j = 1, \ldots, 10$) are listed for the selected inputs (x_{ij}, $i = 1, 2$) and outputs (y_{rj}, $r = 1, 2$).

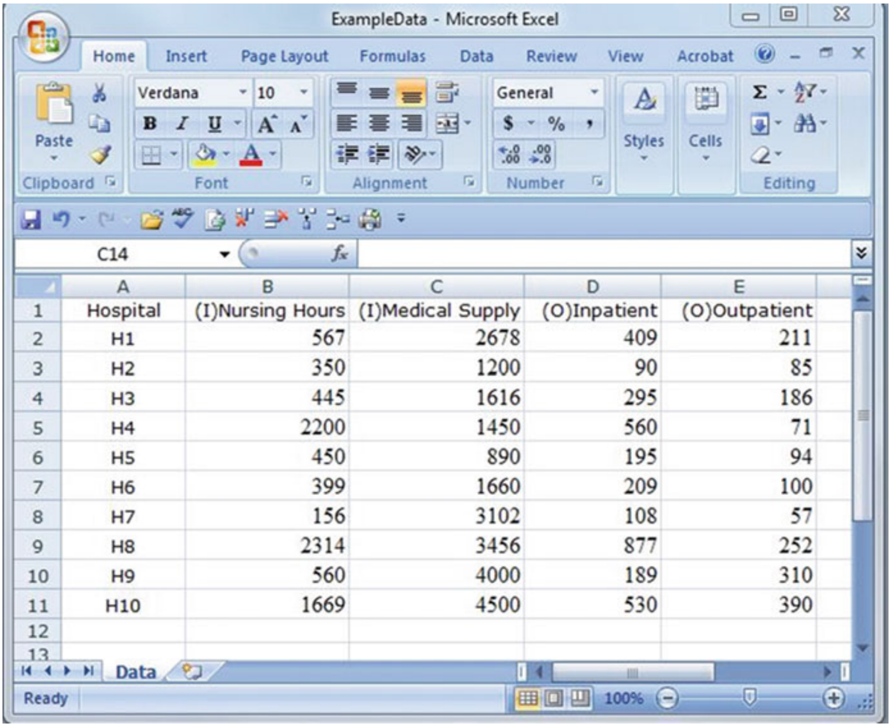
The next step is to enter this information into the DEA Solver software, which runs on an Excel macro-based platform. For information regarding installation of the learning version (LV) of the software and other relevant details, readers are referred to the "Running the DEA Solver-LV" section of the book at the end. The Excel sheet containing the data for DEA analysis is named "Data" and is shown in Fig. 2.4 below.

Please note that the first column is recognized as the DMU identifier (e.g., hospital), followed by two columns of inputs and two columns of outputs. Also note that the top row identifies input and output names, and designation of either "(I)" for input or "(O)" for output precedes the variable names to identify their type (in this case either input or output, however, this will be expanded to other type of variables in ensuing chapters). Before running the model, the Excel file shown in Fig. 2.4 should be saved in a directory on the hard drive.

To run the model, double click on DEA Solver-LV (from hard drive directory), which should bring an empty Excel sheet, as shown in Fig. 2.5 Part A, where the user should activate the macro by clicking on "Options" in the Security Warning

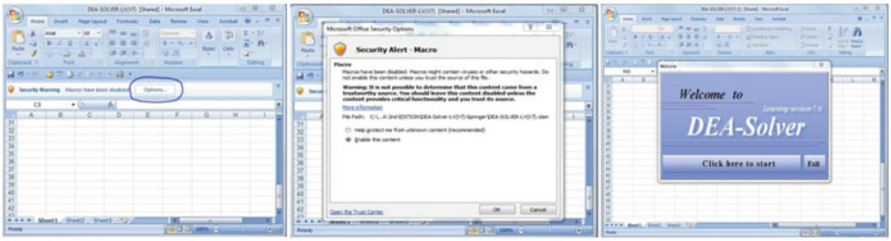
Table 2.5 Hospital inputs and outputs

	Inputs		Outputs	
Hospitals j	Nursing hours x_{1j}	Medical supplies ($) x_{2j}	Inpatient admissions y_{1j}	Outpatient visits y_{2j}
1	567	2,678	409	211
2	350	1,200	90	85
3	445	1,616	295	186
4	2,200	1,450	560	71
5	450	890	195	94
6	399	1,660	209	100
7	156	3,102	108	57
8	2,314	3,456	877	252
9	560	4,000	189	310
10	1,669	4,500	530	390



Hospital	(I)Nursing Hours	(I)Medical Supply	(O)Inpatient	(O)Outpatient
H1	567	2678	409	211
H2	350	1200	90	85
H3	445	1616	295	186
H4	2200	1450	560	71
H5	450	890	195	94
H6	399	1660	209	100
H7	156	3102	108	57
H8	2314	3456	877	252
H9	560	4000	189	310
H10	1669	4500	530	390

Fig. 2.4 DEA solver-LV data setup

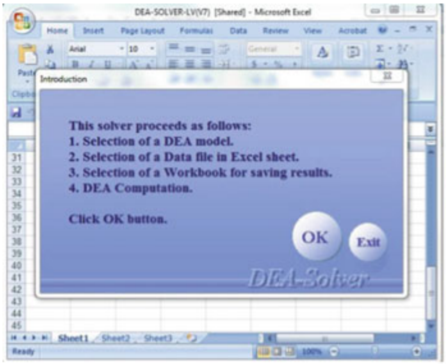


Part A Part B Part C

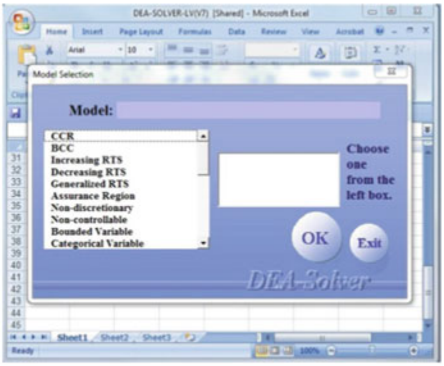
Fig. 2.5 DEA solver-LV macro activation

banner (shown in a circle). When the "Security Alert – Macro" warning pops up, select the radio button "Enable the content" and click "OK" to activate the "DEA Solver-LV" software, Fig. 2.5 Part B.

Once the "DEA Solver" is clicked to start, Fig. 2.5 Part C, a new popup menu appears with four-step instructions as depicted in Fig. 2.6 Part A. To run the CRS model, choose the "CCR Model" option, Fig. 2.6 Part B. This will prompt another instruction to select either CCR-I (input orientation) or CCR-O (output orientation),



Part A



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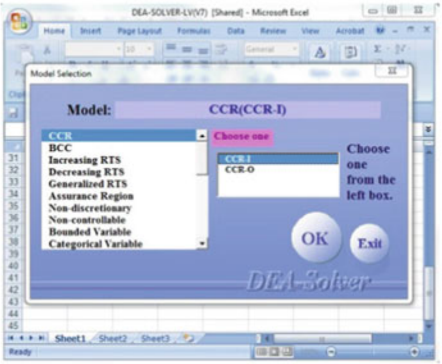

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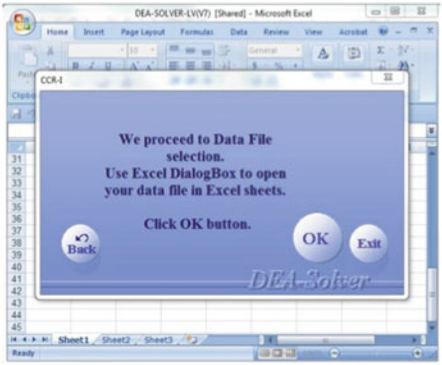

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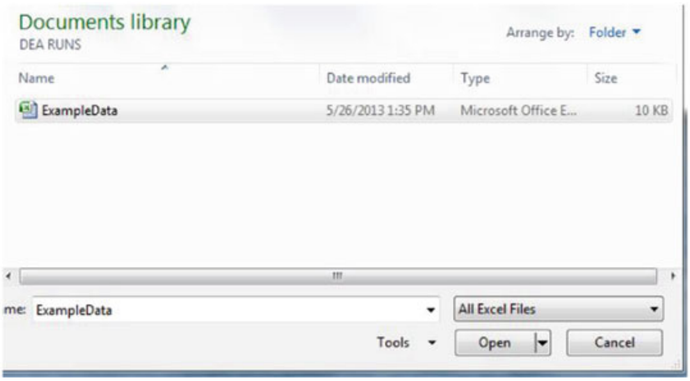
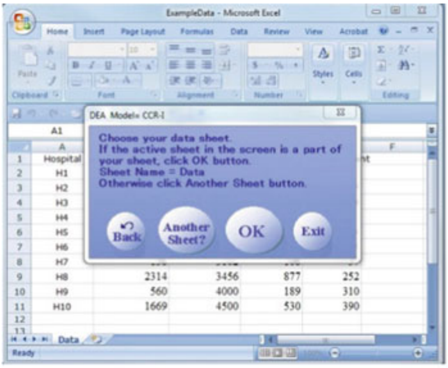
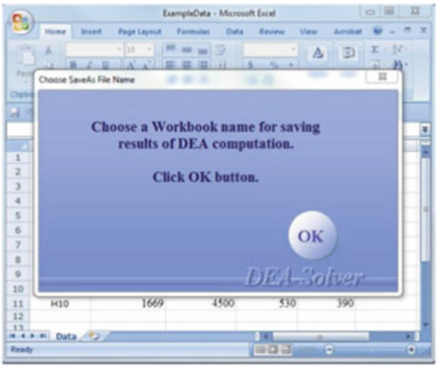


Part E

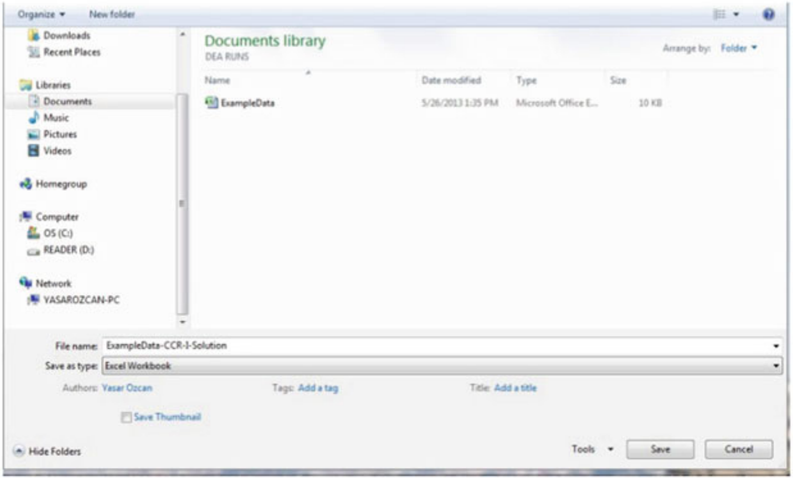
Fig. 2.6 Illustration of DEA solver-LV CCR-I model example run



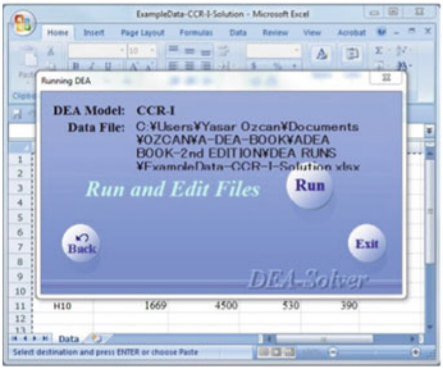
Part F



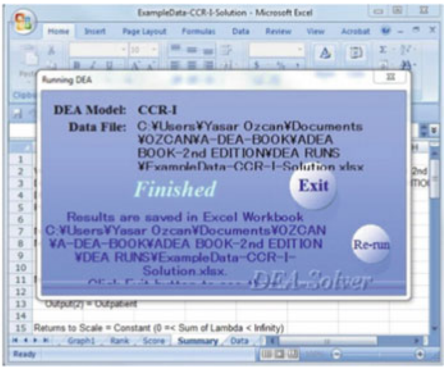
Part G



Part H

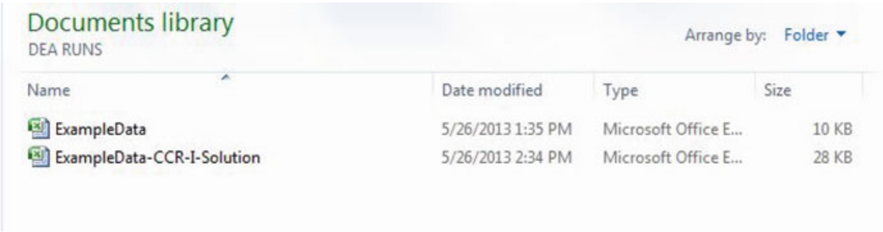


Part I



Part J

Fig. 2.6 (continued)



Part K

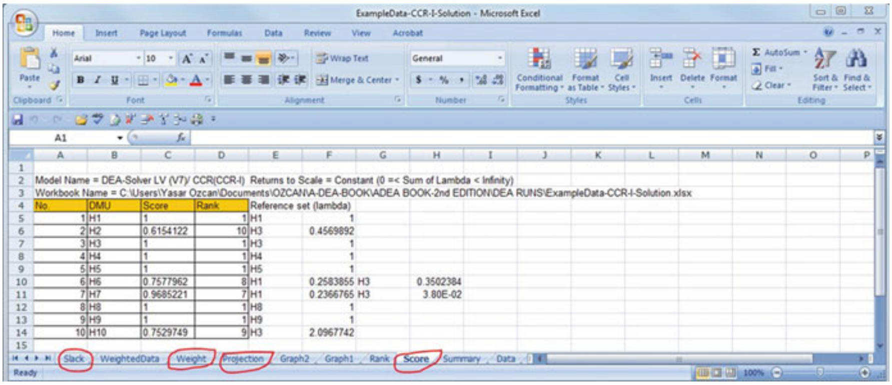
Fig. 2.6 (continued)

Fig. 2.6 Part C. After selecting the CCR-I option, another popup menu will guide the user for the data file selection, Fig. 2.6 Part D. In this illustration, a file named "ExampleData," which was previously saved on "Documents Library," is selected and clicked "open," Fig. 2.6 Part E. Next, the data file will appear in an Excel sheet and a new popup menu will instruct the user to check whether this is the data set that was intended for use in the analysis, Fig. 2.6 Part F. If the data set is the correct one, click "OK" to move to the next menu, which will guide the user to select a workbook name to save the results of DEA computations, Fig. 2.6 Part G. It is recommended that the user save the results file (containing DEA computations) in the same directory where the raw data file is stored and include in the file name the same startup with extension indicating the model type and orientation. For the example shown in Fig. 2.6 Part H, "ExampleData-CCR-I-Solutions" was used as the name to differentiate the solutions from raw data as well as from potential other model runs. Clicking "Save" to create results files will prompt yet another popup menu requiring a click on the "Run" button, Fig. 2.6 Part I. After the program runs, a final popup menu will indicate the "Finish" of the program, as depicted in Part J of Fig. 2.6.

If any of the steps are not followed correctly, the user should start from the beginning of the process to complete computations without any difficulty. The results of the computations will be active on screen for observations, or can be examined later by opening the saved result file (in this case, "ExampleData-CCR-I-Solution") as shown in Fig. 2.6 Part K.

At this stage the user or health care manager can observe many results of the model to identify not only the benchmark hospitals, but also improvement strategies for those hospitals that are currently inefficient.

The results are organized in various Excel worksheets (tabs), as shown at the bottom banner in Fig. 2.7. These worksheets include results of efficiency analysis in the "Score" worksheet, target inputs and outputs in the "Projection" worksheet, and the amount of inefficiencies (slacks) in the "Slack" worksheet. Next we will discuss the results from each of these worksheets.



ExampleData-CCR-I-Solution - Microsoft Excel

Model Name = DEA-Solver LV (V7)/ CCR(CCR-I) Returns to Scale = Constant (0 =< Sum of Lambda < Infinity)
Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\DEA RUNS\ExampleData-CCR-I-Solution.xlsx

No.	DMU	Score	Rank	Reference set (lambda)			
1	H1	1	1	H1	1		
2	H2	0.6154122	10	H3	0.4569892		
3	H3	1	1	H3	1		
4	H4	1	1	H4	1		
5	H5	1	1	H5	1		
6	H6	0.7577962	8	H1	0.2583855	H3	0.3502384
7	H7	0.9685221	7	H1	0.2366765	H3	3.80E-02
8	H8	1	1	H8	1		
9	H9	1	1	H9	1		
10	H10	0.7529749	9	H3	2.0967742		

Slack WeightedData Weight Projection Graph2 Graph1 Rank Score Summary Data

Fig. 2.7 Results of CCR input-oriented model solution

Fig. 2.8 Efficiency report for input-oriented CCR model

CCR-I

DMU	Score
H1	1
H2	0.6154122
H3	1
H4	1
H5	1
H6	0.7577962
H7	0.9685221
H8	1
H9	1
H10	0.7529749

2.9 Interpretation of the Results

Figure 2.8 depicts the abridged version of the efficiency report, where efficiency scores of all ten hospitals are reported. This two-input and two-output model shows that six of the ten hospitals are efficient using these four dimensions. There is no surprise that H1, H3, H4, and H9 all received a score of 1 and are considered efficient. Furthermore, we observe that the efficiency of two additional hospitals, H5 and H8, could not be determined in ratio-based analysis shown in Chap. 1. However, with DEA using multiple inputs and outputs at the same time, we are able to discover them.

Model Name = DEA-Solver LV (V7)/ CCR(CCR-I) Returns to Scale = Constant (0 =< Sum of Lambda < Inf

Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\

No.	DMU	Score	Excess Nursing Hours S-(1)	Excess Medical Supply S-(2)	Shortage Inpatient S+(1)	Shortage Outpatient S+(2)
1	H1	1	0	0	0	0
2	H2	0.6154122	12.03405018	0	44.81182796	0
3	H3	1	0	0	0	0
4	H4	1	0	0	0	0
5	H5	1	0	0	0	0
6	H6	0.7577962	0	0	0	19.663685
7	H7	0.9685221	0	2309.186459	0	0
8	H8	1	0	0	0	0
9	H9	1	0	0	0	0
10	H10	0.7529749	323.6506093	0	88.5483871	0

Slack | WeightedData | Weight | Projection | Graph2 | Graph1 | Rank | Score | Summary

Fig. 2.9 Input and output slacks for input-oriented CCR model

2.9.1 Efficiency and Inefficiency

Hospitals H2, H6, H7, and H10 have scores of less than 1 but greater than 0, and thus they are identified as inefficient. These hospitals can improve their efficiency, or reduce their inefficiencies proportionately, by reducing their inputs (since we run an input-oriented model). For example, H2 can improve its efficiency by reducing certain inputs up to 38.5 % (1.0 − 0.6154122). Similarly, H6 and H10 can do so with approximately 25 % input reduction. However, H7 is closer to an efficiency frontier and needs only a 3.2 % reduction in resources.

This raises the question: which inputs need to be reduced by calculated proportions? These input reductions (or output augmentations in some cases) are called total inefficiencies which comprise not only the amount of proportional reductions, but also an amount called "Slack" for those hospitals that cannot reach their efficiency targets (at frontier) despite the proportional reductions.

2.9.2 Slacks

Figure 2.9 comes from the "Slack" worksheet of the DEA computation results. Mathematical derivation of these slacks is presented in Appendix B of this chapter. Here, we observe that none of the efficient hospitals have any slacks. Slacks exist only for those hospitals identified as inefficient. However, slacks represent only the leftover portions of inefficiencies; after proportional reductions in inputs or increases in outputs, if a DMU cannot reach the efficiency frontier (to its efficient target), slacks are needed to push the DMU to the frontier (target).

It is interesting to note that H2 would be required to reduce its nursing hours by approximately 12 h. However, despite the reduction in this input, it would not achieve efficiency. No other input can be reduced, thus to achieve efficiency, H2 should also augment its inpatient admission by 44.8 %. A similar situation in a different magnitude exists for H10. On the other hand, H6 cannot reduce any inputs, but must augment outpatient visits by 19.7 % or 20 visits. Lastly, H7 should spend $2,309.19 less for medical supplies. Please note that these calculations are the results of Models 2 and 4 executed in succession or Model 5, as explained in the appendices at the end of the chapter.

2.9.3 Efficiency Targets for Inputs and Outputs

We can summarize the efficiency targets by examining the "Projection" worksheet. Here, for each hospital, target input and output levels are prescribed. These targets are the results of respective slack values added to proportional reduction amounts. To calculate the target values for inputs, the input value is multiplied with an optimal efficiency score, and then slack amounts are subtracted from this amount. For detailed formulations of these calculations, the reader is referred to Appendix B, Part 3. Figure 2.10 displays these target values. As the reader can observe, the target values for efficient hospitals are equivalent to their original input and output values.

However, for the inefficient DMUs, the targets for input variables ($\widehat{x}_{io}$) in the CCR input-oriented DEA model will comprise proportional reduction in the input variables by the efficiency score of the DMU minus the slack value, if any, given by the formula:

$$\widehat{x}_{io} = \theta^* x_{io} - s_i^{-*} \quad i = 1, \ldots m \tag{2.3}$$

For example, the target calculations for Nursing Hours (NH) and Medical Supply (MS) inputs of Hospital 2 are calculated as follows:

$$\widehat{x}_{NH,H2} = \theta^* x_{NH,H2} - s_{NH}^{-*}$$

$$\widehat{x}_{NH,H2} = 0.61541 * 350 - 12.03405$$

$$\widehat{x}_{NH,H2} = 203.36022$$

where 0.61541 comes from Fig. 2.8, 350 from Fig. 2.4, and 12.03405 from Fig. 2.9. The reader can confirm the results with Fig. 2.10. Similarly, target calculation of Medical Supply for H2 is:

$$\widehat{x}_{MS,H2} = \theta^* x_{MS,H2} - s_{MS}^{-*}$$

	A	B	C	D	E	F
4	No.	DMU	Score			
5		I/O	Data	Projection	Difference	%
6	1	H1	1			
7		Nursing Hours	567	567	0	0.00%
8		Medical Supply	2678	2678	0	0.00%
9		Inpatient	409	409	0	0.00%
10		Outpatient	211	211	0	0.00%
11	2	H2	0.6154122			
12		Nursing Hours	350	203.36022	-146.6398	-41.90%
13		Medical Supply	1200	738.49462	-461.5054	-38.46%
14		Inpatient	90	134.81183	44.811828	49.79%
15		Outpatient	85	85	0	0.00%
16	3	H3	1			
17		Nursing Hours	445	445	0	0.00%
18		Medical Supply	1616	1616	0	0.00%
19		Inpatient	295	295	0	0.00%
20		Outpatient	186	186	0	0.00%
21	4	H4	1			
22		Nursing Hours	2200	2200	0	0.00%
23		Medical Supply	1450	1450	0	0.00%
24		Inpatient	560	560	0	0.00%
25		Outpatient	71	71	0	0.00%
26	5	H5	1			
27		Nursing Hours	450	450	0	0.00%
28		Medical Supply	890	890	0	0.00%
29		Inpatient	195	195	0	0.00%
30		Outpatient	94	94	0	0.00%
31	6	H6	0.7577962			
32		Nursing Hours	399	302.36067	-96.63933	-24.22%
33		Medical Supply	1660	1257.9416	-402.0584	-24.22%
34		Inpatient	209	209	0	0.00%
35		Outpatient	100	119.66368	19.663685	19.66%
36	7	H7	0.9685221			
37		Nursing Hours	156	151.08945	-4.91055	-3.15%
38		Medical Supply	3102	695.16914	-2406.831	-77.59%
39		Inpatient	108	108	0	0.00%
40		Outpatient	57	57	0	0.00%
41	8	H8	1			
42		Nursing Hours	2314	2314	0	0.00%
43		Medical Supply	3456	3456	0	0.00%
44		Inpatient	877	877	0	0.00%
45		Outpatient	252	252	0	0.00%
46	9	H9	1			
47		Nursing Hours	560	560	0	0.00%
48		Medical Supply	4000	4000	0	0.00%
49		Inpatient	189	189	0	0.00%
50		Outpatient	310	310	0	0.00%
51	10	H10	0.7529749			
52		Nursing Hours	1669	933.06452	-735.9355	-44.09%
53		Medical Supply	4500	3388.3871	-1111.613	-24.70%
54		Inpatient	530	618.54839	88.548387	16.71%
55		Outpatient	390	390	0	0.00%

Slack / WeightedData / Weight / Projection / Graph2 / Graph1

Fig. 2.10 Input and output efficient targets for input-oriented CCR model

$$\widehat{x}_{MS,H2} = 0.61541 * 1200 - 0$$

$$\widehat{x}_{MS,H2} = 738.49462$$

Again, the reader can confirm the result from Fig. 2.10.

In an input-oriented model, efficient output targets are calculated as:

$$\widehat{y}_{ro} = y_{ro} + s_i^{+*} \quad r = 1, \ldots s \tag{2.4}$$

In our ongoing example with H2, Inpatient Admissions (IA) and Outpatient Visits (OV) can be calculated as:

$$\widehat{y}_{IA,H2} = y_{IA,H2} + s_{IA}^{+*} \qquad \widehat{y}_{OV,H2} = y_{OV,H2} + s_{OV}^{+*}$$

$$\widehat{y}_{IA,H2} = 90 + 44.81183 \qquad \widehat{y}_{OV,H2} = 85 + 0$$

$$\widehat{y}_{IA,H2} = 134.81183 \qquad \widehat{y}_{OV,H2} = 85.00$$

The reader can confirm these results from Fig. 2.10 for Hospital 2. The other inefficient hospitals' targets are calculated in the same manner.

2.10 Input-Oriented Model Benchmarks

The "Score" worksheet in Fig. 2.11 provides benchmark results shown in columns E through H. In Chap. 4 we will discuss the use of the information presented here in more detail when more foundational material has been presented. However, here

	A	B	C	D	E	F	G	H	I
1									
2	DEA-Solver LV (V7)/ CCR(CCR-I)			Returns to Scale = Constant (0 =< Sum of Lambda < Infinity)					
3	Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\DEA RU								
4	No.	DMU	Score	Rank	Reference set (lambda)				
5	1	H1	1	1	H1	1			
6	2	H2	0.6154122	10	H3	0.4569892			
7	3	H3	1	1	H3	1			
8	4	H4	1	1	H4	1			
9	5	H5	1	1	H5	1			
10	6	H6	0.7577962	8	H1	0.2583855	H3	0.35023841	
11	7	H7	0.9685221	7	H1	0.2366765	H3	0.03796370	
12	8	H8	1	1	H8	1			
13	9	H9	1	1	H9	1			
14	10	H10	0.7529749	9	H3	2.0967742			
15									

Slack | WeightedData | Weight | Projection | Graph2 | Graph1 | Rank | Score | Summary | Data

Fig. 2.11 Benchmarks for input-oriented CCR model we will explain the benchmark portion of the information presented. The "benchmarks" are created through the DEA computations.

Here, health care managers whose hospital is inefficient can observe the benchmark hospitals to which they need to catch up.

Of course, efficient hospitals may consider themselves to be their own "benchmarks." So, the benchmark for H1 is H1, for H3 is H3, and so on. However, for inefficient hospitals, their benchmarks are one or many of the efficient hospitals. For example, a benchmark for H2 and H10 is H3 (observe that H3 is efficient). A benchmark for H6 and H7 are two hospitals, H1 and H3. This means, to become efficient, H6 and H7 must use a combination from both H1 and H3 in this case (a virtual hospital) to become efficient. How much of H1 and how much of H3 (what combination) are calculated to achieve efficiency and reported next to each benchmark hospital? These are λ (lambda) weights obtained from the dual version of the linear program that is solved to estimate these values. Further formulation details are provided in Appendix A at the end of this chapter. For example, H7 will attempt to become like H1 more than H3 as observed from respective λ weights of H1 and H3 ($\lambda_1 = 0.237$ vs. $\lambda_3 = 0.038$).

2.11 Output-Oriented Models

The essence of output orientation comes from how we look at the efficiency ratios. When we illustrated the input orientation we used the ratios in which inputs were divided by outputs. Hence we can do the opposite by dividing outputs by inputs to create reciprocal ratios. Using the same inputs and outputs from Table 1.2 from Chap. 1, we can calculate these mirror ratios as shown in Table 2.6 below. The first two columns show two different outputs, inpatient admissions and outpatient visits, being divided by the same input, nursing hours. The higher ratio values here would mean better performance for the hospitals.

Table 2.6 Hospital performance ratios

Provider ID	Inpatient admissions/ nursing hours	Outpatient visit/ nursing hours
H1	0.72	0.37
H2	0.26	0.24
H3	0.66	0.42
H4	0.25	0.03
H5	0.43	0.21
H6	0.52	0.25
H7	0.69	0.37
H8	0.38	0.11
H9	0.34	0.55
H10	0.32	0.23

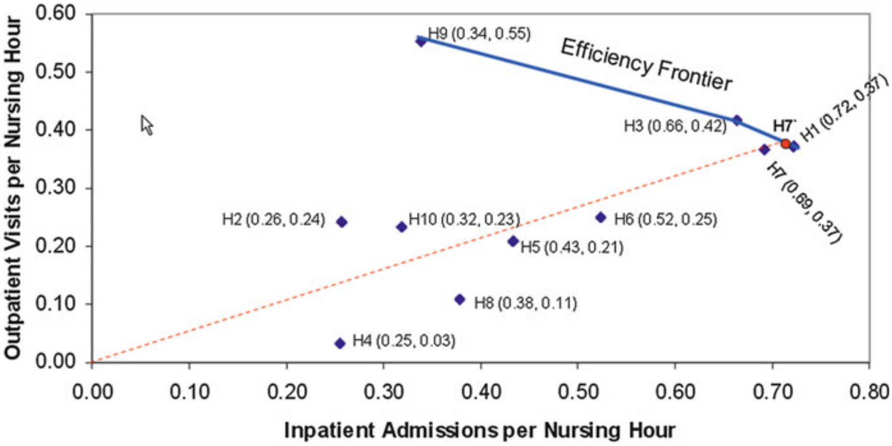


Fig. 2.12 Efficiency frontier for output-oriented model

As can be observed from the first column, H1 has the highest inpatient admissions per nursing hour compared to other providers. However, H9 has the highest outpatient visits per nursing hour as displayed in the second column.

A graphical view of these measures is shown in Fig. 2.12, where H1, H3, and H9 have the highest combination of these ratios when considered together. Here, no other hospital can generate more outputs using the nursing hours as input. However, when other inputs are included in the model using DEA, we may discover other hospitals joining the efficiency frontier.

The reader should also note that H7, an inefficient hospital, can reach this output-oriented frontier by increasing its inpatient admissions and outpatient visits along the direction of the dashed line to H7‘. The distance given by H7‘–H7 defines the amount of inefficiency for H7.

2.12 Output-Oriented CCR [CRS] DEA Model

Using the similar steps in Sect. 2.8, this time we will select “CCR-O” Output-Oriented model from the popup menu as shown in Fig. 2.13.

Again following the similar steps in Sect. 2.8, we get the results shown in Fig. 2.14, which is similar to Fig. 2.7; however, the results are reported as output orientation.

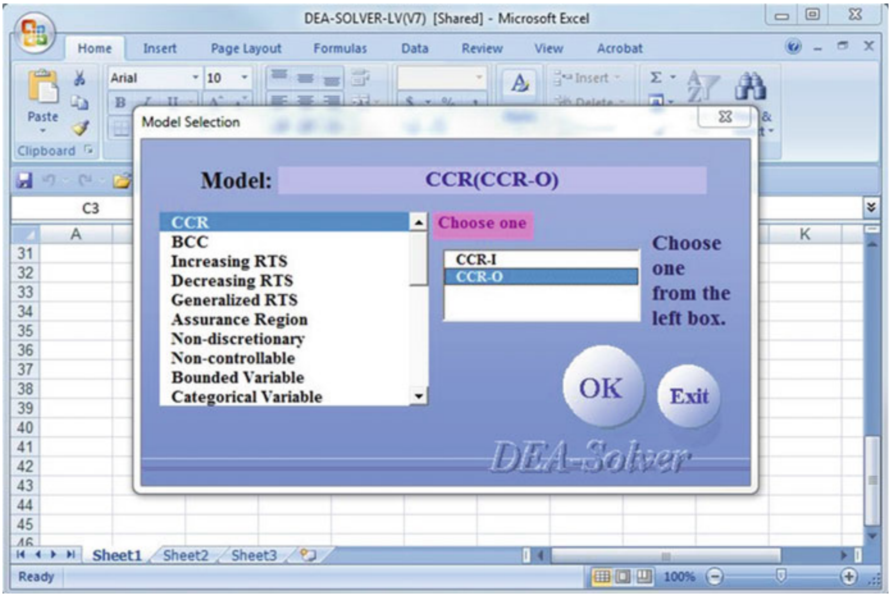
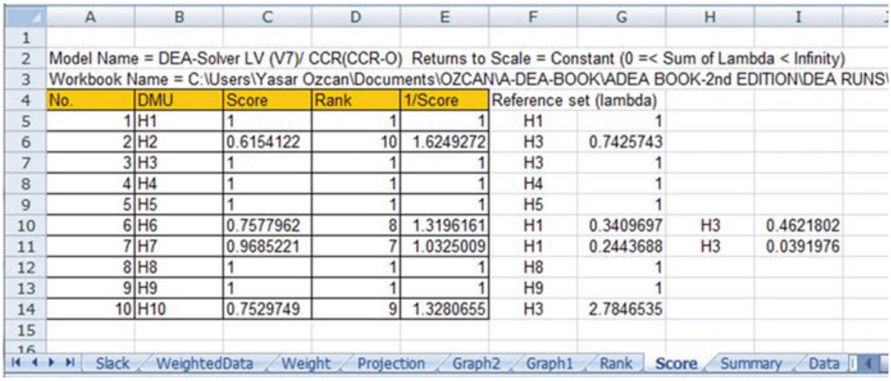


Fig. 2.13 Output-oriented CCR model



Model Name = DEA-Solver LV (V7)/ CCR(CCR-O) Returns to Scale = Constant (0 =< Sum of Lambda < Infinity)
Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\DEA RUNS\

No.	DMU	Score	Rank	1/Score	Reference set (lambda)			
1	H1	1	1	1	H1	1		
2	H2	0.6154122	10	1.6249272	H3	0.7425743		
3	H3	1	1	1	H3	1		
4	H4	1	1	1	H4	1		
5	H5	1	1	1	H5	1		
6	H6	0.7577962	8	1.3196161	H1	0.3409697	H3	0.4621802
7	H7	0.9685221	7	1.0325009	H1	0.2443688	H3	0.0391976
8	H8	1	1	1	H8	1		
9	H9	1	1	1	H9	1		
10	H10	0.7529749	9	1.3280655	H3	2.7846535		

Slack | WeightedData | Weight | Projection | Graph2 | Graph1 | Rank | Score | Summary | Data

Fig. 2.14 Results of output-oriented CCR model

2.13 Interpretation of Output-Oriented CCR [CRS] Results

Figure 2.15 depicts the abridged version of the efficiency report, where efficiency scores of all ten hospitals are reported. This two-input and two-output model shows six of the ten hospitals are efficient using these four dimensions in an output-oriented model.

Fig. 2.15 Efficiency report for output-oriented CCR model

CCR-O

DMU	Score	Rank	1/Score
H1	1	1	1
H2	0.6154122	10	1.6249272
H3	1	1	1
H4	1	1	1
H5	1	1	1
H6	0.7577962	8	1.3196161
H7	0.9685221	7	1.0325009
H8	1	1	1
H9	1	1	1
H10	0.7529749	9	1.3280655

2.13.1 Efficiency and Inefficiency

Observing the "1/Score" column, hospitals H2, H6, H7, and H10 have scores greater than 1; thus they are identified as inefficient in the output-oriented model. These hospitals can improve their efficiency, or reduce their inefficiencies proportionately, by augmenting their outputs (since we run an output-oriented model). For example, H2 can improve its efficiency by augmenting certain outputs up to 62.5 % (1.6249272 – 1.0). Similarly, H6 and H10 can do so with approximately 33 % increases. However, H7 is closer to the efficiency frontier and needs only a 3.3 % increase in outputs.

2.13.2 Slacks

Figure 2.16 comes from the "Slack" worksheet of the DEA run results. Here again we observe that the efficient hospitals have zero ("0") slacks. Slacks exist only for those hospitals identified as inefficient.

It is interesting to note that H2 is required to increase its inpatient admissions by 72.8 patients, after having proportionately increased this output by its efficiency score. However, despite the augmentation in this output, it still would not achieve efficiency. No other output can be increased. Thus, H2 should also reduce its nursing hours by 19.5 h. A similar situation in a different magnitude exists for H10. On the other hand, H6 can augment its outpatient visits by 26. Lastly, H7 cannot augment its outputs at all, but should decrease its medical supplies cost by $2,384.24.

Model Name = DEA-Solver LV (V7)/ CCR(CCR-O) Returns to Scale = Constant (0 =< Sum of Lambda < Infinity)
Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\DEA RUNS'

No.	DMU	Score	Excess Nursing Hours S-(1)	Excess Medical Supply S-(2)	Shortage Inpatient S+(1)	Shortage Outpatient S+(2)
1	H1	1	0	0	0	0
2	H2	0.6154122	19.55445545	0	72.81595807	0
3	H3	1	0	0	0	0
4	H4	1	0	0	0	0
5	H5	1	0	0	0	0
6	H6	0.7577962	0	0	0	25.9485142
7	H7	0.9685221	0	2384.237207	0	0
8	H8	1	0	0	0	0
9	H9	1	0	0	0	0
10	H10	0.7529749	429.8292079	0	117.5980579	0

Slack / WeightedData / Weight / Projection / Graph2 / Graph1 / Rank / Score / Summary / Data

Fig. 2.16 Slacks of output-oriented CCR model

2.13.3 *Efficient Targets for Inputs and Outputs*

Again, we can summarize these finding further by examining the "Projection" worksheet. For each hospital, target input and output levels are prescribed. These targets are the results of respective slack values added onto original outputs, and subtracted from original inputs. To calculate the target values for inputs, the input slacks are subtracted from the inputs. Targets for outputs are calculated by multiplying optimal efficiency scores by the outputs and then adding the slack values to that value. For a detailed formulation of these calculations, the reader is referred to Appendix C, Part 2. Figure 2.17 displays these target values. As the reader can observe, the target values for efficient hospitals are equivalent to their original input and output values.

Health care managers should be cautioned that some of these efficiency improvement options (and the target values) may not be practical. Health care managers can opt to implement only some of these potential improvements at the present time due to their contracts with labor and supply chains and insurance companies.

2.14 Output-Oriented Model Benchmarks

Figure 2.18 displays the results from the initial "Score" worksheet. Here health care managers whose hospital is inefficient can observe the benchmark hospitals.

As in the input-oriented model, the efficient hospitals in the output-oriented model will consider themselves as their own "benchmark." Thus, the benchmark for H1 is H1, for H3 is H3, and so on. On the other hand, for those inefficient hospitals the benchmarks are one or many of the efficient hospitals. For example, the benchmark for H2 and H10 is H3 (observe that H3 is efficient). The benchmark

	No.	DMU I/O	1/Score Data	Projection	Difference	%
6	1	H1	1			
7		Nursing Ho	567	567	0	0.00%
8		Medical Su	2678	2678	0	0.00%
9		Inpatient	409	409	0	0.00%
10		Outpatient	211	211	0	0.00%
11	2	H2	1.624927			
12		Nursing Ho	350	330.4455	-19.55446	-5.59%
13		Medical Su	1200	1200	0	0.00%
14		Inpatient	90	219.0594	129.0594	143.40%
15		Outpatient	85	138.1188	53.11881	62.49%
16	3	H3	1			
17		Nursing Ho	445	445	0	0.00%
18		Medical Su	1616	1616	0	0.00%
19		Inpatient	295	295	0	0.00%
20		Outpatient	186	186	0	0.00%
21	4	H4	1			
22		Nursing Ho	2200	2200	0	0.00%
23		Medical Su	1450	1450	0	0.00%
24		Inpatient	560	560	0	0.00%
25		Outpatient	71	71	0	0.00%
26	5	H5	1			
27		Nursing Ho	450	450	0	0.00%
28		Medical Su	890	890	0	0.00%
29		Inpatient	195	195	0	0.00%
30		Outpatient	94	94	0	0.00%
31	6	H6	1.319616			
32		Nursing Ho	399	399	0	0.00%
33		Medical Su	1660	1660	0	0.00%
34		Inpatient	209	275.7998	66.79976	31.96%
35		Outpatient	100	157.9101	57.91012	57.91%
36	7	H7	1.032501			
37		Nursing Ho	156	156	0	0.00%
38		Medical Su	3102	717.7628	-2384.237	-76.86%
39		Inpatient	108	111.5101	3.510102	3.25%
40		Outpatient	57	58.85255	1.852554	3.25%
41	8	H8	1			
42		Nursing Ho	2314	2314	0	0.00%
43		Medical Su	3456	3456	0	0.00%
44		Inpatient	877	877	0	0.00%
45		Outpatient	252	252	0	0.00%
46	9	H9	1			
47		Nursing Ho	560	560	0	0.00%
48		Medical Su	4000	4000	0	0.00%
49		Inpatient	189	189	0	0.00%
50		Outpatient	310	310	0	0.00%
51	10	H10	1.328065			
52		Nursing Ho	1669	1239.171	-429.8292	-25.75%
53		Medical Su	4500	4500	0	0.00%
54		Inpatient	530	821.4728	291.4728	54.99%
55		Outpatient	390	517.9455	127.9455	32.81%

Slack | WeightedData | Weight | **Projection** | Graph2 | Gr

Fig. 2.17 Efficient targets for inputs and outputs for output-oriented CCR model

Model Name = DEA-Solver LV (V7)/ CCR(CCR-O) Returns to Scale = Constant (0 =< Sum of Lambda < Infinity)

Workbook Name = C:\Users\Yasar Ozcan\Documents\OZCAN\A-DEA-BOOK\ADEA BOOK-2nd EDITION\DEA RUNS

No.	DMU	Score	Rank	1/Score	Reference set (lambda)			
1	H1	1	1	1	H1	1		
2	H2	0.6154122	10	1.6249272	H3	0.7425743		
3	H3	1	1	1	H3	1		
4	H4	1	1	1	H4	1		
5	H5	1	1	1	H5	1		
6	H6	0.7577962	8	1.3196161	H1	0.3409697	H3	0.4621802
7	H7	0.9685221	7	1.0325009	H1	0.2443688	H3	0.0391976
8	H8	1	1	1	H8	1		
9	H9	1	1	1	H9	1		
10	H10	0.7529749	9	1.3280655	H3	2.7846535		

Slack / WeightedData / Weight / Projection / Graph2 / Graph1 / Rank / Score / Summary / Data

Fig. 2.18 Benchmarks for output-oriented CCR model

for H6 and H7 are two hospitals: H1 and H3. This means, to become efficient, H6 and H7 must use a combination of H1 and H3 (a virtual hospital) to become efficient. How much of H1 and how much of H3 are calculated and reported next to each benchmark hospital? These are λ (lambda) weights obtained from the dual version of the linear program that is solved to estimate these values. Further formulation details are provided in the Appendix A. For example, H7 will attempt to become like H1 more than H3, as observed from respective λ weights of H1 and H3 ($\lambda_1 = 0.244$ vs. $\lambda_3 = 0.039$).

2.15 Summary

This chapter introduced the basic efficiency concepts and DEA technique. The model orientation and returns to scale are basic concepts that help health care managers to identify what type of DEA model they should use. We discussed only input- and output-oriented CRS models in this chapter.

Appendix A

A.1 Mathematical Details

Fractional formulation of CCR [CRS] model is presented below:

Model 1

$$Maximize\ \ \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$subject\ \ to\ \ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1$$

$$u_r, v_i \geq 0 \quad for\ \ all\ \ r\ \ and\ \ i.$$

This model can be algebraically rewritten as:

$$Maximize\ \ \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

$$subject\ \ to\ \ \sum_{r=1}^{s} u_r y_{rj} \leq \sum_{i=1}^{m} v_i x_{ij}$$

With further manipulations we obtain the following linear programming formulation:

Model 2

$$Maximize\ \ \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

subject to:

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, \ldots.n$$

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$u_r, v_i \ \ \geq 0$$

A.2 Assessment of the Weights

To observe the detailed information provided in Fig. 2.7, such as benchmarks and their weights (λ), as well as Σλ leading to returns to scale (RTS) assessments, a dual version of Model 2 is needed. The dual model can be formulated as:

Model 3

$$Minimize \;\; \theta_o$$

subject to:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \quad i = 1, \ldots .m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \quad r = 1, \ldots .s$$

$$\lambda_j \geq 0 \quad j = 1, \ldots .n.$$

In this dual formulation, Model 3, the linear program, seeks efficiency by minimizing (dual) efficiency of a focal DMU ("o") subject to two sets of inequality. The first inequality emphasizes that the weighted sum of inputs of the DMUs should be less than or equal to the inputs of focal DMU being evaluated. The second inequality similarly asserts that the weighted sum of the outputs of the non-focal DMUs should be greater than or equal to the focal DMU. The weights are the λ values. When a DMU is efficient, the λ values would be equal to 1. For those DMUs that are inefficient, the λ values will be expressed in their efficiency reference set (ERS). For example, observing Fig. 2.7, H7 has two hospitals in its ERS, namely H1 and H3. Their respective λ weights are reported as $\lambda_1 = 0.237$ and $\lambda_3 = 0.038$.

Appendix B

B.1 Mathematical Details for Slacks

In order to obtain the slacks in DEA analysis, a second stage linear programming model is required to be solved after the dual linear programming model, presented in Appendix A, is solved. The second stage of the linear program is formulated for slack values as follows as:

Model 4

$$Maximize \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta^* x_{io} \quad i = 1, \ldots.m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, \ldots.s$$

$$\lambda_j \geq 0 \quad j = 1, \ldots.n$$

Here, θ^* is the DEA efficiency score resulting from the initial run, Model 2, of the DEA model. Here, s_{i}^- and s_{r}^+ represent input and output slacks, respectively. Please note that the superscripted minus sign on input slacks indicates reduction, while the superscripted positive sign on output slacks requires augmentation of outputs.

In fact, Model 2 and Model 4 can be combined and rewritten as:

Model 5: Input-Oriented CCR [CRS] Model

$$Minimize \ \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i = 1, \ldots.m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1, \ldots.s$$

$$\lambda_j \geq 0 \quad j = 1, \ldots.n$$

The ε in the objective function is called the non-Archimedean, which is defined as infinitely small, or less than any real positive number. The presence of ε allows a minimization over efficiency score (θ) to preempt the optimization of slacks, s_{i}^- and s_{r}^+. Model 5 first obtains optimal efficiency scores (θ^*) from Model 2 and calculates them, and then obtains slack values and optimizes them to achieve the efficiency frontier.

B.2 Determination of Fully Efficient and Weakly Efficient DMUs

According to the DEA literature, the performance of DMUs can be assessed either as fully efficient or weakly efficient. The following conditions on efficiency scores and slack values determine the full and weak efficiency status of DMU:

Condition	θ	θ^*	All s_i^-	All s_r^+
Fully efficient	1.0	1.0	0	0
Weakly efficient	1.0	1.0	At least one $s_i^- \neq 0$	At least one $s_r^+ \neq 0$

When Models 2 and 4 run sequentially (Model 5), weakly efficient DMUs cannot be in the efficient reference set (ERS) of other inefficient DMUs. However, if only Model 2 is executed, then weakly efficient DMUs can appear in the ERS of inefficient DMUs. The removal of weakly inefficient DMUs from the analysis would not affect the frontier or the analytical results.

B.3 Efficient Target Calculations for Input-Oriented CCR [CRS] Model

In input-oriented CCR [CRS] models, levels of efficient targets for inputs and outputs can be calculated as follows:

Inputs: $\widehat{x}_{io} = \theta^* x_{io} - s_i^{-*} \quad i = 1, \ldots.m$
Outputs: $\widehat{y}_{ro} = y_{ro} + s_i^{+*} \quad r = 1, \ldots.s$

Appendix C

C.1 CCR [CRS] Output-Oriented Model Formulation

Since Model 5, as defined in Appendix B, combines the needed calculations for the input-oriented CRS model, we can adapt the output-oriented CRS model formulation using this fully developed version of the model.

Model 6: Output-Oriented CCR [CRS] Model

$$Maximize \;\; \phi - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad i = 1, \ldots.m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{ro} \quad i = 1, \ldots.s$$

$$\lambda_j \geq 0 \quad j = 1, \ldots.n$$

The output efficiency is defined by ϕ. Another change in the formula is that the efficiency emphasis is removed from input (first constraint) and placed into output (second constraint).

C.2 Efficient Target Calculations for Output-Oriented CCR [CRS] Model

In output-oriented CCR models, levels of efficient targets for inputs and outputs can be calculated as follows:

Inputs: $\widehat{x}_{io} = x_{io} - s_i^{-*} \quad i = 1, \ldots.m$

Outputs: $\widehat{y}_{ro} = \phi^* y_{ro} + s_i^{+*} \quad r = 1, \ldots.s.$

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