

Chapter 2

Modeling of Structural, Institutional and Technological Changes

Using the notion main direction when talking about transition might in some sense be misleading. It may provide an impression that well-defined plans were designed at the beginning of transition and that those have been followed like a road map. First of all, because of the many distinctions among states (countries) with the non-market economies in terms of the so-called pre-conditions of transition there is nothing like a road map in transition. It means that the actual paths of reforms can also turn out to be very different from the initial plans. The transition paths can be as much adjusted (tuned) responses to unexpected events which have been overlooked in the initial schedule.

2.1 Main Directions of the Structural Reforms in Economics

Upon the achievement of the financial stabilization, the countries, which have chosen the path to market reforms, faced a complex of significant problems, induced by the alternation of the existing systems of productive and other social and economic relations targeted at the best way leading to the market economy standards. These problems are solved through the implementation of structural, institutional, and technological changes (reforms) (see Sergienko et al. [38]). Their planning is associated with considerable difficulties, since unlike for the initial financial stabilization there are no universal methods. The plan of the reforms should take into account the country's peculiarities and available resources. A large amount of semistructured and unformalized tasks arises in the course of the structural transformations. However, the application of the mathematical models to economy can be an efficient and supportive tool of the decision making in certain aspects of the mentioned reforms.

It should be noted that despite of the specific character of their implementation, the structural reforms in the countries with a transitional economy should be concentrated on several similar directions. An accelerated demonopolization of the economy, i.e., the creation of competitive markets is the first of them. As it was mentioned in Chap. 1, the presence of the imperfect competition, especially in the economic instable environment, can significantly falsify the effectiveness of the market self-regulation mechanisms. By estimating the degree of monopolism in a transitional economy, one should take into account not only the formal, but also informal relations between firms and enterprises. The antimonopoly legal norms, transferred from the legislation of the countries with market economy, would not completely solve the emerged problems under such circumstances, but sometimes they still have a positive effect. Some extra criteria of monopolism are required, first of all, for the price setting analysis. Our mathematical models allow to forecast the price change under different assumptions about the forms of imperfect competition on the given market segment. Comparing the forecasted results with the real world practice, it is possible to determine under which assumptions the most accurate results were obtained, and evaluate the existing situation more adequately on this basis.

The decentralization and disaggregation of the large-scale enterprises are traditionally viewed as an important way toward the development of the competitive market. However, one form of imperfect competition – monopoly – is often replaced by another – oligopoly in such a way. To estimate, to which extend this replacement would comply with the public interest, and to forecast its consequences, one may use the models of the oligopoly pricing. The consequences of the imperfect competition also may be restrained by tools of taxation and customs-tariff policies; our application of this tool is also connected with mathematical modeling.

The second important direction of the structural reforms deals with the creation of the market infrastructure – the development of the stock and money exchange system, insurance market, banking and consulting structures, etc. Our mathematical models can be used to verify the managerial decision and forecast their consequences for the state's economic policy in this direction, as well as to support the management in the newly created economic structures. The development of the market infrastructure demands thoroughgoing alternations of the taxation, budgetary, customs-tariff policies and a fiscal policy as a whole. Such alternations should be systematically based on the quantitative methods of the analysis, including modeling, are widely used in the development of the reform plans in the indicated sphere.

The transformation of a fiscal system is traditionally viewed as one of the most important directions of the structural reforms. The taxation system of post-communist states needs a permanent analysis and perfection, especially in such matters as tax rates, preference grants and their abolition, scheme of taxes payment, etc. The changes in the taxation policy must be connected with budgetary policy. A forecast of the changes in budget incomes, according to which the expenses should be calculated, is required. Some well-known mathematical models are widely used to meet this challenge in the countries with the stable market economy.

The models of KESSU series (see Hetemaki [16]), deserve a special attention – since early 1980s they have been used to support the decision making in the budgetary policy in Finland under the conditions which in some aspects corresponding to the peculiarities of transition economy. Note that Finland has the so-called mixed economy, i.e., an economic system in which both the private sector and state direct the economy, reflecting characteristics of both market economies and planned economies (see, e.g., Bradley [2]).

Most mixed economies can be described as market economies with strong regulatory oversight, and many mixed economies feature a variety of government-run enterprises and governmental provision of public goods.

It should be pointed that, during the development of the fiscal policy, the necessity not only for a checking-up of existed recommendations, but also for the elaboration of new propositions may arise. Optimization models, which were applied in the countries with transitional economy, are being used for this purpose. Their further development needs taking into account the specific features of every country.

The stimulation of changes in existed production technologies is the next important direction of the structural reforms. As it was mentioned in Chap. 1, a large power and material intensity of the production is one of the main reasons for the cost inflation and non-payment crises. These effects can be fully gotten over by significantly reduced production costs per unit, and rejecting the old, outdated technologies. The planning of such changes needs considering of input–output models with variable direct cost matrix. Models of this type started to develop in the 1970s in connection with elaboration of the DISPLAN planning system (see Glushkov [13]) in the former USSR. Their application for the transitional economy requires both a significant revision of the structure of the models, and perfection of the principles for modeling results implementations.

Often, while planning the structural reforms, managers' decisions have to be made by weighting alternative options by means of various indicators (criteria). The mathematical models and methods can also be used here to support the decision making, in particular, the Analytic Hierarchy Process (AHP), proposed by Saati [32], and the methods of fuzzy set theory, developed by Bellman and Zadeh [1]. A choice of alternative for the administrative reform, which is also one of the directions of structural changes in transition economy, is the field of possible application of the above-mentioned approaches.

The current chapter reflects all directions described above. Section 2.2 presents our mathematical models of monopoly and oligopoly pricing, both universal and special-purposed, which allow us to take into account the peculiarities of certain market segments. Section 2.3 presents a mathematical model of the state budget implementation process and a “Budget” modeling complex, created on its basis. The latter has been applied, in particular, to analyze the Project of Ukrainian State Budget. Our results of the conducted analysis are also listed in Section 2.3. A special emphasis is made on the optimization of the fiscal policy standards and solution of the emergent problems of the multi-objective optimization and optimal control with discrete time.

Section 2.4 presents the optimization models for planning and analysis of the changes in production technologies, and reports our results of the calculations made with real world data. These calculations are connected with the solution of complex nondifferentiable and discrete optimization problems which required the development of specialized numerical methods (see Sergienko [35]).

Section 2.5 deals with a huge of alternatives for planning reforms in the transition economy by means of modern information decision support technologies. These technologies are represented by new mathematical models and computational algorithms including their theoretical justification and software development.

The fuzzy set theory and ordinal regression methods are introduced and applied in Sects. 2.6 and 2.7, respectively. These methods taking into account incomplete, weakly structured information applied to define the limits and status of suburban area around the capital of Ukraine – Kiev.

2.2 Modeling of Antimonopoly Measures

We will consider several models of antimonopoly measures based both on tools of financial and industrial policy in this section. Financial tools are first introduced.

A high level of monopoly is one of the reasons for inflation in a transitional economy. In the conditions of unsaturated markets, a monopolist can obtain additional profits by reducing production, which leads to the price increases.

It was shown in Mikhalevich [23] that one possible way to stop such an activity is to increase the interest rate and, connected with it, loans. The loss of profits resulting of production decline cannot be compensated by additional future profits if the interest rate is higher than the inflation rate. Otherwise, the higher interest rate generates a large amount of money in circulation, and hence, makes the market unsaturation deeper and gives the monopolist an additional motivation to increase prices. Such a situation also stimulates savings, but does not direct them to the production purposes. Therefore, the optimum value of the interest rate exists, and this corresponds to the maximum of productive activity of a monopolist. Let us consider the model to obtain this value.

Let us assume that there is a producer who makes only one kind of goods and this producer is a monopolist in his sector. His goal is to maximize his real (with taking into account of discounting) profits in the time period $[0; T]$. Being a monopolist, he can control prices by varying the value of this production, i.e., influencing the supply–demand disequilibrium. We also assume that prices change according to the Walras equation (see Nikaido [30]).

Let us denote the price for the good at the time t as $p(t)$, the share of the producer's profit in the price as $\hat{q}$, the value of the good to be produced at time t as $u(t)$, the producer capacity as $\bar{u}$, the demand for the good as $S(t)$, the initial price at the time instant $t = 0$ as p_0 , the coefficient of the pricing equation as m , the discounting coefficient being further interpreted as the interest rate as ν . For the given assumptions, the producer's activity can be described by the following

problem of optimal control:

$$\begin{aligned} \int_0^T e^{-\nu t} p(t) \hat{q} \min(S(t), u(t)) dt &\rightarrow \max, \\ \frac{dp(t)}{dt} &= m(S(t) - u(t)), \\ 0 \leq u(t) &\leq \bar{u}, \quad t \in [0; T], \\ p(0) &= p_0. \end{aligned} \tag{2.1}$$

Let us denote an optimal solution to this problem obtained for the given ν and $S(t)$ as (p^*, u^*) . Now let us consider the model of the stimulation of a production activity of a monopolist by the state which establishes the value of ν parameter.

The minimization of the supply–demand disequilibrium is the state goal. The demand for goods is assumed to be the proportional to the ratio of consumers' money stock D and the price p . Two sources of consumers' income are considered in our model:

1. the income which consumers receive independently from producer profits (payments from the budget, for instance);
2. the fixed portion q of profits of the producer–monopolist. Let us also assume that the consumers' money stock is discounted with the same coefficient (interest rate) ν .

If the intensity of the consumers' income is denoted as $Q(t)$, the following problem is obtained:

$$\begin{aligned} \int_0^T \max(S(t) - u^*(t), 0) dt &\rightarrow \min, \\ \frac{dD(t)}{dt} &= \nu D(t) + (1 + \nu) \\ &\quad \times \left(Q(t) + \hat{q} q p^*(t) \min(S(t), u^*(t)) - p^*(t) \min(S(t), u^*(t)) \right), \\ \frac{dS(t)}{dt} &= \frac{d}{dt} (D(t) / p(t)), \\ D(0) &= D_0, \quad S(0) = S_0, \end{aligned} \tag{2.2}$$

where D_0, S_0 are the given values of D and S variables for the time instant $t = 0$.

Equations (2.1)–(2.2) are a rather complicated two-level optimal control problem. However, this problem can be simplified.

Let's show that $u^*(t) \leq S(t)$ for every $t \in [0; T]$. Indeed, the producer has no reason to produce goods, which cannot be sold, never bring him any profits and decrease prices, which he tries to increase. Therefore, the equations of the problem (2.2) can be substituted by linear ones:

$$\begin{aligned}\frac{dD(t)}{dt} &= \nu D(t) + (1 + \nu) \left(Q(t) + (\hat{q}q - 1) p^*(t) u^*(t) \right), \\ \frac{dS(t)}{dt} &= \frac{d}{dt} \left(\frac{D(t)}{p^*(t)} \right).\end{aligned}$$

By substituting their solution $S(\nu, p^*, u^*)$ which is obtained for the given ν in problem (2.1), we transform the latter to the former which allows the application of algorithm proposed in Mikhalevich [23] for its solution. If we further substitute the solution of the latter to the objective function of the problem (2.2), we obtain a rather complex, but quite solvable, one-dimensional optimization problem. Its solution ν^* will be the optimum value of the interest rate ν .

Antimonopoly policy can also use tools directly influencing the real sector. Development and application of such measures need the modeling of their consequences. The following model was directed to the solution of this problem.

The goal of the model is to derive the policy prescriptions to be used by a government faced with the problem of regulating the natural monopoly controlled either by the local business groups or by the foreign direct investors. A simplified version of this model suggested by Eaton and Lipsey [9] with the entry deterrence is used for these purposes. Assuming that the market can sustain only one firm in the long run and complete depreciation of capital after two periods, Shy [44] came to the conclusion that when the entry is allowed incumbent overinvests into productive capital to deter entry of any other firm. However, deriving the results of the model Shy [44] implicitly assumed that both incumbent and potential entrant belong to local businesses. Here two cases can be considered. First, a natural monopoly is owned by local business groups and is exposed to a potential entry of either foreign investors or local company. Second, a natural monopoly is owned by the foreign investors and it also faces potential entry. Under the assumption that the goal of the government is to maximize the welfare of society as the whole, the government will asymmetrically treat investments done by local and foreign businesses. In a sense, when the locals own the enterprise, the standard conclusions follow: it might be better for the state if the government blockades the entry to such an industry to prevent the overinvestment of the local firm into unproductive activities. However, when foreigners control the local monopoly, the standard conclusion may not be the best for the state. The government could consider the overinvestment of multinational corporations into the country as a positive phenomenon since it may actually increase the welfare of the host country. It might be better to permit entry into the industry that would lead to overinvestment by the foreigners.

Adapting the assumptions of Shy's [44] simplified version of Eaton and Lipsey [9] model to their framework, it state that there are two parties in the

game. Incumbent natural monopoly owned either by one of local business groups or by foreign investor and potential entrant which could be any firm interested in serving the market. They assume that incumbent and potential entrant produce homogenous product and the cost structure of both players is the same. Without loss of generality, the variable costs of production of both firms can be normalized to zero. The other assumptions are as follows (the denotations of model authors are saving):

1. Each firm can produce in any period t if it has capital in this period t . To build the capital firm has to invest amount B that includes buying new equipment, advertising, promotion, etc.
2. The investment lasts for two periods without depreciation and then completely depreciates, thus to continue producing each firm has to invest again after two periods.
3. If only one firm produces and serves the market, it collects the monopolistic revenue H .
4. If two firms operate in the market then each collects revenue L .
5. The inequality $2L < B < H$ held, which means that the market cannot sustain two firms in the long run, which is a way to impose assumption of natural monopoly.
6. Let $0 < p < 1$ be the same discount factor for both firms.

In that framework the problem for the incumbent monopoly arises from the fact that potential entrant could step into the market at any time. Consequently, the existence of a threat of entry for the incumbent may affect the frequency of its investment.

Action taken by the firm i in time t is denoted as $a_t^i \in \{B, NB\}$ where B —invest and NB not invest in period t and $i \in \{\text{incumbent, potential entrant}\}$. Incumbent is already in the industry (by definition) and thus it is assumed to invest in period $t = -1$ and game starts at period $t = 0$, both players can invest in any period $t = 0, 1, 2, \dots$

Each firm maximizes the sum of its discounted profits:

$$\tilde{I}^i = \sum_{t=0}^{\infty} \rho^t \cdot (R_t^i - C_t^i),$$

where $R_t^i = H$ if one firm has invested and solely operates in the market and $R_t^i = L$ if both firms have invested and together serve the market in period t . $C_t^i = B$ if firm invests at t and $C_t^i = 0$ otherwise.

The introduction of foreign direct investor to the model leads to two distinct cases which have opposite conclusions with respect to the government actions.

Consider the first case when local business group controls the natural monopoly and faces potential entry into the industry. According to our assumptions, if local business group operates in the industry alone it gets the revenue H and if a potential entrant steps into the industry both firms get L . Here the properties of equilibrium strategies that each firm follows to maximize its total profits are determined by the following theorem, which consists of two parts.

Theorem 2.1. 1. If a potential entrant is not allowed entering the market then incumbent invests only once in two periods to renew depreciated capital (a potential entrant clearly stays out of the industry). Hence, the strategy for the incumbent is as follows:

$$a_t^{inc} = \begin{cases} B, & \text{for } t = 1, 3, 5, \dots \\ NB, & \text{for } t = 0, 2, 4, 6, \dots \end{cases}$$

2. If a potential entrant is allowed entering, and if the discount factor is high enough satisfying

$$\frac{B}{H} < \rho < \sqrt{\frac{B-L}{H-L}}(A).$$

Then the following strategies constitute a subgame perfect equilibrium for the game:

$$a_t^i = \begin{cases} B, & \text{if } a_{t-1}^j = NB \\ NB, & \text{if } a_{t-1}^j = B \end{cases}$$

$i, j = \text{incumbent, potential entrant } i \neq j, t = 0, 1, 2, \dots$

Proof. The subgame perfect equilibrium, i.e., the situation where no player finds it beneficial to deviate from his strategy outlined in formulation of Theorem 2.1, is considered. The theorem is proved by contradiction.

First Proposition. Since capital lasts for two periods, the best incumbent can do is to invest in each odd period and minimize its costs. The fact that $B < H$ by assumption makes that investment scheme profitable for incumbent and eliminates incentives to deviate from it.

Second Proposition. Suppose that a potential entrant deviates from the strategy specified in the second part of the theorem then it gets the following profit:

$$\Pi^2 = (1 + \rho)(L - B) + \rho^2 \cdot \frac{H - B}{1 - \rho}$$

If it will not deviate (not enter) then the profit should be less than zero, Hence, if ρ satisfies the considered below condition, potential entrant will not step in:

$$(1 + \rho)(L - B) + \rho^2 \cdot \frac{H - B}{1 - \rho} < 0 \Rightarrow \rho < \sqrt{\frac{B-L}{H-L}}. \quad (2.3)$$

Now for incumbent not to deviate from its optimal strategy profit obtained in one period should be less than all future discounted profits:

$$H < \frac{H - B}{1 - \rho} \Rightarrow \rho > \frac{B}{H}. \quad (2.4)$$

Hence, if ρ satisfies condition (2.4) incumbent will not deviate.

Together (2.3) and (2.4) form the condition which ensures that the entry is deterred by incumbent through overinvesting, i.e., investing every period instead of once in two periods. The Theorem is proved.

Let analyze now the most important and remarkable practical implications of the obtained results.

First note that under quite general conditions specified in assumptions and restrictions on, the model generates the following patterns of investment for incumbent natural monopoly the first five periods.

Essentially, what the theorem tells us is that *if* the potential entrant is allowed to step into the industry (e.g., entry is not blockaded by a government) *then* incumbent will have to carry out excess investment to deter the entry. Since capital lasts for two periods investment in each period ensures that the incumbent will operate in the next period and thus makes the threat of incumbent remaining in industry for one more period credible. However, the excess investment leads to wasting of society resources in a sense that some resources could be invested for production purposes rather than entry deterrence. It causes the reduction in the overall welfare leading to the need of government intervention and prohibition of entry.

According to Table 2.1 local business carries out costly activities to deter the entry. However, the additional resources spent on deterrence could be directed to productive purposes if the government prevents the entry in the first place. Hence, if local business controls the incumbent monopoly it is better for society to prohibit entry in an industry. The threat to the incumbent will be eliminated and it will not misallocate resources. We summarize this with the following policy prescription.

Policy Proposition 1. If the natural monopoly industry is controlled by *local business (that invest only domestically)*, then the government may improve welfare of the country by blockading the entry of other firms into this industry.

Consider now the second case when a foreign investor owns the local incumbent monopoly and is exposed to possible entry of either local firms or other foreign investors into an industry. Theorem 2.1 tells us that if entry is blockaded foreign investor will invest only in odd periods following pattern number 1 in Table 2.1. The outcome will be efficient amount of investment from the foreign investor's point of view. However, in that case government misses the opportunity to improve social welfare in a country though making foreigner incur additional investment. If the entry is allowed the foreign investor will spend money on its deterrence

Table 2.1 Investment patterns under different policy rules

Period	-1	0	1	2	3	4	5
1. Entry is blockaded	B	0	B	0	B	0	B
2. Entry is not blockaded	B	B	B	B	B	B	B

and thus it will invest additional money in a country every even period following pattern number 2 in Table 2.1. Consequently, investments in a country spent on maintaining monopoly status by foreigner will double in our model. Since local government welcome more investment it will perceive overinvestment as positive phenomenon, which leads to the redistribution of monopoly profits in favor of locals and increases overall welfare. Therefore, the government should, in contrast to the first case, behave in completely different way—it should not prohibit the entry as it was in the case with the local business control and make foreign investors carry out additional investment activities. This can be summarized with the following policy prescription.

Policy Proposition 2. If the natural monopoly industry is controlled by *foreign* business, then the government may improve welfare of the country by making the *entry free* for other firms into this industry.

Using the simplified version of Eaton and Lipsey [9] model, it is demonstrated that the government should provide asymmetric regulation of natural monopolies under different ownership structure. If local business group controls natural monopoly, government should blockade entry to an industry preventing the misallocation of resources to unproductive activities. On the other hand, government should allow entry to an industry if the owner is foreign investor. In that case, social welfare will be increased through additional foreign investments to buying new equipment, advertising and promotion in a country and thus increasing employment in it. Further, investments in promotion may increase consumer surplus and welfare through decrease in monopoly prices.

Of course, there are possible dangers of using the above proposed policy prescriptions without further considerations of each particular case. The most important one is the possibility of increase in direct bribing of local politicians and governmental officials by foreign investor to maintain its monopoly status. Another one is the possibility of violation of assumption of identical technologies. If the entry is blockaded by the government, the old production technology might be chosen by local businesses. Without competitive pressure and possibility of entry of investors with better technologies a country could be worse off in the long run; this is a subject of another study. Some aspects of technology development in transition economy are considered in detail in Sect. 2.4.

Models of real sector with the account of imperfect competition at some of its segments and externality impact are considered in Cavalletti [5]. They also can be applied for the development of anti-monopoly policy in transition economy.

A two-sector model of economy is considered. A competitive sector (S_X) produces an appropriable good (X) by means of a single-output return to scale technology, using as primary inputs labor and capital. In the non-competitive sector (S_G) two firms – a private and a public owned one – produce a differentiated product, sharing the same constant return to scale technology. Each unit of this market good produced by the private and the public firm generates a unit of public characteristic, which enters the utility function of consumer. Consumers maximize utility over characteristics to their constraints.

Market goods are denoted as X , G_z and G_h ; the primary factors are L and K . Market prices for produced good are indexed by $i = x, z, h$ while factor prices are denoted as w and r for labor and capital services, respectively.

There are two productive sectors or industries denoted as S_X and S_G , producing the good X and goods G_z and G_h , respectively. The market goods G_z and G_h can be interpreted as the production of a differentiated product by two firms active in the same industry.

Let X is produced by means of a single-output, Cobb–Douglas [6] technology whose unit cost function can be written as:

$$C_x(p, w, r, 1) = \phi_x^{-1} \left(\frac{r}{\beta} \right)^\beta \left(\frac{w}{1 - \beta} \right)^{1-\beta} \quad (2.5)$$

where p is a vector of good prices.

There are two firms in sector S_G , each producing a differentiated product denoted as G_z and G_h for the public and the private firm, respectively. Each unit of goods G_z and G_h , generates one unit of public characteristic Y which represents the positive external effect on the utility of the consumers.

Firms share the same technology, which for a given level of inputs produces multiple outputs in the fixed proportion. A simple representation of the technical constraint is obtained for a technology of the following form:

$$F(G_i, Y_i, K_i, L_i) = g(G_i, Y_i) - f(K_i, L_i); \quad i = z, h. \quad (2.6)$$

In this case the functions $g(\cdot)$ and $f(\cdot)$ are linear homogeneous in outputs and inputs respectively, hence, it is possible to separate the producer's problem into two separate optimizations of conditional outputs and inputs. In the considered model, $g(\cdot)$ represents a fixed-coefficient combination of outputs, G_i and Y_i ; $f(\cdot)$ is a Cobb–Douglas aggregate of factors. For any given output vector $z_i = [G_i, Y_i]$, the unit cost function is defined as follows:

$$C_i(w, r, z, (p, w, r, 1)) = \phi_i^{-1} \left(\frac{r}{\alpha_i} \right)^{1-\alpha_i} \left(\frac{w}{1 - \alpha_i} \right)^{\alpha_i}; \quad i = z, h. \quad (2.7)$$

The second important characteristics of the commodity produced in sector G is that firms do not act as the price takers. Each firm's behavior is then defined in terms of some pricing rule, which maps the relation between prices and quantity decisions. The quantity produced by each firm – and therefore the total supply of the sector – depends on a number of factors: the strategy adopted by the firms, the degree of product differentiation, the number of active firms in each sector and the objective pursued by the firms. In our model we assume that private firms act as price competitors a la Bertrand and firms charge an ad valorem mark-up rate on marginal cost μ_{gi} . The mark-up rate is a function of the elasticity of individual demand ξ_{gi} :

$$\mu_{gi} = \frac{1}{|\xi_{gi}| - 1}; \quad i = z, h, \quad (2.8)$$

which, in turn, depends on the price-elasticity of aggregate demand at the equilibrium and other factors such as the number of firms active in the sector considered, the strategy adopted by the firms, their degree of product differentiation, the structure of preferences.

Expenditure decisions are taken by a representative consumer j and in our model we use the Lancaster's [21] characteristic approach. Two are the goods available for consumption, namely X – a private appropriable good – and G an impure public good. Each unit of X – traded at price P_x – generates one characteristic x and has no effect on the utility of other individuals. Each unit of G – traded at price P_g – is generating α units of a private characteristic g and β units of a public characteristic Y . The utility function of this consumer is defined over the space of characteristics x, g, Y ; it is strictly increasing, quasi-concave, continuous and everywhere twice differentiable, and can be written as follows:

$$U_j = u_j(x_j, g_j, Y) \quad (2.9)$$

where the trade off between the different characteristics is determined by the choice of u which summarizes individual preferences. Even without his personal contribution, each consumer can consume units of the public characteristic generated by the consumption of the others. Consumers' demands derive from the solution of a standard maximization problem. At the top level utility is a Cobb–Douglas function of the private good (X), the private characteristic of the impure public good (G) and the public component (Y):

$$U_j = X_j^{\alpha_{xj}} G_j^{\alpha_{gj}} Y_j^{\alpha_{yj}}, \quad (2.10)$$

where Y_j is treated as a public good, i.e., the public characteristic depends, for each consumer, on the total consumption of good G . To represent this characteristic, the consumption of good Y is aggregated by a Leontief technology (a production function in which no substitution between inputs is possible) which transforms one unit of Y into one unit of Y_j , for $j = 1, \dots, n$, so that:

$$Y = Y_1 = \dots = Y_n \quad \text{and} \quad P_y = \sum_{j=1}^n P_i. \quad (2.11)$$

At the bottom level the utility function is defined as a constant elasticity of substitution aggregate of private and public production of the impure public good:

$$G_j = \left(\delta G_{zj}^{\sigma-1/\sigma} + (1-\delta) G_{hj}^{\sigma-1/\sigma} \right)^{\sigma/\sigma-1}, \quad (2.12)$$

where δ is the share parameter and σ is the inter-industry elasticity of substitution.

The budget constraint for consumers is given by the sum of his earnings: consumers supply a fixed amount of labor $\bar{L}_j$ at the price w , earn capital rents $r\bar{K}_j$ and collect the profits distributed by the private firm. His income I_j , gross of the externality effect, is then defined as:

$$I_j = r\bar{K}_j + w\bar{L}_j + \theta_j \left(\sum_{i=z,h} \pi_i \right), \quad (2.13)$$

where π_i is defined as $\pi_i = P_{gi}G_i - rk_i - wL_i$ and θ_j is the share of total profits redistributed to the consumer j .

The consumption pattern of each individual does not depend only on his decisions since he can also consume the public characteristic of the quantity of good G bought by other consumers. This is the case of positive externality since the total positive external effect $Y = Y_z + Y_h$ enters the utility function of the representative consumer, but the consumer does not compensate the producers for the value of the external effect produced. This creates an interesting problem from an Applied General Equilibrium (AGE) modeling point of view (see e.g., Cardenete et al. [4]). The codes for an AGE model in fact are based on balanced Social Account Matrices where consumers have to pay for each commodity they receive and firms ought to be paid a price according to their marginal cost of production. A positive external effect (Y) is produced by sector S_G and enters the utility functions of the consumers, but the consumers do not compensate sector S_G for the value of the external effect produced.

To represent this situation a fictitious tax is levied on the output of sector S_G . The rate of this tax is chosen to make its revenue equal to the value of the external effect. The revenue is used to subsidize the consumer for the 'purchase' of Y . In this way the externality effect causes a net transfer of income from the producers (which are charged with the fictitious taxation) to the consumers (which are subsidized for the same amount).

The modeling of the subsidy to the consumer takes explicitly into account the fact that the externality affects directly his utility by modifying his unit expenditure function (utility price).

Let us define the unit expenditure function of the j -th consumer as:

$$e_j(w, r, p) = \phi^{-1} \left(\frac{p_x}{\alpha_x} \right)^{\alpha_x} \left(\frac{p_Y}{\alpha_y} \right)^{\alpha_y} \left[\frac{\left(\delta^\sigma p_{gz}^{1-\sigma} + (1-\delta)^\sigma p_{gh}^{1-\sigma} \right)^{1/1-\sigma}}{\alpha_g} \right]^{\alpha_g} \quad (2.14)$$

The unit expenditure function can be used to define income in terms of indirect utility as:

$$e_j V_j = r\bar{k}_j + w\bar{L}_j + \theta_j \sum_{i=z,h} \pi_i + p_j Y_j \quad (2.15)$$

As shown in Eq. (2.15) the consumer is refunded for his purchase of good Y : the positive external effect causes the unit expenditure function to be lower than it would have been without externality; if s_j is defined as $p_j Y_j / V_j e_j$, the consumer's income, net of the externality effect, is equal to

$$M_j = e_j = (1 - s_j) V_j = r \bar{k}_j + w \bar{L}_j + \theta_i \sum_{i=z,h} \pi_i, \quad (2.16)$$

where $e_j(1 - s_j)$ is the unit expenditure function net of the external effect (the net price of utility).

The explicit considerations of combined market failures such as oligopoly and externality in applied analysis can greatly improve our understanding of specific policies concerning the provision of public goods and the correction of externalities in non-competitive markets.

The literature suggests subsidizing the price of the impure public good for an amount which corresponds to society's evaluation of the public good produced. In the specific case presented above, G_z and G_h produces the public characteristic at the same rate, hence they should be subsidized at a uniform rate.

On the other hand, the literature on market regulation to reduce the inefficiencies caused by imperfect competition suggests that the public firm to whom the regulator can impose marginal pricing rules should increase its production in order to reduce the monopolistic power of the private competitor. In both cases and when only one market failure applies to a specific institutional context the use of these rules are expected to improve welfare.

The model (2.5)–(2.16) allows to verify if these rules are still valid when several market failures, namely the presence of oligopoly (hence a monopolistic rent) and a merit good affect the same market. For our numerical exercise we have aggregated consumers into one single representative consumer; the value of the exogenous parameters and elasticities are summarized in Table 2.2. The observed unit price for a price competitor a la Bertrand depends on the calibrated value of the substitution elasticity between differentiated goods in the oligopolistic sector (i.e., the calibrated value of σ). It seems that these values turned out to be 8.0 for 0.2 of observed unit profit of the private firm and 6.0 of the individual elasticity of demand.

Table 2.2 Calibrated elasticities for the model (2.5)–(2.16)

Calibrated elasticities	Value
Elasticity of input substitution of firm X	1.0
Elasticity of input substitution of firm z in sector G	1.0
Elasticity of input substitution of firm h in sector G	1.0
Elasticity of substitution between G_z and G_h	8.0
Elasticity of individual demand of firm h in sector G	6.0
Unit average profit of firm h in sector G (μ)	0.2

In this first exercise the cost of the public characteristic of the impure public good is financed by general taxation and the optimal production is induced through a subsidy to sector G .

In particular, the model is calibrated for an initial equilibrium in which no charge is made on the consumer for the use of the public good (the externality effect is a net income transfer from producers to the consumer). Starting from this equilibrium, the optimal subsidy schemes S_z and S_h which internalize the external effect associated with the total consumption of the impure public good are obtained. The cost of the subsidy is financed by a proportional tax on total income, which due to the assumption of fixed labor supply can be considered as a first-best instrument. The results of this first simulation are presented in Table 2.3.

Two countervailing equilibria are presented for alternative scenarios regarding the behavior of the private firm. In the first case, it is assumed that the private firm producing G_h can act as a Bertrand competitor where the value of the mark-up is sustained by the elasticity of substitution between G_z and G_h calibrated in the initial equilibrium; in the second case, the firm is regulated since it has to charge a price equal to its marginal cost, i.e., the firm has not a profit. This second case differs from a perfectly competitive market since the two qualities of goods produced G_z and G_h are not a perfect substitute. The introduction of the subsidy (case 1) is slightly welfare improving, as shown by the equivalent variations (EV) in Table 2.3.

Table 2.3 Correcting the externality with a subsidy^a

	$\sigma = 8.0$		
	Bench	1	2
X	1	0.888526	0.873164
G_h	1	0.829519	2.64792
G_z	1	1.20237	0.615821
w	1	1	1
r	1	0.90032	0.88755
P_x	1	0.929133	0.919888
P_{gz}	1	0.968989	0.964846
P_{gh}	1.2	1.16063	0.964846
P_y	0.3	0.223293	0.21441
S_z		0.230439	0.222222
S_h		0.19239	0.222222
s	0.127659	—	—
e	1	0.822985	0.7850748
v	2.35	2.35735	2.40429
μ	0.2	0.1977	0
Tax revenue		0.300933	0.306931
EV		0.007347	0.054291

^a 1 = an optimal subsidy with the private firm being a Bertrand competitor;

2 = optimal subsidy with the private firm regulated with the price = marginal cost

The positive EV is the result of a reduction in the unit expenditure function (e) and an increase in the utility index (v). The demand for X is decreasing as we might expect. In the benchmark, in fact, the price paid by the consumer to buy from sector G was not optimal since it included the cost to produce the public good. The introduction of a subsidy allows the consumer to increase its purchase of G , in particular the variety produced by the public sector which is cheaper since there is no mark-up on it. The subsidy rate is not uniform and it is higher for the private producer. If the hypothesis of mark-up pricing (case 2) is removed, i.e., the private producer charges a price equal to its marginal cost, there is a more substantial improvement in welfare, an increase in the variety produced by the private sector and the subsidy rate is uniform. The results presented in Table 2.3 allow us to make the following conclusions:

- the presence of oligopoly is a sufficient condition to make uniform rates a sub-optimal policy; and
- the same conclusion applies to welfare gains and losses, which are represented by the equivalent variations (EV).

The second type of issue can be discussed in relation with the market regulation. The second-best optimal policies to apply in a mixed oligopoly, i.e., a market in which a private firm and a publicly controlled one are coexisting have been intensively studied. The monopolistic power of the private firm derives from the imperfect substitution nature of the goods produced by the two firms and cannot be removed through regulation. The problem for the public firm is to determine an optimal price/quantity policy for its product given the structure of the market in which it operates, i.e., given the reactions of the private producer. In general, the private firm is going to reduce the quantity offered in response to an increase in the quantity provided by the public firm in order to maximize its profit. If, as it is assumed in neoclassical models, all available resources are used, the final composition of the supply will be biased toward public production.

This is a typical second-best problem for which the traditional literature on pricing policies points out that the simple rule price equals the marginal cost (MPR), although being welfare improving, it might not be the optimal second-best policy, and concludes that a price higher than marginal cost could in fact be a superior policy to the marginal cost pricing rule. A priori, one could expect that a marginal pricing rule, in the presence of externalities could be a superior instrument. In this case, in fact, the public production would increase, but since the commodity traded is a merit good, it could still be underprovided because of the externality element.

In the first simulation (case 1), the public producer corrects its price policy taking into account the external effects. A subsidy on the marginal cost of production is levied at a rate S_z such that the subsidy matches the value of the external effect. In this way, the market price of good G_z reflects the unit cost of production of the appropriable characteristic only. The subsidy is financed with a general tax on income while, in order to gain some understanding from the comparison with the previous exercises, the tax revenue is kept at the original level of the previous simulations and results are shown in Table 2.4.

Table 2.4 Simulations using regulation^a

	$\sigma = 8.0$	
	Bench	1
X	1	0.876397
G_h	1	0.239923
G_z	1	1.38588
w	1	1
r	1	0.87633
P_x	1	0.91173
P_{gz}	1	0.743125
P_{gh}	1.2	1.11032
P_y	0.3	0.218045
S_z	0	0.226855
s	0.127659	–
e	1	0.82159
v	2.35	2.28550
μ	0.2	0.15517
Tax revenue	0	0.300933
EV	0	–0.0645

^a 1 = marginal cost pricing rule for the public producer

It should be noted that the simple marginal pricing rule in this context produces a welfare loss, i.e., it creates a deadweight loss that is greater than the inefficiency it tries to correct. This is an important result since such a rule, although not being perhaps optimal, it is said to produce welfare gains in a monopolistic market context. This result is due to the presence of the externality element. The negative sign is caused by an excess of public production compared to both the private production of the same good and the private production of the numeraire good, which, in turns, results from the way private and public characteristics interacts in the model. The public characteristic is produced at the same rate by G_z and G_h ; the optimal rule in this market should then be to buy this good only from the producer that sells at the lowest cost. However, on the private market the appropriable characteristic is not a perfect substitute hence both goods are produced. The final equilibrium will depend on a balance between these two points, both of which are far from optimal. This in turn causes the welfare loss. These effects are obviously inversely related to the degree of substitution between the private and the public produced merit good.

The model (2.5)–(2.16) allows us to derive important conclusions on optimal second best policy in the presence of market power and externalities. Now we want to demonstrate that the use of subsidy and regulation instruments depends on the environment in which the policy has to be taken.

For this reason, the results of the simulations previously presented in Table 2.3 are displayed for alternative levels of σ in Table 2.5; the special attention is paid to the analysis of changes of the degree of substitution between G_z and G_h which in turn influences μ .

Table 2.5 Sensitivity analysis

	$\sigma = 10$	$\sigma = 8^a$	$\sigma = 5.7$	$\sigma = 4.5$	$\sigma = 1.4$
X	0.88864	0.80885	0.88830	0.88113	0.88605
G_h	0.28265	0.82951	0.83189	0.83390	0.89392
G_z	1.20264	1.20237	1.20186	1.20144	1.19682
w	1	1	1	1	1
r	0.90041	0.90032	0.90013	0.89972	0.89825
P_x	0.92920	0.92913	0.92899	0.92888	0.92763
P_{gz}	0.96902	0.96898	0.96892	0.96887	0.96832
P_{gh}	1.11654	1.16063	1.25640	1.35219	2.01436
P_y	0.22336	0.22329	0.22316	0.22305	0.22183
S_z	0.23050	0.23043	0.23031	0.23021	0.22909
S_h	0.20004	0.19239	0.17761	0.16491	0.01101
S_h	0.82242	0.82298	0.82420	0.82542	0.91238
μ	0.154	0.2	0.3	0.4	20
Tax revenue	0.30120	0.30093	0.30036	0.29983	0.27025
EV	0.00940	0.00734	0.00294	-0.00136	-0.7236

^a Simulation of Table 2.3

The welfare gains deriving from the subsidy on goods G_z and G_h are larger the smaller the mark-up is. It is worth remembering that in the considered model the mark-up is determined by the product differentiation, hence a small mark-up implies that the two varieties of G are relatively high substitutes so that the consumer has a smaller reduction in his utility from the oligopoly. A higher mark up implies that the distortion caused by oligopoly is relatively more important than the presence of externality. The subsidy can only remove the second source of market failure hence its effectiveness will be positively related to the relative importance of the failure it is correcting. For the same reasons, the welfare gain is greater the greater is σ . In the presented exercise, the sign of the equivalent variation changes for a value of σ is equal to 4.5. This means that if the degree of substitution between the two varieties is less than or equal to 4.5 giving a subsidy is not a first best policy. The following conclusions implies from obtained results:

- the optimal subsidy rates are inversely related to the degree of monopoly power which, in turn, depends on the degree of product differentiation, captured in the model by μ , given the number of active firms on the market; and
- the same trend occurs in the magnitude of welfare gains and losses, which are represented by the equivalent variations (EV).

Table 2.6 reports the results of the simulations presented in Table 2.4 for alternative levels of σ . It should be pointed out that, if σ is relatively high ($\sigma = 10$), the two goods are not highly differentiated, so that the increase in the production of the public variety of good G creates a distortion that is smaller than the market failure created by the positive externality it is correcting. The increase in public production brought about by this type of regulation is then welfare improving.

Table 2.6 Sensitivity analysis

	$\sigma = 10$	$\sigma = 8^a$	$\sigma = 5.7$	$\sigma = 1.4$
X	0.85830	0.87639	0.87471	0.87429
G_h	0.15673	0.23992	0.38857	0.86809
G_z	1.85543	1.38588	1.32548	1.14218
w	1	1	1	1
r	1.065	0.87633	0.86866	0.85670
P_x	1.04552	0.91173	0.90614	0.89738
P_{gz}	0.83110	0.74312	0.74074	0.73543
P_{gh}	1.13832	1.11032	1.19081	1.87881
P_y	0.18816	0.21804	0.21789	0.21922
S_z	0.18460	0.22685	0.22730	0.22963
e	0.80019	0.82159	0.82082	0.86932
μ	0.11680	0.15517	0.24218	18.6804
Tax revenue	0.30120	0.30093	0.30036	0.27025
EV	0.28402	-0.0645	-0.08164	-0.71161

^a Simulation of Table 2.4

When the two goods are highly differentiated, the first effect (substitution of private variety with public variety) has worse consequences than correcting for the externality effect, i.e., this policy is Pareto-inferior.

These results are confirmed by the sensitivity analysis on φ , the scale parameter for the production of the impure public good. The value of σ , the elasticity of substitution between private and public production constant, is kept out and the value of φ_h , the share of the private production of the impure public good, is alternatively increased and decreased.

If we compare the results in Tables 2.5 and 2.6 with those presented in Tables 2.3 and 2.4, we can note that there is an inverse correlation between the scale parameter and the signs of the equivalent variations. An increase in the market share, other things being equal, has the same impact on welfare as a decrease in the benchmark elasticity of substitution: the producer has a greater monopolistic power on the variety, which results, other things being equal, in a higher benchmark value of μ compatible with a pre-determined value of σ . Subsidy and regulation are not welfare improving policies in this case, but the regulation has an even worse impact, i.e., it is a Pareto-inferior instrument.

2.3 Models for Fiscal System Optimization

The economic situation in many post-communist countries (first of all, in CIS countries) does not fit the assumptions on which market and centrally planned economies are based.

The characteristic of economy in transition, such as a dominance of formerly state-owned property, strong monopoly influence, the absence of the centralized

control of industry and prohibitive technology costs result in the complexity of the problem of supporting decisions in the budgetary, monetary and financial policy. Both experience of experts (this mainly refers to the pre-reform period) and analogies with market economies are not suitable in this case. A detailed investigations of transitional economy as a special economic system using quantitative and qualitative methods is necessary. These research require the mathematical modeling of transitional processes in financial and budget spheres.

The models of financial sphere and its essential components (such as pricing) were elaborated by Cagan [3], Sargent and Wallace [33], Johansson [19]. Input-output models were applied in the analysis of influence of the existing technological structure on prices and profitability of the different sectors of economy in such papers as Moroney and Toevs [29], Gandolfo [12] and Hahn [15]. It should be noted that all mentioned authors deal with a well-developed market economy and their results cannot be applied to the transition period without modification.

The models for the state budget consist of a special class of financial models. The procedures of development and realization of budget are determined by well-developed national legislation; therefore they are very special in every country. General ideas and details of models are the subject of such publications, which have been collected in Wallis and Whitley [49], Hetemaki and Kaski [17].

The problem of fiscal policy parameters optimization is especially important in such conditions. The model which can connect the budget, tax system, and other main macroeconomic parameters (processes) is necessary for these investigations. However, several problems appear when an optimization approach is applied to budgetary modeling. The construction of goal (objective) function is one of them. Therefore, the complex consideration of models, optimization methods, and necessary computer tools seems to be urgent.

The development of rational budget is one of the vital issues of economic stabilization. This possesses some special features in transition economy. The structure of expenditures is one of them. The financial stability of formerly state-owned enterprises (i.e., soft private sector) cannot be ensured without large state subsidies if the current prices are rapidly changing and consumer's demand is decreasing. Therefore, such subsidies become the main budget expenditures (about 40–59 % of total expenditures in the Ukraine in 1993 and 20–25 % in Russia at the moment of writing this text). Transformation of industrial structure and implementation of new technologies also need the state investments with long lag of revenues. Values of such subsidies and investments are strongly determined by the current prices for manufactured and consumer goods production. Control of prices and stimulation of activity in the main industrial sectors are other important goals of direct and indirect financial support from the state budget. Beside the direct impact the budget also produces a fundamental and indirect influence on prices through money supply increases to cover the deficit, through taxes which can change the cost of production and through social payments from the budget which change consumer demand. Therefore, the accurate forecast of the development of budget needs cannot be done without information about the structure and values of main expenditures and revenues of the budget. These two problems must be solved simultaneously.

The budget deficit has been traditionally considered among the most important reasons of inflation. It should be noted that the reduction of the budget deficit is important not only for the period of planning as a whole, but also for every subperiod. For example, the budget deficit in Ukraine in 1993 covered about 15 % of total expenditures. But for several months of this year it was more than 50 % of total income (Mikhalevich, Podolev [25]). Of course, it was the strong reason for hyperinflation. The analysis of dynamics of the budget is necessary for accounting of such effects.

The well-balanced budget is necessary, but not sufficient for financial stability in the transition economy. The dominance of enterprises which are managed independently, but bear no responsibility for their results, and the absence of a working bankruptcy mechanism together with the pressure of managers on state bodies, create possibilities for large out-of-budget outlays of money and credits, such as took place in Ukraine in 1993 and in the first half of 1994 (for all details see Mikhalevich, Podolev [25]).

The estimation of the impact of the budget on production is also important. High taxes intensify the industrial decline and decrease the real budget incomes. This produced a higher budget deficit than had been planned. The joint analysis of all these aspects needs modern methods, including mathematical modeling and system researches.

It should be noted that the modeling of state budget is far from being complete even for market economies (see Wallis and Whitley [49]). The construction of such models for transition economy needs special approaches. It is difficult to construct econometric models for such purposes due to the lack of appropriate statistical data. Input–output models seem to be more suitable for such purposes.

One of such models is considered in this section. It can be used to analyze the consequences of the tax policy and decisions about budget outlays. To provide an explanation of the impact of the budget deficit or surplus on the dynamics of prices and, consequently, on the budget revenues, is the main goal of the model. It takes into account all the above-mentioned peculiarities of the problem of budget development in transition.

Note that the effect of the government budget on prices in a transition economy is very complex and multifaceted. It includes the imbalance of supply and demand caused by budgetary reallocation of producer and consumer incomes, the effect of taxes on production costs, and the consequences of emissions intended to cover the budget deficit. It is difficult to allow for all these factors in a relatively simple analytical model. We accordingly propose a simplified simulation model for analyzing the consolidated state budget, which is based on following assumptions.

1. We consider a system of n material production industries plus a nonproductive sector, m consumer groups, and the government budget.
2. The revenues of the government budget derive from tax receipts from the industries, the income tax paid by consumers, fixed payments from industries and consumers, and other sources.
3. The model considers three types of taxes on industries:

- (a) value added tax charged at a rate $q_j^{(1)}$, which is possibly differentiated by industry;
 - (b) tax on profit charged at a basic rate $q_j^{(2)}$, which also may be differentiated by industry;
 - (c) excise taxes on some industry products, which constitute a fraction $q_j^{(3)}$ of the final price.
4. The actual budget revenue from these taxes is proportional to the volume of sales of the industries in current prices.
 5. The revenue from income tax on consumers is proportional to the nominal income of consumers with proportionality coefficient q_j^4 , which is differentiated by consumers groups.
 6. The budget revenue from fixed payments and other sources is assumed given.
 7. The main budget expenditures include the following:
 - (a) subsidies to industries;
 - (b) indexed and unindexed payments to consumers;
 - (c) other expenditures.

The expenditures of the first group are proportional to the production volume of the industries eligible for subsidies. Payments to consumers are indexed in proportion to the weighted price index calculated from data on the volume of sales of industry products in current and constant prices. Other expenditures are assumed given.

8. The model is based on the following pricing mechanism. Price is determined by the cost and monetary factors according to the econometric model presented in Mikhalevich, Podolev [25]. Three types of product prices are considered:
 - (a) fixed price, when products are sold at a given (constant) price and the difference between industry costs and sales revenues is made up from the budget;
 - (b) controlled prices, when the budget compensates a part of industry costs through subsidies;
 - (c) free prices, when no subsidies are paid.

We denote the set of industries with fixed prices by Ω_1 , the set of industries with controlled prices by Ω_2 , and the set of industries with free prices by Ω_3 .

9. The time in the model is discrete with an increment Δt in the interval $[0, T]$. Let us give the detailed description of the model starting from model inputs and variables which are recalculated during the model runs.

a_{ij}	is the direct input in constant prices of industry i product consumed to produce a unit of industry j product ($i, j = 1, \dots, n$);
$\bar{q}_j$	is the share of labor costs in the price of industry j product ($j = 1, \dots, n$);

$\widehat{q}_j$	is the standard profit margin in the price of industry j product ($j = 1, \dots, n$);
q_j^+	is the share of other value added components in the price of industry j product ($j = 1, \dots, n$);
$q_j^{(1)}$	is the rate of value added tax for industry j ($j = 1, \dots, n$);
$q_j^{(2)}$	is the rate of tax on profit for industry j ($j = 1, \dots, n$);
$q_j^{(3)}$	is the share of excise taxes in the price of industry j product ($j = 1, \dots, n$);
$x_j^{(t)}$	is the predicted output of industry j product at time $t \in [0, T]$ in constant prices; this quantity may be viewed as the upper bound on the sales volume produced by the available assets and capacities ($j = 1, \dots, n$);
$B_j^{(t)}$	is the production cost per unit industry j product at time t , which does not depend directly on internal prices ($j = 1, \dots, n, t \in [0, T]$);
$W_j(t)$	is the excess profit per unit output of industry j at time t ($j \in \Omega_2 \cap \Omega_3, t \in [0, T]$);
$\overline{W}_j(t)$	are the subsidies per unit output of industry j at time t ;
$H(t)$	is the budget revenue from fixed payments at time t ;
$\widehat{H}(t)$	is the budget revenue from other sources at time t ;
$\overline{p}_j(t)$	is the fixed price of industry j product at time t ($j \in \Omega_1, t \in [0, T]$);
g_j	is the proportion of industry j product used for consumption ($j = 1, \dots, n$);
β_{jk}	is the proportion of industry j product in the bundle of goods and services used by consumers of group k ($j = 1, \dots, n, k = 1, \dots, m$);
$Q_k^{(1)}(t)$	is the amount of indexed payments from the budget to consumers of group k at time t in constant prices ($k = 1, \dots, m, t \in [0, T]$);
$Q_k^{(2)}(t)$	is the amount of unindexed payments from the budget to consumers of group k at time t in constant prices ($k = 1, \dots, m, t \in [0, T]$);
$G(t)$	are other budget expenditures at time t ;
c_{jk}	is the share of group k consumers in the labor costs of industry j ;
q_k^4	is the income tax rate for group k consumers;
$D_k^{(0)}$	are the money balances for group k consumers ($k = 1, \dots, m$);
$\overline{M}(t)$	is the predicted quantity of money at time t assuming zero budget deficit in the time interval $[0, T]$;
$p_j(t)$	is the relative price (measured in relation to the base price) of industry j product at time t ($j \in \Omega_2 \cup \Omega_3, t \in [0, T]$);
$\tilde{p}_j(t)$	is the price component determined by production costs;
$y_j(t)$	is the industry j product in constant prices used for consumption ($j = 1, \dots, n, t \in [0, T]$);

$z_j(t)$	is the sales volume of industry j product at time t in constant prices;
$\bar{\Pi}_j(t)$	is the income of industry j at time t ;
$\bar{\Pi}_j(t)$	is the income earned by consumers of all groups from work in industry j at time t ;
$\widehat{p}$	is the weighted average price index of goods and services;
$S_j^k(t)$	is the cash-paying demand of group k consumers for industry j product ($j = 1, \dots, n, k = 1, \dots, m$);
$M(t)$	is the quantity of money at time t (allowing for emission to cover the budget deficit);
$D_k^{(t)}$	are the money balances of group k consumers at time t ($k = 1, \dots, m, t \in [0, T]$);
$A_0^{(t)}$ and $B_0^{(t)}$	are the budget revenue and expenditure at time t , respectively.

Before starting the model simulation, we calculate the value added percentage q_j and the price influence coefficient $\bar{a}_{ij}$ from the formulas (see Mikhalevich V.S. et al. (1993)):

$$q_j = \frac{\bar{q}_j + q_j^+}{1 - q_j^{(1)}} + \frac{\widehat{q}_j}{(1 - q_j^{(1)})(1 - q_j^{(2)})} + q_j^{(3)}, \quad j = \overline{1, n},$$

$$\bar{a}_{ij} = \frac{a_{ij}}{1 - q_j}, \quad i, j = \overline{1, n}.$$

We also calculate the elements $\bar{a}_{ij}$ of the inverse of the matrix $(E - A)$, $A = \{a_{ij}\}$, E is the identity matrix. Then we do the following for each time instant $t = 0, \Delta t, 2\Delta t, \dots$

1. Calculate the relative prices of the industry products:

$$\begin{aligned} \tilde{p}_j(t + \Delta t) = & \sum_{i \in \Omega_1} \bar{a}_{ij} \bar{p}_i(t) + \sum_{i \in \Omega_2 \cup \Omega_3} \bar{a}_{ij} p_i(t) + B_j(t + \Delta t) \\ & + \frac{1 - q_j^{(2)}(1 - q_j^{(1)})}{1 - q_j^{(1)}} W_j(t + \Delta t) - v_j(t + \Delta t), \\ & j \in \Omega_2 \cup \Omega_3, \end{aligned}$$

where

$$v_j(t + \Delta t) = \begin{cases} \bar{W}_j(t + \Delta t), & \text{if } j \in \Omega_2, \\ 0, & \text{if } j \in \Omega_3, \end{cases}$$

$$p_j(t + \Delta t) = \max \left(\tilde{p}_j(t + \Delta t), \exp \left((\ln p_j(t) - \ln M(t) + \gamma) \times \right. \right.$$

$$\times e^{-\alpha^{-1} \Delta t} + \ln M(t) - \gamma \Bigg), \quad j \in \Omega_2 \cup \Omega_3.$$

The last equality implies from above-mentioned pricing mechanisms.

2. Calculate the subsidies needed to sustain fixed prices:

$$\begin{aligned} \overline{W}_j(t + \Delta t) &= \sum_{i \in \Omega_1} \overline{a}_{ij} \overline{p}_i(t) + \sum_{i \in \Omega_2 \cup \Omega_3} \overline{a}_{ij} p_i(t) + B_j(t + \Delta t) \\ &+ \frac{1 - q_j^{(2)}(1 - q_j^{(1)})}{1 - q_j^{(1)}} W_j(t + \Delta t) - \overline{p}_j(t + \Delta t), \quad j \in \Omega_1. \end{aligned}$$

3. Calculate the volume of each industry product used for consumption:

$$y_i(t + \Delta t) = (x_i(t + \Delta t) - \sum_{j=1}^n a_{ij} x_j(t + \Delta t) g_i; \quad i = \overline{1, n},$$

and also the cash-paying demand of consumers:

$$S_j^k(t + \Delta t) = \frac{D_k(t) \beta_{jk}}{p_j(t + \Delta t)}; \quad j = \overline{1, n}, \quad k = \overline{1, m}.$$

4. Determine the sales volume of industry products:

$$\begin{aligned} z_i(t + \Delta t) &= \sum_{j=1}^n \widehat{a}_{ij} \left(\min \left(\sum_{k=1}^m S_j^k(t + \Delta t), y_j(t + \Delta t) \right) + \tilde{z}_j(t + \Delta t) \right), \\ i &= \overline{1, n}, \end{aligned}$$

where $\tilde{z}_j(t + \Delta t)$ are the uses of product for purposes other than consumption and production.

5. Calculate the weighted price index:

$$\widehat{p}(t + \Delta t) = \frac{\sum_{j \in \Omega_1} \overline{p}_j(t + \Delta t) z_j(t + \Delta t) + \sum_{j \in \Omega_2 \cup \Omega_3} p_j(t + \Delta t) z_j(t + \Delta t)}{\sum_{j=1}^n z_j(t + \Delta t)}.$$

6. Determine the industry income and profits received by consumers:

$$\Pi_j(t + \Delta t) = p_j(t + \Delta t) z_j(t + \Delta t) - \left(\sum_{i \in \Omega_1} \overline{a}_{ij} \overline{p}_i(t) + \sum_{i \in \Omega_2 \cup \Omega_3} \overline{a}_{ij} p_i(t) \right)$$

$$\begin{aligned}
& + B_j(t + \Delta t) + \frac{1 - q_j^{(2)}(1 - q_j^{(1)})}{1 - q_j^{(1)}} \\
& \times W_j(t + \Delta t) - v_j(t + \Delta t) \Big) x_j(t + \Delta t), \quad j \in \Omega_2 \bigcup \Omega_3, \\
\Pi_j(t + \Delta t) &= \widehat{q}_j z_j(t + \Delta t) \overline{p}_j(t + \Delta t), \quad j \in \Omega_1, \\
\overline{\Pi}_j(t + \Delta t) &= \begin{cases} \overline{p}_j(t + \Delta t) \overline{q}_j z_j(t + \Delta t), & j \in \Omega_1, \\ p_j(t + \Delta t) \overline{q}_j z_j(t + \Delta t), & j \in \Omega_2 \bigcup \Omega_3. \end{cases}
\end{aligned}$$

7. Calculate the consumer money balances:

$$\begin{aligned}
& D_k(t + \Delta t) \\
&= D_k(t) + \sum_{j=1}^n c_{jk} \overline{\Pi}_j(t + \Delta t) + \widehat{p}(t + \Delta t) Q_k^{(1)}(t + \Delta t) \\
&+ Q_k^{(2)}(t + \Delta t) - \sum_{j \in \Omega_1} \overline{p}_j \min \left(S_j^k(t + \Delta t), y_j(t + \Delta t) \frac{S_j^k(t + \Delta t)}{\sum_{i=1}^m S_j^i(t + \Delta t)} \right) \\
&- \sum_{j \in \Omega_2 \bigcup \Omega_3} p_j(t + \Delta t) \min \left(S_j^k(t + \Delta t), y_j(t + \Delta t) \frac{S_j^k(t + \Delta t)}{\sum_{i=1}^m S_j^i(t + \Delta t)} \right).
\end{aligned}$$

8. Determine the budget revenues:

(a) from value added tax:

$$\begin{aligned}
h^{(1)}(t + \Delta t) &= \sum_{j \in \Omega_1} q_j^{(1)} q_j \overline{p}_j(t + \Delta t) z_j(t + \Delta t) \\
&+ \sum_{j \in \Omega_2 \bigcup \Omega_3} q_j^{(1)} q_j p_j(t + \Delta t) z_j(t + \Delta t);
\end{aligned}$$

(b) from tax on industry profits:

$$h^{(2)}(t + \Delta t) = \sum_{j=1}^n q_j^{(2)} \Pi_j(t + \Delta t);$$

(c) from excise taxes:

$$h^{(3)}(t + \Delta t) = \sum_{j \in \Omega_1} q_j^{(3)} \bar{p}_j(t + \Delta t) z_j(t + \Delta t) + \sum_{j \in \Omega_2 \cup \Omega_3} q_j^{(3)} p_j(t + \Delta t) z_j(t + \Delta t);$$

(d) from income tax on consumers:

$$h^4(t + \Delta t) = \sum_{k=1}^m q_k^4 \times \left(\sum_{j=1}^n c_{jk} \bar{\Pi}_j(t + \Delta t) + \widehat{p}(t + \Delta t) Q_k^{(1)}(t + \Delta t) + Q_k^{(2)}(t + \Delta t) \right).$$

9. Calculate the total budget revenues:

$$B^0(t + \Delta t) = \sum_{l=1}^4 h^{(l)}(t + \Delta t) + H(t + \Delta t) + \widehat{H}(t + \Delta t).$$

10. Determine the expenditure on subsidies to industries:

$$\bar{W}(t + \Delta t) = \sum_{j \in \Omega_1 \cup \Omega_2} \bar{W}_j(t + \Delta t) x_j(t + \Delta t).$$

11. Calculate the total budget expenditure:

$$A^0(t + \Delta t) = \bar{W}(t + \Delta t) + \sum_{k=1}^m (\widehat{p}(t + \Delta t) Q_k^{(1)}(t + \Delta t) + Q_k^{(2)}(t + \Delta t)) + G(t + \Delta t).$$

12. Determine the change of money quantity:

$$M(t + \Delta t) = M(t) + (\bar{M}(t + \Delta t) - \bar{M}(t)) + (A^0(t + \Delta t) - B^0(t + \Delta t)).$$

The values of the model variables at time $t = 0$ are determined as follows. The values of $D_k(0)$, $k = \overline{1, m}$, $p_j(0)$, $j \in \Omega_2 \cup \Omega_3$, $\bar{W}_j(0)$, $j \in \Omega_1$, are assumed given. The values of $y_i(0)$, $S_j^k(0)$, $\Pi_j(0)$ are calculated from the formulas

$$\begin{aligned}
y_j(0) &= \left(x_i(0) - \sum_{j=1}^n a_{ij} x_j(0) \right) g_i, \quad i = \overline{1, n}, \\
S_j^k(0) &= \frac{D_k(0) \beta_{jk}}{p_j(0)}, \quad k = \overline{1, m}, \quad j \in \Omega_2 \cup \Omega_3, \\
S_j^k(0) &= \frac{D_k(0) \beta_{jk}}{\bar{p}_j(0)}, \quad j \in \Omega_1; \\
\Pi_j(0) &= p_j(0) z_j(0) - \left(\sum_{i \in \Omega_1} \bar{a}_{ij} \bar{p}_i(0) + \sum_{i \in \Omega_2 \cup \Omega_3} \bar{a}_{ij} p_i(0) + B_j(0) \right. \\
&\quad \left. + \frac{1 - q_j^{(2)}(1 - q_j^{(1)})}{1 - q_j^{(1)}} W_j(0) - v_j(0) \right) x_j(0), \quad j \in \Omega_2 \cup \Omega_3.
\end{aligned}$$

The other model variables are calculated from the previously given formulas setting $t + \Delta t = 0$.

The considered model has been implemented in the decision support system (DSS) UKRAINIAN BUDGET (further—BUDGET), which models the effect of the state budget proposal on the main macroeconomic and interindustry characteristics and develops recommendations concerning efficient macroeconomic policies. The system is realized by PC using “Windows” environment. It does not demand the special knowledge in computer programming and coding. User can operate in multiscreen mode, open and close “Windows” with necessary information. The BUDGET system is equipped with multifunctional table and graphical editor and provides the SQL inquires. The results of model simulations (including various alternatives of macroeconomic policy) can be presented in tabular or graphic form. BUDGET produces a forecast of price dynamics, domestic sales volumes, budget revenues and expenditures. The modeling results can also explain the deviations of the predicted values from initial assumptions of the draft budget. Using hierarchical menus and computer graphics, the user of system (an expert in macroeconomic analysis) can select for each branch the pricing mechanisms and other model assumptions that are most essential in his view. Eliminating some factors, the user can estimate their influence at observed processes. An important function of the system is checking the effectiveness of various strategies of direct and indirect subsidies to producers and developing recommendations for reasonable taxation of production. A set of different optimization models can be applied for this purpose. Some problems concerning with their development is the subject of further consideration.

The development of simulated budget processes depends on several parameters which we can change in the model. First of all, the tax rates $q_j^{(1)}$, $q_j^{(2)}$, $q_j^{(3)}$ and q_j^4 can be considered as controlled variables. The subsidies $\bar{W}_j(t)$ and fixed prices $\bar{p}_j(t)$ are also the same. In some cases the budget problems can be solved changing the value and structure of budget expenditures $Q_k^{(1)}(t)$, $Q_k^{(2)}(t)$ and $G(t)$. Therefore the

problem of optimal search of these parameters from the set of their admissible values appears. If the objective function is known this problem can be formulated as the optimal control problem for systems with discrete time; see Ermoliev et al. [10]. However, the construction of objective function in mentioned problem is rather difficult because we have to take into account the different nature goal indicators. Among them:

- the total value of budget deficit which is the commonly used indicator of well-developed budgetary policy:

$$F_1 = \sum_{t=1}^T A^0(t) - \sum_{t=1}^T B^0(t) \rightarrow \min;$$

- the values of budget deficit at some time moments $t \in \overline{\mathfrak{T}}$ can essentially impact on general financial situation as it was mentioned above, so they must be taken into account separately:

$$F_2^t = A^0(t) - B^0(t) \rightarrow \min, \quad t \in \overline{\mathfrak{T}};$$

- the stimulation of economic growth is one of the main goals of macroeconomic policy; therefore the value of total production F_3 and rates of economic growth F_4 at the end of observed time interval $[0; T]$ we consider as important economic indicators:

$$F_3 = \sum_{t=1}^T \sum_{i=1}^n Z_i(t), \quad F_4 = \frac{\sum_{i=1}^n Z_i(t)}{\sum_{i=1}^n Z_i(1)};$$

- the price stability is necessary condition for social development; taking this fact into account we consider the following criterion:

$$F_5 = \frac{\widehat{p}(T)}{\overline{p}(1)} \rightarrow \min.$$

Another important economic figures such as total incomes of consumers and producers, total cash-paying demand are also criteria for consideration.

Thus we obtain the multicriteria optimization problem. The changes in values of some its criteria cannot be fully compensated by other criteria. Such fact creates some difficulties for the objective function construction. Taking this all into account, we apply the methods of ordinary regression to solve this problem; see Gruber [14].

Let us consider the vector $F = (F_1, \dots, F_K)$, where K is the number of criteria. We write the following analytical form

$$U(F) = \min_{k=1, \overline{K}} (\alpha_k F_k - \beta_k) + \sum_{k=1}^K \gamma_k F_k,$$

where α_k , β_k , γ_k are unknown parameters, satisfied by means of the above mentioned conditions.

The values of these parameters can be identified as the solution of the next inequality system:

$$U(F^{(i)}) \geq U(F^{(j)}), \quad (2.17)$$

where $F^{(i)}$ is the value of criteria vector for alternative i , $F^{(j)}$ is the value of criteria vector for alternative j and it is known that alternative i is no worse than j .

The methods of nondifferentiable optimization (see Shor [40]) have been applied to find the system solution (or pseudo-solution). When we substitute the parameters by their known values we receive the objective function. The best solution in the budget model can be obtained as the maximal point of mentioned function under the constraints from previous section. The gradient method for optimal control problems solution [10] can be used to find this point.

The model simulations were made using the above mentioned approach applied to the basis of real data obtained from [48] *Ukrainian Economics in Figures (1994–1996)* and [7] *Consolidated Interindustry Balances (1990–1995)*. The search of rational tax rates values was the main goal of these experiments. The first step was to construct the objective function. Taking into account the monetary reform in the Ukraine in 1996 which changed the name of national currency, the scale of prices and the mechanisms of financial control, we present the indicators of budget deficit and total production in related from. Thus, the considered objective function depends on the following criteria:

- related value (to total budget expenditures) of budget deficit (in %):

$$F_1 = \frac{\sum_{t=1}^T A^0(t) - \sum_{t=1}^T B^0(t)}{\sum_{t=1}^T A^0(t)} 100\% \rightarrow \min.$$

- rates of production total growth/decline related to known basic variant (in %):

$$F_2 = \left(\frac{\sum_{t=1}^T \sum_{i=1}^n Z_i(t)}{\sum_{t=1}^T \sum_{i=1}^n \tilde{Z}_i(t)} - 1 \right) \cdot 100\% \rightarrow \max,$$

where $\tilde{Z}_i(t)$ is the value of production of branch i at time moment t according to the basic variant. The variant in which the total growth, compared with 1995, was considered as the basic;

- the rates of total economic growth/decline at the last month of the year related to the its first month (in %):

$$F_3 = \left(\frac{\sum_{i=1}^n Z_i''}{\sum_{i=1}^n Z_i(1)} - 1 \right) \cdot 100 \% \rightarrow \max .$$

- the rate of inflation:

$$F_4 = \frac{\widehat{p}(T)}{\widehat{p}(1)} \rightarrow \min .$$

The analytical form of function

$$U(F, \alpha, \beta, \gamma) = \min_{i=1,4} (\alpha_i F_i - \beta_i) + \sum_{i=1}^4 \gamma_i F_i$$

considered in the previous section was applied for this function construction by methods of ordinary regression. Unknown parameters α , β , γ were identified to construct this function on the basis of comparison of eight alternatives. The values of criteria for these alternatives were presented in Table 2.7. Alternatives I–VI are the result of different approaches to the development of fiscal and structural economic policy. Alternatives VII and VIII were introduced to emphasize the growth of objective function by its F_2 variable and its decline by F_4 variable for sufficiently large inflation rates. This demand is the natural restriction for the economy after hyperinflation and connected with it essential GDP decline.

Table 2.7 Criteria values for the considered alternatives

Alternatives	Criteria			
	F_1	F_2	F_3	F_4
I	7.6	−9.1	−7.3	1.42
II	4.7	1.5	0.5	1.36
III	7.6	−9.1	−7.1	1.42
IV	5.2	−7.4	−6.1	1.23
V	7.0	−9.1	−7.1	1.42
VI	7.7	2.3	1.0	1.54
VII	7.6	−10.0	−7.3	1.42
VIII	7.6	−10.0	−7.3	1.55

The alternatives were compared by experts from analytical divisions of central governmental bodies. According to their consolidated mind, alternative II is the best one, alternative VI with high inflation but positive rates of economic growth is more preferable than the low inflationary alternative IV.

Alternative ν with low budgetary deficit is better than alternatives III and I which deficit values are greater. Therefore all alternatives can be ranked from more to less preferable as follows:

II, VI, IV, V, III, I, VII, VIII.

The identification of parameters was made by special computer program SPOF on the basis of above-mentioned data. The parameters values were obtained as the solution of inequality system (2.17). Thus, constructed objective function is the following:

$$\begin{aligned} U(F_1, F_2, F_3, F_4) \\ = \min \left(F_1 - 16.209, F_2 - 2.038, F_3 + 13.588, F_4 - 1.387 \right) \\ - 57.986F_1 + 85.197F_2 + 64.921F_3 - 0.769F_4. \end{aligned}$$

The values of function $u(\cdot)$ for the considered alternatives are presented in Table 2.8. It should be noted that constructed function decreases with F_1 variable growth and increases with F_2 and F_3 variables growth. For small values of inflation rates it slowly increases with F_4 growth, but for large values it is decreased by this variable. Therefore this function is rather adequate to preferences existed in macroeconomic planning for transition economy.

The rational values of tax and subsidies rates were received as the maximum point of constructed objective function under the restrictions considered in Sect. 2.3. The obtained optimization model was solved as the optimal control problem with discrete time. The special version of quasigradient method [10] was applied for this purpose.

The calculations to be done show the necessity of deep differentiation of VAT and profit tax rates for different kinds of economic activity. This is connected with the great impact of the cost inflation scenario called [25] as the “Structural hyperinflation crisis” on the processes of budget realization. The structural disproportions in the economy – large rates of cost growth in some branches – are the source of such crisis. In this case all other pricing mechanisms would be, sooner or later, substituted by cost pricing. The rates of indirect taxes must be close to minimal to eliminate the

Table 2.8 Values of constructed objective function for the considered alternatives

Alternatives	VIII	VII	I	III	V	IV	VI	II
Values of objective function	−1761	−1760	−1684	−1671	−1636	−1321	−171.5	−99.23

cost growth in branches with largest industrial expenditures. Low taxes in last ones can be compensated by tax increase for economic activities with low cost and quick money turnover. Thus, obtained rates of VAT change from 5–7 % for fuel and energy production branches to 22–25 % for trade and public supply.

Real data runs show the low efficiency of the “frozen” price policy for high cost branches which initiate the structural hyperinflation crisis. The inflationary impact of large money outlays which are necessary to provide subsidies, is no less in this case than the consequences of cost increases.

But the policy of “Low taxes, no subsidies” leads to the high rates of total cost growth in the branches under cost inflation. The demand restriction (usual strong monetary anti-inflation measures) leads to an additional industrial decline. It can decrease the rates of price growth and even made them less than rates of cost growth, but under the absence of effective bankruptcy mechanism enterprises try to subsidize themselves at the expense of their suppliers. Deep payments crisis observed in majority of CIS countries is the result of such activity. The best policy, according to the model simulations, is to subsidize the sector of economy weakly connected with branches which provoke the cost growth. Applying to Ukrainian data for 1996, this subsidies can cover up 17–22 % of the total cost in food industry, agriculture, transport, and telecommunications.

The forecasted values of main macroeconomic indicators for economy with obtained tax and subsidies rates are presented in Table 2.9 as “constructed variant.” They can be compared with real values of these indicators for existing tax rates which are presented as “real values.” Analyzing the information from this table, it is possible to assume that the proposed changes in fiscal policy would essentially change the general economic situation in the last half of 90th.

The following conclusion can be elaborated by the results of our research.

Table 2.9 Values of main macroeconomic indicators

Indicators	Constructed variant	Real values in 1996
Related value of budget deficit (%)	7.0	12.1
Rates of total production growth (%)	−4.6	−9.0
Rates of total sales growth during the year (%)	2.26	−6.2
Rates of inflation (GDR deflator)	0.91	1.214
Rates of consumer real incomes growth (real values = 100)	125.8	100
Rates of producer real incomes growth (real values = 100)	119.5	100
Real money supply (real values = 100)	120.2	100

1. The development of anti-inflationary policy in a transition economy must take into account the effect of inflationary processes both inside and outside the money sphere. Therefore, pure monetary measures in the framework of this policy should be supplemented by deep structural–technological changes and corrections in fiscal policy, in particular as a part of government-directed program of structural reforms. The main priority of this program is to reduce production costs in branches enmeshed in a structural hyperinflation crisis.
2. Streaming of the state budget is one of the main stabilization mechanisms in a transition economy. However, elimination of budget deficit is not the only goal: it is necessary to take into account the effect of taxes on prices and product sales, positive and negative consequences of differential subsidies to some industries, etc. Hence, the multicriteria approach and construction of objective function are necessary for budget optimization.
3. The changes in the utility born by decrease of some arguments of objective function cannot be fully compensated by means of increase of its other arguments in the model of budget optimization. Therefore the objective function in this model is nondifferentiable. Its construction can be done by methods of ordinary regression.
4. According to model runs the tax system in a transition economy should be different from that under stable conditions. The tax system should rely on taxes assessed independently of business results (taxes on land, property, etc.). The tax burden should be 20–30 % of the profit of an average business entity. The sum of other payments to the budget should not exceed 30–40 % of these taxes.

The following studies will be made for further investigation:

- the account of internal impact on national economy and joint optimization the customer tariff and tax rates;
- the analysis of influence of random processes in considered model;
- the development of computer tools for model runs and objective function construction;
- the real-data model runs using the current economic information for different countries with a transition economy.

The application of smooth approximations of objective function and comparison of obtained results also seems to be interesting. For example, quadratic objective functions for our model can be constructed on the basis of ordinal data.

Our analysis of the existing fiscal system in Ukraine was done by means of using the BUDGET system.

It is well known that the existing tax system is one of the major factors of the economic crisis that lasts in Ukraine. Destructive influence on manufacture exercises not so much overestimate of existing taxes rates as their macro and a microeconomic nonagreement, unsystematic character, indirectivity on stimulation of economic growth, ambiguity of charging taxes rules, an opportunity of evasion from their payment and other general systematic lacks. So, reforming of tax system in Ukraine can be considered as the major compound of system-structural transformations, it

should be preceded by versatile scientific researches with application of various quantitative and qualitative methods, including mathematical modeling.

During scientific researches on this subject, mathematical modeling has been applied for revealing the mutual influence of existing kinds of taxes and specifications of the taxation, on one hand, and macroeconomic processes—on the other one. For a solution of this problem the simulation dynamic model from the BUDGET system has been applied, which displays movement of means on separate compounds of the system of national accounts. Except for the tax system, the real sector of economy, the state budget, end-consumers (households), the consolidated account of the external economic operations, bank system are considered in the models. These models provide a more detailed description of any of the specified subsystems, for example, sectorized division is taken into account in the real sector. Consideration of simultaneous or alternative actions of various mechanisms of the pricing, inherent in transition economy, and various strategies of direct or mediate influences of the state on the prices and final consumer demand are the main features of our models. In particular, our models may allow for subsidizing of manufactures and social payments to consumers from the budget, influence on the prices of different ways of covering the deficit of the state budget and changes in monetary policy, etc.

With the purpose of our analysis of influence of the suggested taxation rates changes on volumes of inland revenues, our alternative computer modeling has been carried out on the basis of the imitating model considered above. Taking into account the available information, complete character of the budget for the period of its adoption and some other factors, the modeling was conducted for data of the state budget of Ukraine for 1998. An overall objective was to estimate the influence of taxation rates reductions on filling of the budget and on the course of macroeconomic processes.

Calculations were carried out according to the following variants.

Variant I. Upon specifications acting at that time and conditions of the taxation it was supposed that the production of manufactured volumes and sales will respond to macroeconomic forecasts on the basis of which the budget project has been developed. In particular, 3 % reduction of real GDP were supposed. This variant was considered for an estimation of accuracy of modeling which would be characterized by approximation of values of the indicators received during modeling to values in the project specified.

Variant II. Upon specifications acting at that time and conditions of the taxation it was supposed that the production of manufactured volumes and sales are determined by effective consumers' demand. This variant responds to a real world situation which has developed together with fulfilment of the state budget upon fiscal policy acting at that time.

Variant III. Under assumptions of a previous variant concerning factors which influence volumes of manufacture and sales of production that changes concerning reduction of tax pressure on the manufacturer and rationalization of system of taxes will be carried out. In particular it is supposed that the rate of the VAT will

Table 2.10 Results of alternative modeling of fulfilment of the basic indicators of the state budget of Ukraine for 1996 (in % to planned values)

Indicators	Variants		
	I	II	III
Budget incomes from:			
(a) VAT	103.9	67.1	58.2
(b) Profits tax	81.6	46.7	45.4
(c) Payment fund taxes	96.7	81.2	78.1
Aggregate budget incomes	97.9	77.2	85.3
Aggregate budget expenses	101.1	98.4	100.5

be decreased from 20 % down to 16 %, and the cumulative rate of the taxation of a payment fund will be decreased from 52 % down to 32 %. Also an introduction of the fixed tax to activity of small enterprises and the uniform land tax with the abolishment of other direct taxes for agricultural producers was supposed. This complex of preconditions covers the majority of constructive suggestions concerning changes in the taxation expressed in the middle of the 90th. Our results of modeling according to this variant, displayed in Table 2.10, have shown how the specified suggestions can influence the economic situation in the country.

Let's analyze the received results. The deviation of values of the majority of indicators according to variant from planned in the project of the budget does not exceed 3–4 %. So, there is some ground to consider this size as a rough estimate of modeling accuracy. Comparison of variants and II gives the basis to assume that during macroeconomic forecasting reduction of a final demand which, according to variant II, should serve as the reason of recession of GDP at a rate of 9 % have been inadequately appreciated. (Real reduction of GDP in 1996 has barely exceeded 10 % that completely falls into the modeling accuracy specified above.) Under these conditions, reduction of tax pressure can become one of the major factors of economic stabilization. The fact that in variant III in comparison to variant II inpayments from the profits taxes practically have not decreased under essential reduction of the tax base (owing to the abolishment of this tax for small and agricultural enterprises) testifies of probable increase of profitability in the majority of branches of economy. It is necessary to pay a special attention to the fact that reduction of inpayments from payment fund taxes in variant III are much more smaller in comparison with reduction of rates of these taxes. It testifies of overvaluation of such rates in the working tax laws. Though in variant III essential budgetary deficit remains, it nevertheless would be less, than in variant II. So, it is possible to draw a conclusion that the reductions of taxes mentioned above as a whole would exercise positive influence on process of fulfilment of the state budget.

An even greater positive influence they could exercise on the general macroeconomic situation. Comparison of some macroeconomic indicators according to variants II and III is carried out in Table 2.11.

Thus, there are all grounds to state that absence of positive changes in tax system is one of the major factors of long economic decline. The carried out researches allow to determine such mainstreams of necessary improvement of this system.

Table 2.11 Value of the main macroeconomic indicators for variants II and III

Indicators	Variants	
	II	III
Increase (decline) of GDP (%)	−9.0	−6.1
including for the last quarter	−5.5	1.7
Cumulative real incomes of consumers (% to values for var. II)	100	128.7
including group with the lowest incomes	1000	113.1
Incomes of the enterprises (% to values for var. II):		
(a) For group with the highest incomes	100	101.5
(b) For group with the lowest incomes	100	102.8
Consumer price index (% to the beginning of the year)	1,281.4	120.7

1. The question concerning expediency of attraction to the state budget of finance in volume of 35–40 % of GDP remains debatable. On one hand, this specification is, certainly, too big under conditions of weak economy, in which market relations are still developing and the public sector starts to function. On the other hand, taking into account the structure of final consumption on needs of the state, the state budget is one of the major factors of creation of permanent effective demand for end production, and consequently, one of the factors of deceleration of economic decline. In addition, Ukraine has developed social infrastructure that is financed from the state budget destruction of which will have unforeseen consequences. In our opinion, the main flaw of fiscal system existing in Ukraine is not the fact that tax rates are too big, but the fact that the existing system of taxation does not assist economic growth, in particular, suppresses the final effective demand.
2. Taking this into account, the most negative influence on economy have indirect taxes on fund of payment. These taxes can be reduced to 10–20 %. If extricated current assets are directed on increase of a level of wages, it will exercise appreciable positive influence both on dynamics and on structure of GDP. Thus, the general inpayments from the specified taxes will decrease much less, than reduction of their rates, and financing of target expenses on which they were directed, will not make especial problems.
3. Introduction and the further development of taxes the volumes of which do not depend on results of economic activities is necessary: the land tax, the property tax, the rent payments, the fixed taxes, etc. Inpayments from these taxes should make at least 20–25 % from volume of the general inpayments to the consolidated budget of the country.
4. The existing custom duties do not exercise such destructive influence on economy as taxes do. The majority of them can be considered close to the optimum under conditions which have developed. Increase of the custom duties concerning some groups of consumer goods and services (polygraphic production, products

of light industry, luxury goods) at the same time is possible. But the part of these kinds of production in the general import is rather small. An increase of import tariffs concerning production of intermediate use is inexpedient as it intensifies processes of cost-push inflation to which there are already some preconditions.

5. The further improvement of tax system demands carrying out of complex economic researches with attraction of experts of different directions.

2.4 Optimization Model of Structural and Technological Changes

The experience with overcoming high inflation in Ukraine in 1992–1994 has focused attention on both the unavoidable use and the limited effect of traditional measures, such as reduction of money emission, reduction of the budget deficit, elimination of subsidized enterprise credit, and the active exchange-rate policy of the National Bank. The price increases were a consequence of deep structural distortions in the economy inherited from the earlier decades and significantly amplified in the initial stages of the transition. Under these conditions, monetary stabilization methods not accompanied by radical structural adjustment simply transformed these distortions into a different form, e.g., inflation was replaced with an increase of accounts payable, but the origins of the distortions were not eliminated. Once relative financial stability has been achieved, it is essential to proceed with a structural reform of the economy. The main tasks include restructuring of the fiscal system and the social security system, changes in property relations, development of a market infrastructure, encouragement of small- and medium-size entrepreneurship, and transformation of the technological structure of the economy.

Structural transformations are much more difficult task than initial stabilization of the quantity of money. No standard solutions are available in this area, and much depends on the numerical characteristics describing the features of the economy. This is particularly relevant for the technological structure, which is unique for each country. The elucidation of technological–structural distortions that affect crisis phenomena in the economy and analysis of techniques for the elimination of these distortions require the use of quantitative methods, in particular, mathematical modeling. In this section, these methods are applied to plan technological changes for increased production efficiency.

The existing technological system in Ukraine is the result of several decades of cumulative influence of the following factors:

- low prices of natural resources (especially low energy prices), which encouraged wasteful use;
- low prices of basic consumer commodities combined with low wages, which resulted in a low share of value added in the consumer prices of most goods;

- a pervasive system of budgetary subsidies, which distorted the interindustry relative prices, artificially lowering the prices of some goods while raising the prices of others;
- the closed nature of the socialist economy, which did not require competitiveness of domestic producers. Today the economy is functioning under fundamentally different conditions. The prices of energy imports have risen to the world level, and in some cases actually exceed the world prices. Energy prices were one of the main factors driving the cost inflation in 1992–1994 [25,47]. The inflationary process was characterized by increases of production costs, which led to price increases long before consumer incomes had a chance to catch up. This reduced the consumer demand, lowered the share of value added in the prices of goods, and triggered Keynesian macroeconomic processes [20], when the price is insensitive to demand and the production volume is insensitive to price. Under these conditions, one of two alternative scenarios is observed, depending on the strictness of the monetary policy:
 - (a) lax payment discipline leading to an increase of accounts payable and accelerated decrease of production, or
 - (b) accelerated cost inflation. To overcome this situation, it is necessary to increase the disposable income of the consumers (primarily households) while at the same time reducing production costs.

Decrease of production costs is thus a major goal of technological–structural change. It can be achieved in a twofold way: first, by changing the technology so as to reduce inputs of commodities, materials, and other manufactured products per unit of output, and second, by gradually reducing the use of energy- and resource-inefficient technologies, until they are completely phased out. Both these approaches involve changes at the interindustry level, which explains the choice of interindustry models as an analytical tool. Note that these changes are manifested in the reduction of the input matrix elements of the aggregated interindustry balance. Indeed, the direct input coefficients a_{ij} representing the quantity of industry i product required to produce a unit of industry j product is representable

$$a_{ij} = \sum_{k_1=1}^{l_i} \sum_{k_2=1}^{l_j} \tilde{a}_{ij}^{k_1 k_2} W_j^{k_2}, \quad (2.18)$$

where

- k_1 are the indices of the technologies and subindustries included in industry i ,
- k_2 are the indices of the technologies and subindustries included in industry j ,
- l_i, l_j is the number of such technologies and subindustries,
- $\tilde{a}_{ij}^{k_1 k_2}$ is the consumption of products manufactured by technology k_1 in industry i to produce a unit of product by technology k_2 in industry j ,
- $W_j^{k_2}$ is the share of technology k_2 in industry j .

Then the first of the two approaches to technological–structural change mentioned above will reduce the values of $\bar{a}_{ij}^{k_1 k_2}$, whereas the second approach will reduce the values of $W_j^{k_2}$ for the largest $\bar{a}_{ij}^{k_1 k_2}$. We see from (2.18) that both approaches reduce a_{ij} . This explains the choice of interindustry aggregated balance models as the tool for analyzing and planning technological–structural change. One such a model is considered in this section.

Assume that the country's economy consists of n aggregated industries and $A = \{a_{ij}\}$ is the direct input matrix for these industries. We assume that labor cost is a linear function of production volume in each industry. Let q_i be the share of labor costs in the price of industry i product. Denote by y_i and x_i respectively the net and the gross product of industry i in constant prices. These quantities are linked by the relationship

$$x_i = \sum_{j=1}^n a_{ij} x_j + y_i, \quad i = \overline{1, n}, \quad (2.19)$$

and the real consumer income D is given by

$$D = \sum_{i=1}^n q_i x_i. \quad (2.20)$$

The problem is to determine what changes in the elements of the matrix A and the vector $q = \{q_1, \dots, q_n\}$ will maximize the consumer income D without triggering additional inflationary forces.

Conditions excluding outburst of cost inflation under the impact of endogenous factors are considered in Mikhalevich [23]. Using the coefficients of the matrix A , we can write these conditions in the form

$$\sum_{i=1, i \neq j}^n \frac{a_{ij}}{1 - a_{jj} - \bar{q}_j} < 1, \quad j = \overline{1, n}, \quad (2.21)$$

where $\bar{q}_j$ is the share of value added in the price of industry j product.

We assume that the net product consists of two parts: a part that depends on D and a part independent of D . Assuming linear dependence of the first part on consumer income, we obtain

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, n}, \quad (2.22)$$

where the coefficients α_i , mainly reflect the structure of individual consumption and internal investment, while h_i is determined by the export–import balance of the industries and by public consumption.

We use these relations to express D in terms of A and q . From (2.19), $x = (E - A)^{-1}y$, where $x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n)$. Therefore, $D = (q, x) = (q, (E - A)^{-1}y)$. From the last equality and (2.22) we obtain

$$D = \frac{q^T (E - A)^{-1}h}{1 - q^T (E - A)^{-1}\alpha},$$

where $h = (h_1, \dots, h_n)$, $\alpha = (\alpha_1, \dots, \alpha_n)$.

Also note that $\bar{q}_j$ is representable in the form

$$\bar{q}_j = l_j q_j + d_j,$$

where l_j and d_j are given coefficients.

Allowing for errors in input data and for the maintenance of a reserve to support a noninflationary increase of the value-added components that are independent of q , we replace (2.21) with the conditions

$$\sum_{i=1, i \neq j}^n \frac{a_{ij}}{1 - a_{jj} - (l_j q_j + d_j)} \leq \beta, \quad i = \overline{1, n}, \quad (2.23)$$

where $\beta < 1$ is a prespecified threshold value. Based on these remarks, the model takes the following form. Find the changes Δa_{ij} and Δq_j ($i = \overline{1, n}$, $j = \overline{1, n}$) in the current elements of the matrix A and the vector q so as to maximize the function

$$f(\Delta A, \Delta q) = \frac{(q + \Delta q)^T (E - (A + \Delta A))^{-1}h}{1 - (q + \Delta q)^T (E - (A + \Delta A))^{-1}\alpha} \rightarrow \max, \quad (2.24)$$

where $\Delta q = (\Delta q_1, \dots, \Delta q_n)$, $\Delta A = \{\Delta a_{ij}\}_{ij}^{nn}$ subject to the constraints

$$\sum_{i=1, i \neq j}^n \frac{a_{ij} + \Delta a_{ij}}{1 - (a_{jj} + \Delta a_{jj}) - (l_j (q_j + \Delta q_j) + d_j)} \leq \beta, \quad j = \overline{1, n}, \quad (2.25)$$

$$0 \leq q_j + \Delta q_j \leq 1, \quad 0 \leq a_{ij} + \Delta a_{ij} \leq 1, \quad i, j = \overline{1, n}, \quad (2.26)$$

$$a_{jj} + \Delta a_{jj} + l_j (q_j + \Delta q_j) + d_j \leq 1, \quad j = \overline{1, n}. \quad (2.27)$$

These constraints may be augmented with bounds on allowed changes of direct input coefficients imposed by the existing technologies

$$\gamma_{ij} \leq \Delta a_{ij} \leq \bar{\gamma}_{ij}, \quad i, j = \overline{1, n}. \quad (2.28)$$

and bounds imposed by the availability of resources for technological–structural changes,

$$\sum_{j=1}^n \sum_{i=1}^n b_{kij} \max(0, -\Delta a_{ij}) \leq B_k, \quad k = \overline{1, K}, \quad (2.29)$$

where

- K is the number of different resources,
- B_k is the quantity of resource k allocated for reduction of production costs,
- b_{kij} is the consumption of this resource to implement measures that lead to a unit reduction in the consumption of industry i product to produce a unit of industry j product.

Note that aggregated interindustry models are widely used for planning and analyzing technological–structural change in the economy. In particular, the proposed model may be regarded as further development of [13], where Glushkov has considered the changes in the matrix A that ensure attainment of the production target (x, y) subject to the given productive resources. Although this is a standard problem for a centrally planned economy, it remains relevant for a transition economy highly dependent on critical imports of energy, raw materials, and other industrial inputs.

The optimization problem (2.24)–(2.29) is a difficult mathematical programming problem. The functional (2.24) and the system of constraints (2.25)–(2.29) lead to a number of difficulties even when we try to find a local extremum of problem (2.24)–(2.29). These difficulties impose certain requirements on the choice of the solution method. The simplest requirement is associated with the constraint system (2.25)–(2.29), where constraints (2.12) are nonsmooth, and the group of constraints (2.25) are fractional-linear. In constraint j from (2.25) the denominator is independent of i , and by (2.27) we conclude that it is positive. We can thus rewrite the group of constraints (2.25) in the form

$$\sum_{i=1, i \neq j}^n (a_{ij} + \Delta a_{ij}) + \beta(a_{jj} + \Delta a_{jj}) + \beta(l_j(q_j + \Delta q_j) + d_j) \leq \beta, \quad j = \overline{1, n}. \quad (2.30)$$

Then the constraint system (2.25)–(2.29) can be replaced with a single convex nonsmooth constraint

$$F(\Delta A, \Delta q) = \max \{f_1(\Delta A, \Delta q), \dots, f_5(\Delta A, \Delta q)\} \leq 0, \quad (2.31)$$

where

$$\begin{aligned}
 f_1(\Delta A, \Delta q) &= \max_j \left\{ \sum_{i=1, i \neq j}^n (a_{ij} + \Delta a_{ij}) + \beta(a_{jj} + \Delta a_{jj}) \right. \\
 &\quad \left. + \beta(l_j(q_j + \Delta q_j) + d_j) - \beta \right\}, \\
 f_2(\Delta A, \Delta q) &= \max_{i,j} \{a_{ij} + \Delta a_{ij} - 1, -a_{ij} - \Delta a_{ij}, q_j + \Delta q_j - 1, -q_j - \Delta q_j\}, \\
 f_3(\Delta A, \Delta q) &= \max_j \{a_{jj} + \Delta a_{jj} + l_j(q_j + \Delta q_j) + d_j - 1\}, \\
 f_4(\Delta A, \Delta q) &= \max_{i,j} \{\Delta a_{ij} - \bar{\gamma}_{ij}, \gamma_{ij} - \Delta a_{ij}\}, \\
 f_5(\Delta A, \Delta q) &= \max_k \left\{ \sum_{j=1}^n \sum_{i=1}^n b_{kij} \max(0, -\Delta a_{ij}) - B_k \right\}.
 \end{aligned}$$

The functional (2.24) leads to greater difficulties, because it is not continuous for all Δa_{ij} and Δq_j , $i, j = \overline{1, n}$. To avoid discontinuities, we need to satisfy the following conditions:

- (a) the matrix $E - (A + \Delta A)$ is nonsingular;
- (b) $(q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha \neq 1$.

Inequalities (2.21) imply that A is a production matrix, and therefore the first condition is satisfied for Δa_{ij} from the feasible region of (2.25)–(2.29) and from some sufficiently large neighborhood of the feasible region. The product $q^T (E - A)^{-1} \alpha$ may be regarded as a multiplier for “increase of production-increase of consumer incomes.” Increase of production requires additional cost savings even if resource-efficient technologies are used. Moreover, the newly created value added is never used entirely for consumption, and for actual technologies we always have $(q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha \neq 1$. Therefore, in the region where ΔA and Δq satisfy constraint (2.24), the function $f(\Delta A, \Delta q)$ is continuously differentiable and its gradient is computed by the following formulas

$$\begin{aligned}
 \frac{\partial f(\Delta A, \Delta q)}{\partial \Delta q_j} &= \frac{1}{1 - (q + \Delta q, \tilde{\alpha})} (e_j, \tilde{h}) \\
 &\quad + \frac{(q + \Delta q, \tilde{h})}{(1 - (q + \Delta q, \tilde{\alpha}))^2} (e_j, \tilde{\alpha}), \quad j = \overline{1, n},
 \end{aligned} \tag{2.32}$$

$$\begin{aligned} \frac{\partial f(\Delta A, \Delta q)}{\partial \Delta a_{ij}} &= \frac{1}{1 - (q + \Delta q, \tilde{\alpha})} (e_i, \tilde{q})(e_j, \tilde{h}) \\ &+ \frac{(q + \Delta q, \tilde{h})}{\left(1 - (q + \Delta q, \tilde{\alpha})\right)^2} (e_i, \tilde{q})(e_j, \tilde{\alpha}), \quad i, j = \overline{1, n}. \end{aligned} \quad (2.33)$$

Here we use the following notation:

$$\begin{aligned} \left(E - (A + \Delta A)\right)^{-1} h &= \tilde{h}, \quad \left(E - (A + \Delta A)\right)^{-1} \alpha = \tilde{\alpha}, \\ \left(E - (A + \Delta A)^T\right)^{-1} (q + \Delta q) &= \tilde{q}, \end{aligned}$$

e_i is the n -dimensional vector whose i -th component is 1 and all other components are 0.

When evaluating the gradient of $f(\Delta A, \Delta q)$ with respect to the variables Δa_{ij} , we use the following differentiation rule for the inverse matrix X^{-1} , where $X = \{x_{ij}\}_{ij}^{nn}$:

$$\frac{\partial X^{-1}}{\partial x_{ij}} = -X^{-1} \frac{\partial X}{\partial x_{ij}} X^{-1} = -X^{-1} e_i e_j^T X^{-1}.$$

This is a consequence of differentiating the identity $XX^{-1} = E$.

Yet another difficulty is associated with the large dimension of problem (2.24)–(2.29). Thus, the number of variables in this problem is $n^2 + n$ (if we need to find the changes of all the elements of the matrix A , which is fairly large even for n of the order of 10–20).

Thus, to find a local extremum in problem (2.24)–(2.29) we need an efficient extremum-seeking method for nonsmooth functions. The $r(\alpha)$ -algorithm [39, 43] meets this requirement. This is an efficient practical method from the family of subgradient type methods with space dilation in the direction of the difference of two successive subgradients. First, it can handle the nonsmoothness of the constraint (2.31). Second, given the specific features of the objective function (2.24), we can use its modified version [42] evaluating the gradient of the function $f(\Delta A, \Delta q)$ only in the region of its continuous differentiability. Third, experience shows that the $r(\alpha)$ -algorithm is efficient for functions with several hundreds of variables (including functions with deep narrow valleys). The multi-extremum nature of problem (2.24)–(2.29) can be allowed for (to a certain extent) by running the calculations from different starting points with a small initial step h_0 .

The $r(\alpha)$ -algorithm is the basis of the program MULSTR for PC. This is a Fortran program to find a local extremum of problem (2.24)–(2.29). The program allows control of the following parameters:

- (a) the initial approximation,
- (b) the penalty multiplier (used to reduce problem (2.24)–(2.29) to an unconstrained optimization problem);

- (c) the parameters of the $r(\alpha)$ -algorithm, which include the initial stepping multiplier h_0 ,
- (d) the maximum number of iterations *maxitn*, stopping parameters specified by accuracy in the argument and the gradient norm ε_x and ε_g .

Other parameters of the $r(\alpha)$ -algorithm are chosen following the recommendations of [42] (see also [39]). In particular, the space dilation factor is taken at $\alpha = 2$, and for the adaptive step control parameter we take $q_1 = 0.95$, $q_2 = 1.1$, $n_h = 3$. The objective function gradients are calculated from formulas (2.32), (2.33) using the numerically stable subprograms DECOMP and SOLVE [11] to solve the systems of linear equations. These programs are based on the Gaussian elimination technique with partial choice of the pivot element.

The program MULSTR has been applied to run test calculations for real-life data with the model (2.24)–(2.29). These test calculations are described below.

Our numerical experiments had three objectives: first, to assess the efficiency of the proposed algorithm when applied to real-life data with problems of real-life size; second, to anticipate difficulties with data and computation procedures that may arise in the model; and third, to analyze the recommendations for management decisions produced by the model.

The program MULSTR has been used at the Institute of Cybernetics to run test calculations using the data from [18]. We consider the aggregated 18-industry balance and make the following changes in the model (2.24)–(2.29).

1. As no reliable information is available for some components of the vector h and other components are unstable, we replace (2.24) with the following objective function:

$$F(\Delta A, \Delta q) = (q + \Delta q)^T \left(E - (A + \Delta A) \right)^{-1} \alpha.$$

The function (2.24) obviously increases with the increase of the function $F(\Delta A, \Delta q)$. In this function, α are the industry shares in the net product components dependent on consumer income (individual consumption and investment). The values of α were calculated using the data of [45].

2. In the constraints (2.25) we took $\beta = 0.85$. Given the accuracy of the input data, this ensured at least a 5 % increase in the share of value added in the price of industry products.
3. Only input coefficients greater than 0.001 were allowed to vary (within 50 % of their initial value); the remaining input coefficients were kept unchanged. The number of variables in problem (2.24)–(2.29) was thus 200 (of which 182 were the changes of input matrix elements and 18 the changes in the components of the vector q).
4. As the constraint (2.29) we used the bound on investments attracted for technological–structural change. Alternative calculations were carried out for different investment volumes. The coefficients and the right-hand side of the constraint (2.29) were based on expert estimates provided by various advisory and analytical divisions of the Cabinet of Ministers of Ukraine.

Three scenarios were considered. According to the first scenario, the constraint (2.29) permits reducing the ratio between intermediate consumption and GDP by 10–15 % of the 1995 level. According to the second scenario (which requires double the capital investment), the ratio can be reduced by 15–20 %. The third scenario assumed a fixed share of labor costs in the price of industry products, while keeping all the assumptions of the first scenario. This scenario was regarded as reflecting the impact of labor costs on final outcomes.

The three scenarios led to the following results.

There is a group of “critical” elements in the matrix A , which are reduced with any investment volume. These critical elements are the electric power inputs for the production of a unit of product in coal and food industries; inputs of all forms of fuel to produce a unit of product in the power generating industry, in transport, and in agriculture; metal inputs in machine building; and transport costs in most aggregated heavy industries. The reduction of these components ranges from 20 to 50 % of their initial value, depending on the allocated investment resources. The remaining elements of the matrix A can be reduced with a sufficiently large investment, and they are grouped as follows.

Power Efficiency. In addition to previously mentioned industries, it is advisable to reduce the electric power costs in trade and catering, in transport, in machine building, in ferrous and nonferrous metallurgy, in chemical and petrochemical industry. The unit inputs of liquid fuel and gas should be reduced in light and food industry, in trade and catering; the unit inputs of coal should be reduced in ferrous metallurgy.

Resource Efficiency. It is advisable to reduce (by 20–40 %) the rate of input use from the main material manufacturing industries (ferrous and nonferrous metallurgy, chemical and petrochemical industry) by the power-generating industry, machine building, and transport. It is also required to reduce the rate of input use from machine building and wood and lumber industry in most other material manufacturing industries. At the same time, the second scenario allows these input rates to be increased in construction (this provides evidence that the prices of construction services are maintained at an artificially high level).

Reduction of Accompanying Costs. Almost all industries should reduce the rate of use of services from trade and catering and from transport. This signifies, in particular, reduction of trade and transport markups.

Tables 2.12 and 2.13 present the number of coefficients in each block that have to be adjusted under each scenario. Note that scenario 3 requires much greater technological–structural changes than scenarios 1 and 2, and the optimal objective function value for this scenario is approximately half that for the other two scenarios. This indicates that achievement of real economic growth requires not only radical changes of technology, but also deep restructuring of the existing system of labor remuneration.

The observed reduction of material costs makes it possible to increase the share of labor cost in the product price of most industries from the present level of

Table 2.12 Results of calculations using various scenarios for the aggregated interindustry model

Scenarios and blocks	Number of varied coefficients	Decrease				Increase
		Less than 10 %	10–25 %	25–47 %	Over 47 %	
Scenario 1	182	6			13	40
<i>Blocks</i>						
Energy efficiency	45	2	1	4		8
Resource efficiency	54	1		7		12
Reduction of accompanying costs	30	1	2	2		12
Scenario 2	182	9	14	32	39	8
<i>Blocks</i>						
Energy efficiency	45	2	1	4		8
Resource efficiency	54	1		7		12
Reduction of accompanying costs	30	1	2	2		12
Scenario 3	182	9	14	32	39	8
<i>Blocks</i>						
Energy efficiency	45	2	1	4		8
Resource efficiency	54	1		7		12
Reduction of accompanying costs	30	1	2	2		12

Table 2.13 Objective function value for optimal solution under each scenario

Scenario	Value of $q^T(E - A)^{-1}\alpha$	Remarks
Scenario 1	0.41579	–
Scenario 2	0.48513	–
Scenario 3	0.21178	–
<i>Note</i>		
For the 1995 direct input matrix	0.22669	Does not satisfy constraints (2.29)

3–7 % to 15–25 %. The recommended increase is different for different industries, ranging from a high of 10–17 % in agriculture, light, and food industry, and machine building to a low of 3–4 % in chemical and petrochemical industry and in wood and lumber industry. In ferrous and nonferrous metallurgy and in construction, this indicator actually should be decreased.

Note that the proposed model is primarily intended to identify the main directions of change of technological coefficients. It has to be modified to examine specific reform measures, especially on the micro level.

Measures intended to produce technological–structural changes are usually implemented as (industry-specific) innovation programs. The set of all possible projects is usually finite, but the number of projects is so large that it is impossible to choose the appropriate projects by simple enumeration. It is thus necessary to consider a discrete version of the model (2.24)–(2.29).

Let N be the number of projects, and for each project we know what changes Δa_{ij}^t of the matrix A and Δq_j^t of the vector q it will produce ($i, j = \overline{1, n}$, $t = \overline{1, N}$). It is required to find a list of projects that ensures the maximum increase of real consumer income subject to constraints that avoid cost inflation and subject to bounds on resource availability z_t .

Introducing the variables

$$z_t = \begin{cases} 1 & \text{if project } t \text{ is implemented,} \\ 0 & \text{otherwise,} \end{cases}$$

we rewrite the model in the form

$$\frac{q(z) \left(E - A(z) \right)^{-1} h}{1 - q(z) \left(E - A(z) \right)^{-1} \alpha} \rightarrow \max,$$

where

$$\begin{aligned} q(z) &= (q_1(z), \dots, q_n(z)), \quad A(z) = \{a_{ij}(z)\}_{ij}^{nn}, \\ q_j(z) &= q_j + \sum_{t=1}^N \Delta q_j^t z^t, \quad a_{ij}(z) = a_{ij} + \sum_{t=1}^N \Delta a_{ij}^t z^t, \end{aligned}$$

subject to

$$\sum_{\substack{i=1 \\ i \neq j}}^n \frac{a_{ij}(z)}{1 - a_{ij}(z) - l_j(q_j(z) + d_j)} \leq \beta, \quad j = \overline{1, n},$$

$$0 \leq a_{ij}(z) \leq 1, \quad 0 \leq q_j(z) \leq 1, \quad i, j = \overline{1, n},$$

$$\sum_{t=1}^N b_{kt} z_t \leq B_k, \quad k = \overline{1, K}.$$

Here K is the number of resources, B_k is the volume of resource k allocated for the projects of technological–structural change, and b_{kt} is the consumption of this resource by project t .

We can also add constraints specifying mutual exclusion of some projects:

$$\sum_{(t_1, t_2) \in R} z_{t_1} z_{t_2} = 0,$$

where R is the set of mutually exclusive projects.

This model leads to a discrete optimization problem classifiable as a multi-dimensional knapsack problem with a nonlinear nonadditive objective function and nonlinear constraints. The latter complicates solution of the problem by the branch-and-bound method and algorithms based on sequential analysis of partial solutions (such as the Bellman method). Certain difficulties are also attributable to the existence of multiple extrema of the objective function. The problem should thus be considered as a combinatorial optimization problem on the space of combinations and solved by the descent vector method [34], which is a simple iterative algorithm for approximate solution of such problems.

If we define the unit neighborhood in the combination space [34, 36] and restrict the local search to such neighborhoods, then the calculation of the descent vector components requires bounding the objective function changes when project t_1 is added to the set of previously accepted projects t_m or when project t_1 is excluded from the previous set. As we have noted above, innovation projects are inherently sectoral, and therefore all the changes in the components of the matrix $A(z)$ are concentrated in a particular column $j(t_1)$. This fact can be exploited to bound the objective function. Let

$$\begin{aligned} A^{T_m} &= \left\{ a_{ij}^{T_m} \right\}_{ij}^{nn}, & a_{ij}^{T_m} &= a_{ij} + \sum_{t \in T_m} \Delta a_{ij}^t, \\ q^{T_m} &= \left\{ q_j^{T_m} \right\}_j^n, & q_j^{T_m} &= q_j + \sum_{t \in T_m} \Delta q_j, \end{aligned}$$

$$A_j^{t_1} \text{ is the column vector } \begin{pmatrix} \Delta a_{ij(t_1)} \\ \vdots \\ \Delta a_{nj(t_1)} \end{pmatrix}.$$

We have the following theorem.

Theorem 2.1. *Let $H_{t_1}^+$, $H_{t_1}^-$ be the objective function increments following the inclusion (elimination) of project t_1 in (from) the set t_m , t_m^+ and t_m^- are the resulting new sets of projects. Then*

$$H_{t_1}^+ = \frac{\overline{H}h \left(1 - q^{T_m} (E - A^{T_m})^{-1} \alpha \right) + \overline{H} \alpha q^{T_m} (E - A^{T_m})^{-1} h}{\left(1 - q^{T_m} (E - A^{T_m})^{-1} \alpha \right)^2 + \overline{H} \alpha \left(1 - q^{T_m} (E - A^{T_m})^{-1} \alpha \right)},$$

$$H_{t_1}^- = \frac{\hat{H}h(1 - q^{T_m}(E - A^{T_m})^{-1}\alpha) + \hat{H}\alpha q^{T_m}(E - A^{T_m})^{-1}h}{(1 - q^{T_m}(E - A^{T_m})^{-1}\alpha)^2 + \hat{H}\alpha(1 - q^{T_m}(E - A^{T_m})^{-1}\alpha)},$$

where

$$\bar{H} = \left((q + \Delta q_{j(t_1)e_{j(t_1)}}) \left(\frac{1}{1 - g_{j(t_1)}} \bar{G} \right) + \Delta q_{j(t_1)e_{j(t_1)}} \right) (E - A^{T_m})^{-1},$$

$$\hat{H} = \left((q + \Delta q_{j(t_1)e_{j(t_1)}}) \left(\frac{1}{1 + g_{j(t_1)}} \bar{G} \right) - \Delta q_{j(t_1)e_{j(t_1)}} \right) (E - A^{T_m})^{-1},$$

$\bar{G} = (E - A^{T_m})^{-1} A_j^{t_1}$, g_j is the j -th element of $\bar{G}$, e_j is the j -th unit vector.

Here

$$(E - A^{T_m^+})^{-1} = (E - A^{T_m})^{-1} + \frac{1}{1 - g_i} \bar{G} (E - A^{T_m})^{-1}, \quad (2.34)$$

$$(E - A^{T_m^-})^{-1} = (E - A^{T_m})^{-1} - \frac{1}{1 + g_i} \bar{G} (E - A^{T_m})^{-1}. \quad (2.35)$$

Proof. By definition,

$$H_{t_1}^+ = \frac{(q + \bar{\Delta}q) (E - A^{T_m^+})^{-1} h}{1 - (q + \bar{\Delta}q) (E - A^{T_m^+})^{-1} \alpha} - \frac{q (E - A^{T_m})^{-1} h}{1 - q (E - A^{T_m})^{-1} \alpha}, \quad (2.36)$$

where $\bar{\Delta}q = \Delta q_{j(t_1)e_{j(t_1)}}$.

Using the estimates from [39] for the inverse matrix when one of the columns of the original matrix is changed, we obtain

$$(E - A^{T_m^+})^{-1} - (E - A^{T_m})^{-1} = \frac{1}{1 - g_i} G (E - A^{T_m})^{-1}, \quad (2.37)$$

where $G = (E - A^{T_m})^{-1} \Delta A^{T_m}$, g_j is the j th element of G , ΔA^{T_m} is the vector of changes made in column j of the matrix A^{T_m} . If the project $A_j^{t_1}$ is added, the column $j(t_1)$ is changed, and the associated changes form the vector $A_j^{t_1}$. We thus have $G = (E - A^{T_m})^{-1} A_j^{t_1} = \bar{G}$, and equality (2.34) follows from (2.37). Let us estimate

$$\begin{aligned} \bar{H} &= (q + \bar{\Delta}q) (E - A^{T_m^+})^{-1} - q (E - A^{T_m})^{-1} \\ &= q \left((E - A^{T_m^+})^{-1} - (E - A^{T_m})^{-1} \right) + \bar{\Delta}q (E - A^{T_m^+})^{-1}. \end{aligned}$$

From (2.37) and (2.34) we obtain

$$\bar{H} = \left((q + \bar{\Delta}q) \left(\frac{1}{1 - g_{j_{t_1}}} \bar{G} \right) + \bar{\Delta}q \right) (E - A^{T_m})^{-1}.$$

Using this estimate, we reduce the right-hand side of (2.19) to a common denominator and simplify the resulting expression. The end result is the sought equality for $H_{t_1}^+$.

The equality for $H_{t_1}^-$ and relationship (2.35) are derived similarly, noting that when the project t_1 is eliminated the changes in the column $j(t_1)$ of the matrix A^{T_m} will have opposite signs and

$$H_{t_1}^- = \frac{(q - \bar{\Delta}q) (E - A^{T_m^-})^{-1} h}{1 - (q - \bar{\Delta}q) (E - A^{T_m^-})^{-1} \alpha} - \frac{q (E - A^{T_m})^{-1} h}{1 - q (E - A^{T_m})^{-1} \alpha}.$$

□

These estimates enable us to calculate the components of the descent vector without inverting the matrices $E - A^{T_m^-}$ and $E - A^{T_m^+}$, which essentially simplifies the construction of an iterative procedure for an integer model. The integer model, combined with the solution method of the continuous problem (2.24)–(2.29) by modeling [13] appropriate information base, and a good user interface could provide a basis for a special-purpose DSS for the advisory and analytical services of the Cabinet of Ministers, whose responsibility would include analysis and screening of proposals for improvement of the technological structure of the economy and development of technical policy recommendations. Solution of these problems would also require analysis of interindustry processes affecting employment.

2.5 Models and Information Technologies for Decision Support During Structural and Technological Changes

Structural reforms are one of the basic problems solved during market transformation of the economy. Planning such reforms involves severe difficulties, since it should rest upon the productive forces available and should account for the interests of various social groups and strata formed in the transition economy and turning the existing social and economic structure to their advantage. In contrast to the early stages of transition economy, where there were no alternatives for reforms in the majority of postsocialist countries, structural changes under such circumstances

should be chosen from a great number of possible alternatives. The great number of such alternatives and aspects to assess them and the strong effect of the risk and uncertainty on the feasibility of the plan of reforms necessitate modern information decision support technologies to be applied. The development of mathematical models and relevant computational algorithms is an integral part of the development of such technologies. In this book, we consider such models and methods for the decision support of planned structural and technological changes, being one of the most important components of structural reforms.

The current system of industrial technologies was developed during a long time under the conditions of centralized economy. It was assumed at that time that prices were controlled, production was scheduled and unprofitable with losses covered from the state budget, and industrial expenses in some branches were high. Such preconditions contradict the fundamentals of the functioning of market economy and cause a wide use of “soft budgetary constraints” [20], i.e., covering a part of expenses of some economic entities from the financial resources of the whole society. Such a practice hampers structural reforms, it generates corruption and “paternalistic” relationships between the authority and particular economic entities, which objectively makes decision making at the nation-wide level non-transparent and cast some doubt on the necessity of the further democratization of the society. Thus, structural and technological changes to eliminate the above-mentioned disproportions play an important role in the social stabilization. Reducing the industrial expenses, which creates preconditions for the increase of the payment for labor and profitability on the inflation-free basis and thus decreasing the motivation of applying “soft budgetary constraints,” should become the main trend of these transformations.

Two important aspects related to the general decrease of industrial expenses are noteworthy. One is the necessity of intensifying energy- and resource-saving under fast increase in the prices for resources (especially energy ones) in the world markets. Under conditions of artificial underevaluation of fuel and electric energy in the former USSR in the 1970–1980s, the energy intensity of production was several times greater than that in developed countries with market economy. It is only radical reduction of energy consumption that can compensate for the consequences of a quick growth in prices for hydrocarbon fuel. There are two basic ways of energy saving:

- (i) direct energy saving, where the introduction of new technologies, modernization of the equipment available, refusal of the most power-consuming industries, and replacements of their production with import reduce the specific industrial expenses of energy resources;
- (ii) indirect energy saving, where the general consumption of energy decreases due to replacing power-consuming industrial products with less power-consuming analogs and reducing the consumption of some power-consuming products with preserving (and may be increasing) the total volume of output.

The capabilities of the indirect energy saving are as a rule greater; however, the great variety of alternatives of its implementation requires various, including quantitative,

methods of economic studies (including mathematical economic modeling) to be applied for management decision support in choosing this way.

The second important aspect related to the reduction of production costs is the change of the expenditure pattern of the state budget. Rejecting the practice of “soft budgetary constraints” allow concentrating the financial resources on the solution of actual social problems. At the same time, structural and technological changes involve significant budgetary resources. Estimating their efficiency is also of great importance.

In the first decade of the twenty-first century, the authors have developed a series of models to determine structural and technological changes and to solve the above-mentioned problems. In particular, to determine the main trends of reducing production costs, the studies [27, 37] have proposed an intersectoral optimization model with variable coefficients of direct costs. To search for changes in the structure of export and import that reduce the total energy consumption, the linear model considered in [27, 28], and using the equations of intersectoral balance has been developed. The influence of risk and uncertainty factors on the solution of the above problems was studied in [26, 28].

The further development of the models, first of all, of the intersectoral optimization model [27, 37] has allowed using them as a basis to create prototype versions of information technologies of management decision support in carrying out structural and technological changes. Let us consider the most important (in our opinion) aspects of these technologies—the basis model and its modifications, numerical methods of computations and their implementation on modern computers, and some aspects of the implementation of model computations.

The section is structured as follows. Section 2.5.1 reviews briefly the results on modeling structural and technological changes [27, 37], considers some problems of carrying out model computations, and analyzes the properties of the intersectoral optimization model that can be used in this case. Section 2.5.2 presents an approach (alternative to that proposed in [27, 37]) to performing computations based on this model. Section 2.5.3 briefly describes the IOMSTC program system, which is a basis of the information technology of management decision support. Section 2.5.4 considers the modifications of the intersectoral optimization model. The final subsection formulates conclusions and proposes fields for further studies.

2.5.1 Optimization Model of Planning Structural and Technological Changes

To determine the structural and technological changes that would reduce the manufacture costs and thus would increase the incomes of ultimate customers and make the economy more dynamic the following optimization model has been proposed in [27, 37].

Let us consider the economy formed by n pure industries manufacturing only one type of products. Let i, j , be the numbers of these branches ($i, j = \overline{1, n}$). Denote by a_{ij} the value of direct production costs of the branch i for manufacturing a unit of production of the branch j . This quantity can be expressed in both natural and cost measures depending on the information available. The matrix $A = \{a_{ij}\}$, $i, j = \overline{1, n}$ is called the matrix of the coefficients of direct costs (the matrix of technological coefficients, the Leontief matrix) [22].

Assume that the incomes D of ultimate customers are proportional to the volume of production:

$$D = \sum_{i=1}^n q_i x_i = (q, x),$$

where q_i is the share of ultimate incomes (payment for labor, social transfers, and profit) in the price of the production of the branch i , x_i is the gross output of this branch, $q = (q_1, \dots, q_n)$, $x = (x_1, \dots, x_n)$. In the model, we also assume that the final output y_i of each branch is proportional to ultimate incomes

$$y_i = a_i D + h_i, \quad i = \overline{1, n},$$

where the coefficients a_i reflect mainly the structure of individual consumption and internal investments, and h_i is determined from the export/import balance of branches and the structure of public consumption. Let $h = (h_1, \dots, h_n)$, $a = (a_1, \dots, a_n)$.

One more assumption of the model is a linear relationship between the share of the added cost $\tilde{q}_i$ in the price of the product of the branch i and the share of ultimate incomes in the price of this product

$$\tilde{q}_i = l_i q_i + d_i, \quad i = \overline{1, n},$$

where l_i is a fiscal multiplier of ultimate incomes [8], and d_i is the share of other components of the added cost in the price of the product of the i th branch.

The problem is to determine the structural and technological changes so as to maximize either the ultimate incomes or the multiplier “increase of incomes–increase of output,” which would provide the dynamic property of the economy.

In the model from [27, 37], these changes are reflected as the changes Δa_{ij} and Δq_i of the coefficients a_{ij} and q_i , which may be both positive (increase in the corresponding coefficient) and negative (its decrease). A decrease in the technological coefficients requires some limited assets (financial, material, or intellectual). The model presented below implies a linear relationship between the decrease in the coefficients and expenditures of the assets; however, more complicated dependences are possible.

In view of the assumptions above, we obtain the following optimization model.

It is necessary to determine Δa_{ij} and Δq_i ($i, j = \overline{1, n}$) such that would satisfy the following:

- the constraints that exclude the intensification of the inflation of costs

$$\sum_{\substack{i=1 \\ i \neq j}}^n \frac{a_{ij} + \Delta a_{ij}}{1 - (a_{jj} + \Delta a_{jj}) - (l_j (q_j + \Delta q_j) + d_j)} \leq \beta, \quad j = \overline{1, n}, \quad (2.38)$$

where $0 < \beta < 1$ is a preset confidential parameter;

- the relationships that follow from the physical meaning of the coefficients Δa_{ij} and Δq_j :

$$0 \leq q_j + \Delta q_j \leq 1, \quad 0 \leq a_{ij} + \Delta a_{ij} \leq 1, \quad i, j = \overline{1, n}; \quad (2.39)$$

- the balance of the expenses and added cost

$$a_{jj} + \Delta a_{jj} + l_j (q_j + \Delta q_j) + d_j \leq 1, \quad j = \overline{1, n}; \quad (2.40)$$

- constraints for the possible ranges of variation of the coefficients due to specific features of the technologies available

$$\begin{aligned} \underline{\gamma}_{ij} &\leq \Delta a_{ij} \leq \overline{\gamma}_{ij}, \quad i, j = \overline{1, n}; \\ \underline{q}_i &\leq \Delta q_i \leq \overline{q}_i, \quad i = \overline{1, n}, \end{aligned} \quad (2.41)$$

where $\underline{\gamma}_{ij}, \overline{\gamma}_{ij}$ are the lower and upper limits of the possible variation in the technological coefficients, $\underline{q}_i$ and $\overline{q}_i$ are the lower and upper limits of the possible variation in the share of ultimate incomes;

- the resource constraints

$$\sum_{j=1}^n \sum_{i=1}^n b_{kij} \max(0, -\Delta a_{ij}) \leq B_k, \quad k = \overline{1, K}, \quad (2.42)$$

where K is the number of resources, B_k is the volume of the k th resource intended to carry out structural and technological changes, b_{kij} is the expenditure of this resource in taking the measures that provide a unitary decrease in the expenses of the production of the branch i to produce a unit of production of the branch j .

The variations Δa_{ij} and Δq_i from the set satisfying constraints (2.38)–(2.42) should be chosen so as to maximize the ultimate incomes of consumers

$$D = f_1(\Delta A, \Delta q) = \frac{(q + \Delta q)^T (E - (A + \Delta A))^{-1} h}{1 - (q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha} \longrightarrow \max, \quad (2.43)$$

where $\Delta q = (\Delta q_1, \dots, \Delta q_n)$, $\Delta A = \{\Delta a_{jj}\}_{j=1}^n$, $\Delta q = (\Delta q_1, \dots, \Delta q_n)$, $h = (h_1, \dots, h_n)$, $\alpha = (\alpha_1, \dots, \alpha_n)$, E —is a unit matrix, and T denotes transposition.

Another objective function is the multiplier “increase of incomes—increase of production”

$$f_2(\Delta A, \Delta q) = (q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha^1 \longrightarrow \max, \quad (2.44)$$

where $\alpha^1 = (\alpha_1^1, \dots, \alpha_n^1)$ is the vector of the structure of ultimate internal consumption.

For a detailed substantiation of constraints (2.38)–(2.42) and objective functions (2.43), (2.44) see [27, 37].

The optimization model may be considered as a one-criterion problem (with one of the functions (2.43) or (2.44)) or in a multicriterion case (with both objective functions considered earlier). Since a limited consumption of some scarce imported energy resources (e.g., natural gas) is important, the list of criteria may be supplemented with their consumption

$$f_3(\Delta A, \Delta q) = g^T (E - (A + \Delta A))^{-1} (\alpha D + h) \longrightarrow \min, \quad (2.45)$$

where $g^1 = (g_1, \dots, g_n)$ is the vector of specific industrial consumption of this resource.

The resultant multicriterion problem can be solved using the convolution method by maximizing the function

$$F(\Delta A, \Delta q) = \tilde{\beta}_1 f_1(\Delta A, \Delta q) + \tilde{\beta}_2 f_2(\Delta A, \Delta q) + \tilde{\beta}_3 f_3(\Delta A, \Delta q), \quad (2.46)$$

where the weight coefficients $\tilde{\beta}_i$, $i = \overline{1, 3}$ can be determined by serial-regression methods based on expert estimates [27].

Since the criteria (2.43) and (2.45) are opposite, it is expedient to complement constraints (2.38)–(2.42) with a constraint for the minimum level of ultimate incomes

$$f_1(\Delta A, \Delta q) \leq \hat{f}_1, \quad (2.47)$$

where $\hat{f}_1$ is a desirable level of incomes. Note that the problem (2.38)–(2.42), (2.46), (2.47) has no basic differences from the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44); therefore, in what follows, we will consider the two last problems.

Solving problems (2.38)–(2.43) or (2.38)–(2.42), (2.44) involves some difficulties. First, since the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$ are nonconvex, these problems are multiextreme despite the convexity of the set of solutions X specified by (2.38)–(2.42). Second, the objective functions of these problems are not definite for all the ΔA and Δq . For the function $f_2(\Delta A, \Delta q)$ to be definite, the matrix $E - (A + \Delta A)$ should be nonsingular. For $f_1(\Delta A, \Delta q)$ to be definite, it is additionally required that the condition $(q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha \neq 1$ be satisfied. Third, the left-hand sides of constraints (2.42) are continuous yet not everywhere differentiable convex functions, and the left-hand sides of constraints (2.38) are discontinuous. Note that the last two problems can be solved. If constraints (2.41) and (2.42) do not reduce all $a_{ij} + \Delta a_{ij}$ up to zero for some j (such a situation is typical for economy since it is impossible to produce anything without material inputs), the denominator of the left-hand side of (2.38) will be non-negative for all the ΔA and Δq satisfying (2.40). The left-hand side of (2.38) increases without limit if this denominator is close to zero; therefore, this constraint will not be satisfied. Thus, for admissible solutions and solutions from some neighborhood of X , the denominator of the left-hand side of (2.38) will be positive, and these constraints can be rearranged as

$$\sum_{\substack{i=1 \\ i \neq j}}^n (a_{ij} + \Delta a_{ij}) \leq \beta \left(1 - (a_{jj} + \Delta a_{jj}) - (l_j (q_j + \Delta q_j) + d_j) \right), \quad j = \overline{1, n}, \quad (2.48)$$

whence the sufficient existence condition $(E - (A + \Delta A))^{-1}$ known as the efficiency condition [22] follows for the matrix $(A + \Delta A)$. It can be shown in a similar way [37] that the condition $(q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha \neq 1$ is satisfied for all the solutions from X and some its neighborhood, and the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$ are defined for all the admissible solutions of problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) and also for the inadmissible solutions insignificantly differing from admissible ones. Thus, the objective functions of the problems specified are defined and continuous on some rather small neighborhood X .

Note that constraint (2.48) is linear, in contrast to (2.38). To solve the problem thus transformed, well-known methods such as the method of nonsmooth penalty functions [39] can be applied. This does not complicate the problem due to nonsmooth constraints (2.42). The extremum of the penalty function can be found by any non-differentiable optimization method; in particular, the r -algorithm [43] was used in [27, 37]. Taking the penalty factor sufficiently large always allows localizing the search in a rather small neighborhood of X , where the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$ are continuous, and the replacement of constraint (2.38) with inequality (2.48) is equivalent.

Implementing this approach involves the dimension problem and of the differentiation of the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$, which employ the operation of matrix inversion. The total number of variables equals $n^2 + n$, which is rather large even for n of order 20 $\Rightarrow$.40. To differentiate the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$, the authors have proposed [37] formulas that follow from the componentwise differentiation of the matrix function $\bar{F}(Y)$ given implicitly by the relationship $\bar{F}(Y)Y = E$, where Y is an arbitrary $n \times n$ matrix.

The dimension of the problem being solved can be substantially reduced if we exclude from the consideration the positions (i, j) , where $|\underline{\gamma}_{ij} - \bar{\gamma}_{ij}| < \varepsilon$ (ε is a sufficiently small number) and also Δa_{ij} and Δq_i such that for any ΔA and Δq , the inequalities $\left| \frac{\partial f_1(\Delta A, \Delta q)}{\Delta a_{ij}} \right| < \varepsilon$ and $\left| \frac{\partial f_1(\Delta A, \Delta q)}{\Delta q_i} \right| < \varepsilon$ hold for the problem (2.39)–(2.43), (2.48) and the inequalities $\left| \frac{\partial f_2(\Delta A, \Delta q)}{\Delta a_{ij}} \right| < \varepsilon$ and $\left| \frac{\partial f_2(\Delta A, \Delta q)}{\Delta q_i} \right| < \varepsilon$ hold for the problem (2.39)–(2.42), (2.44), (2.48). In particular, as the computations based on the intersectoral balances composed in Ukraine at the end of the 1990s—beginning of the 2000s show, it was possible to consider no more than 130 variables in the case of $n = 18$, and no more than 250 for $n = 38$.

The fact that the problem being solved is multiextreme can be taken into account by carrying out independent computations by the r -algorithm from different initial points. In particular, initial points can be selected from a uniform partition of the diagonal of the hypercube

$$\left\{ (\Delta q, \Delta A) : \underline{\gamma}_{ij} \leq \Delta a_{ij} \leq \bar{\gamma}_{ij}, \quad \underline{q}_i \leq \Delta q_i \leq \bar{q}_i \right\}$$

i.e. the points $(\underline{q}_1, \dots, \bar{q}_n, \underline{\gamma}_{11}, \dots, \bar{\gamma}_{nn})$ and $(\bar{q}_1, \dots, \bar{q}_n, \bar{\gamma}_{11}, \dots, \bar{\gamma}_{nn})$.

Note that the results of computations based on the optimization models (2.38)–(2.43) and (2.38)–(2.42), (2.44) cannot be directive under conditions of transitive economy. They can be used to obtain a desirable structure of industrial technologies that intensify the social and economic development of the country, to reveal the ways of reducing the existing structure to the desirable one, to evaluate the necessary resources, etc. Of importance is the analysis of the variation of the technological coefficients a_{ij} in time during the last several years, revealing the tendencies of the approximation (or removal) of the real values of a_{ij} to the desirable values obtained from the computation using the above-mentioned models. From this point of view, obtaining a series of local extrema of functions (2.43) or (2.44) with the values sufficiently close to their global maxima is more preferable to the subsequent application than searching for the global extremum.

The values of some parameters in the models (2.38)–(2.43) and (2.38)–(2.42), (2.44) may be determined ambiguously. First of all, these are the parameter β in constraints (2.38), possible ranges of variation of the technological coefficients and of the share of added value in the price of a production in inequalities (2.41),

and some coefficients and the right-hand sides of the resource constraints (2.42). Performing dialogue computations on the models (2.38)–(2.43) and (2.38)–(2.42), (2.44) for different values of the above parameters is therefore of interest. It is expedient that the time interval between the beginning and the end of a current series of computations, i.e., between the formation of a version of the problem with certain values of its parameters and obtaining a series of its locally optimal solutions, does not exceed 35 min. As numerical experiments show, the run time on a Pentium IV computer using the model (2.39)–(2.43), (2.48) and the r -algorithm for one initial approximation for $n = 38$ varies from 0.3–0.4 min (if the approaches considered earlier are used to reduce the problem dimension) to 1.3–1.5 min (if the computations are performed for a problem of full dimension). If 10–15 initial approximations are taken, the above-mentioned time constraints may not be satisfied. A way out may be the use of modern multiprocessor devices. A local extremum for each initial approximation is sought using a separate processor. The values of the objective function at the found points are passed to one of the processors, where solutions with insufficiently large values of the objective functions are rejected and other alternatives are ordered and relayed to decision-makers for further analysis. Other formal criteria of rejecting the solutions are also possible. The number of initial approximations for a sufficiently large number of processors can be increased up to several tens or even hundreds, the total run time remaining almost the same as if one extremum is searched for on a one-processor computer, i.e., being quite comprehensible to support the system-user dialogue. The SKIT-2 and SKIT-3 clusters developed recently at the Institute of Cybernetics of the National Academy of Sciences of Ukraine are suitable for computations according to the above-stated scheme.

When searching for an extremum by maximizing the penalty function using the r -algorithm, additional problems may arise. As noted earlier, the functions $f_1(\Delta A, \Delta q)$ and $f_2(\Delta A, \Delta q)$ are defined on the sets of admissible solutions of problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) and for some neighborhoods of these sets. Nevertheless, when an extremum is searched for, the sequence of approximations obtained using the r -algorithm may go beyond the limits of these neighborhoods. This was observed on the average for 6–7 of 10 initial approximations during the numerical experiments. As a result, the computational procedure terminates at an inadmissible point, where the objective function of the problem is not defined. To avoid this, it is proposed to apply a modification of the r -algorithm [42], which assumes that the subgradient of the objective function is computed only at admissible points, or [42], which possesses the same properties. From this point of view, of interest is to transform the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) to a form that avoids the use of functions not defined everywhere, which has underlain the alternative approach to the development of computational algorithms to be considered in the next subsection.

The above-mentioned numerical experiments have shown one more feature of the problems being solved. Different initial approximations have yielded a series of solutions with almost equal values of the objective function (deviations from the

maximum value in the series are 0.3–1.0 %), but with several substantially differing components of the solution. The statement below explains this effect to some extent.

Theorem 2.2. *Let $(\Delta A^{(1)}, \Delta q^{(1)})$ and $(\Delta A^{(2)}, \Delta q^{(2)})$ be arbitrary admissible solutions of the problem (2.38)–(2.43) or (2.38)–(2.42), (2.44) for which the equality*

$$\left(E - (A + \Delta A^{(1)})^T\right)^{-1} (q + \Delta q^{(1)}) \left(E - (A + \Delta A^{(2)})^T\right)^{-1} (q + \Delta q^{(2)}) \quad (2.49)$$

holds componentwise. Let also

$$(\Delta A_\lambda, \Delta q_\lambda) = \lambda (\Delta A^{(1)}, \Delta q^{(1)}) + (1 - \lambda) (\Delta A^{(2)}, \Delta q^{(2)}),$$

where $0 < \lambda < 1$ is an arbitrary number.

Then the equality

$$f_i = (\Delta A^{(1)}, \Delta q^{(1)}) = f_i (\Delta A_\lambda, \Delta q_\lambda) = f_i (\Delta A^{(2)}, \Delta q^{(2)}), \quad i = \overline{1, 2},$$

is true and $(\Delta A_\lambda, \Delta q_\lambda)$ is an admissible solution of the above problems.

The proof of the theorem is based on the statement considered in the next subsection and will be presented after this statement. Let us dwell on some corollaries of the theorem.

Admissible points having identical values of the objective function and belonging to the set satisfying (2.49) form structures similar to a set of intervals of straight lines. If the extreme points of one of such intervals differ in only several components, then all the internal points of the interval also differ from each other in only the values of these components. If the question is values close to the global maximum, the r -algorithm for the set of initial points will result in one of such structures, which corresponds to the results of numerical experiments.

Note that the approach proposed to solve problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) allows obtaining their local solutions in a comprehensible time; however, analytic capabilities are rather limited in this case. This necessitates the development of an alternative approach to the solution of these problems.

2.5.2 Alternative Approach to Developing a Computational Algorithm

Let us rearrange the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) by introducing new variables

$$z = \left(E - (A + \Delta A)^T\right)^{-1} (q + \Delta q),$$

whence

$$\left(E - (A + \Delta A)^T\right)z = q + \Delta q. \quad (2.50)$$

Then the problem of maximizing ultimate incomes of consumers can be formulated as the problem of finding $(\Delta A, \Delta q, z)$ that maximize the function

$$\bar{f}_1(z) = \frac{z^T h}{1 - z^T \alpha} \quad (2.51)$$

under the constraints (2.39)–(2.42), (2.48), (2.50). The problem of maximizing the multiplier “increase of incomes–increase of output” is formulated as the problem of maximizing the function

$$\bar{f}_2(z) = z^T \alpha^1 \quad (2.52)$$

under the same constraints.

Note that the objective functions and constraints of these problems are defined for any values of $(\Delta A, \Delta q, z)$ (except for z such that $z^T \alpha = 1$ for the problem with the objective function $\bar{f}_1(z)$).

Like (2.39)–(2.43), (2.48) and (2.39)–(2.42), (2.44), (2.48), the problems obtained can be solved by the method of nonsmooth penalty functions with the use of the r -algorithm. Certain difficulties may arise because of the equality constraints (2.50) that cause the nonconvexity of the set of admissible solutions. Therefore, it is possible to apply here the same approach as that used to solve previous problems—choosing several initial approximations for the r -algorithm.

The following statement is true for the problems under study.

Theorem 2.3. *Let $x^{(1)} = (\Delta A^{(1)}, \Delta q^{(1)}, z^*)$ and $x^{(2)} = (\Delta A^{(2)}, \Delta q^{(2)}, z^*)$ be arbitrary admissible solutions of problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52) with the identical values of the variables $z = z^*$. Let also $x(\lambda) = \lambda x^{(1)} + (1 - \lambda)x^{(2)}$, where $0 < \lambda < 1$ is an arbitrary number. Then the values of the functions $\bar{f}_i(z)$, $i = 1, 2$ are identical for the solutions $x^{(1)}$, $x^{(2)}$, and $x(\lambda)$, $x(\lambda)$ being an admissible solution for any $0 < \lambda < 1$.*

Proof. Let us show a point $x(\lambda)$ satisfies constraints (2.39)–(2.42), (2.48), and (2.50) for any $0 < \lambda < 1$. Indeed, constraints (2.39)–(2.41), and (2.48) are linear inequalities, and the left-hand side of constraints (2.42) is a convex function. Therefore, the above constraints specify a convex set. If the points $x^{(1)}$ and $x^{(2)}$ belong to this set, then their convex combination $x(\lambda)$ also belongs to it, i.e., it satisfies the relationships (2.39)–(2.42), (2.48). Further, since $x^{(1)}$ and $x^{(2)}$ are admissible, the following equalities are true:

$$\left(E - (A + \Delta A^{(1)})^T\right)z^* = (q + \Delta q^{(1)}),$$

$$\left(E - (A + \Delta A^{(2)})^T\right) z^* = (q + \Delta q^{(2)}).$$

Multiplying the first of these equalities by λ and the second by $1 - \lambda$ and adding them yield

$$\left(E - (A + \lambda \Delta A^{(1)} + (1 - \lambda) \Delta A^{(2)})^T\right) z^* = q + \lambda \Delta q^{(1)} + (1 - \lambda) \Delta q^{(2)}. \quad (2.53)$$

Taking into account $z_\lambda = \lambda z^* + (1 - \lambda) z^* = z^*$, we can rearrange Eq. (2.53) as

$$\left(E - (A + \Delta A_\lambda)^T\right) z^* = q + \Delta q_\lambda, \quad (2.54)$$

where $(\Delta A_\lambda, \Delta q_\lambda, z_\lambda)$ are components of the solution $x(\lambda)$. According to (2.54), the point $x(\lambda)$ satisfies Eq. (2.50) and is an admissible solution of the problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52).

Note that the values of the variables z for the solution $x(\lambda)$ coincide with the values of these variables for the solutions $x^{(1)}$ and $x^{(2)}$. Since the objective functions $\bar{f}_1(z)$ and $\bar{f}_2(z)$ of these problems depend only on z , their values at the points $x^{(1)}$, $x^{(2)}$, and $x(\lambda)$ coincide too, whence the validity of the theorem follows.

Having proved Theorem 2.3, let us get back to the proof of Theorem 2.2.

Note that the admissibility of the solutions $(\Delta A^{(1)}, \Delta q^{(1)})$ and $(\Delta A^{(2)}, \Delta q^{(2)})$ yields the existence of matrices $\left(E - (A + \Delta A^{(i)})^T\right)^{-1}$, $i = \overline{1, 2}$. A unique admissible solution $(\Delta A^{(i)}, \Delta q^{(i)}, z^{(i)})$ of the problems (2.39)–(2.42), (2.48), (2.50), (2.51) or (2.39)–(2.42), (2.48), (2.50), (2.52) corresponds to each admissible solution $(\Delta A^{(i)}, \Delta q^{(i)})$, $i = \overline{1, 2}$, of the problems (2.38)–(2.43) or (2.38)–(2.42), (2.44), the relationship $z^{(1)} = z^{(2)} = z^*$ being true by virtue of (2.49) and (2.50).

The matrix $E - (A + \Delta A_\lambda)$ is also nonsingular, and a unique admissible solution $(\Delta A_\lambda, \Delta q_\lambda, z_\lambda)$ of the transformed problems also corresponds to the admissible solution $(\Delta A_\lambda, \Delta q_\lambda)$ of the problems (2.38)–(2.43) or (2.38)–(2.42), (2.44). Therefore, it is possible to apply Theorem 2.3 for $x^{(1)} = (\Delta A^{(1)}, \Delta q^{(1)}, z^*)$, $x^{(2)} = (\Delta A^{(2)}, \Delta q^{(2)}, z^*)$ and $x(\lambda) = (\Delta A_\lambda, \Delta q_\lambda, z_\lambda)$. According to this theorem, $\bar{f}_i(z^*) = \bar{f}_i(z_\lambda)$, $i = \overline{1, 2}$. Since the corresponding problems are equivalent in their original and rearranged forms, the equality $f_i(\Delta A^{(i)}, \Delta q^{(i)}) = \bar{f}_i(z^*)$, $i = \overline{1, 2}$ should hold. Herefrom, $f_i(\Delta A^{(i)}, \Delta q^{(i)}) = f_i(\Delta A_\lambda, \Delta q_\lambda)$, $i = \overline{1, 2}$, which proves the theorem.

According to Theorem 2.3, the set X_1 of the values of variables ΔA and Δq admissible for given values of z is convex. This simplifies both constructing this set and searching for extremum points of some additional criteria on it, for example, of the function $f_3 = (\Delta A, \Delta q)$ defined according to (2.45).

Implementing, from several initial points, the procedure of the subgradient method with space dilatation using values of the objective function [39] ($\bar{f}_i(z)$, $i = \overline{1, 2}$, are known if z is given), we obtain several points of the above-mentioned

set. By virtue of Theorem 2.3, their convex linear combination also belongs to this set. Constructive algorithms of enumerating various points from X_1 can also be constructed based on subgradient methods that use external approximation of the set of extrema by ellipsoids, for example, on the method of finding an admissible point of a system of convex inequalities [41], which provides an accelerated convergence to boundary points of the set X_1 . The approximation of X_1 by an ellipsoid, constructed at the previous step of this method, can be used to find the next boundary point of X_1 .

Note that the components of the vector z reflect the structure of ultimate incomes obtained from various forms of economic activity. Such a structure determines a parity of interests (existing or desirable) among various economic entities. Performing alternative computations (for different z) and comparing the resultant values with the solutions of the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) will make it possible to estimate the influence of partial interests on the efficiency of economic development and to predict whether economic entities accept or oppose a solution (globally optimal or close to it). The models (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52) considered for given z are models with fixed objectives, where the results are determined but a set of tools (i.e., of measures to achieve these results) should be constructed. Using such models is important for both a preliminary analysis of the situation and final selection of solutions obtained earlier. From this standpoint, generating the set of admissible solutions of the problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52) with the values of objective functions sufficiently close to the global maximum and carrying out interactive computations play an important role. Multiprocessor computers (SKIT-2 and SKIT-3 clusters) and a computation-parallelizing scheme according to which extremum from each initial approximation is searched for at a separate processor can be used.

Rearranging the problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52), it is possible to obtain sufficiently exact estimates of their objective functions at the point of global extremum. Such information is extremely valuable in selecting the solutions obtained.

Changing the variables $\Delta a_{ij} = \Delta a_{ij}^+ - \Delta a_{ij}^-$ and rearranging the objective function (2.51) as $v(1 - z^T \alpha) = z^T h$, $\bar{f}_1 = v \rightarrow \max$ make it possible to consider the problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52) as special cases of the problem of maximizing a linear function

$$Q(x) = (c, x) \rightarrow \max \quad (2.55)$$

under quadratic and linear constraints of the form

$$x^T H_1 x + (b_1, x) = q_1, \quad (2.56)$$

$$x^T H_2 x + (b_2, x) = q_2, \quad (2.57)$$

$$x \geq 0, \quad (2.58)$$

where H_1 and H_2 are some square matrices, b_1 and b_2 are vectors, and x is the vector of variables.

To obtain upper estimates of the maximum value of the objective function Q^* of the problem (2.55)–(2.58), Lagrange dual estimates ψ^* can be used. Finding them is reduced to minimizing nonsmooth convex functions defined on a family of nonpositive definite symmetric matrices linearly depending on Lagrangian multipliers that correspond to constraints (2.56) and (2.57). The duality may discontinue, i.e., $\Delta^* = \psi^* - Q^*$ will be positive and the estimate ψ^* be insufficiently exact.

One of the ways of reducing Δ^* involves functionally redundant constraints (which may also increase the number of variables of the problem (2.55)–(2.58)). According to [40] functionally redundant are the constraints that do not change the sets of admissible solutions of the problem (2.55)–(2.58). However, this changes the form of the Lagrangian function, which may decrease Δ^* .

An elementary example of functionally redundant constraints is quadratic corollaries of linear constraints, for example, a quadratic constraint

$$(b_1^T x + c_1)(b_2^T x + c_2) \geq 0$$

follows from two linear inequalities $b_1^T x + c_1 \geq 0$ and $b_2^T x + c_2 \geq 0$. Numerous linear constraints present in the problems considered earlier facilitate the application of such an approach to solve them. More complicated functionally redundant constraints considered in [46] may also be applied. In particular, if there are two-sided constraints for three variables of the problem, $0 \leq x_1 \leq 1$, $0 \leq x_2 \leq 1$, and $0 \leq x_3 \leq 1$, functionally redundant constraints can be written as the inequalities

$$\begin{aligned} x_1 x_2 + x_1 x_3 + x_2 x_3 - x_1 - x_2 - x_3 &\geq -1, \\ -x_1 x_2 - x_1 x_3 + x_2 x_3 + x_1 &\geq 0, \\ -x_1 x_2 + x_1 x_3 - x_2 x_3 + x_2 &\geq 0, \\ x_1 x_2 - x_1 x_3 - x_2 x_3 + x_3 &\geq 0. \end{aligned}$$

This approach can easily be generalized to the case of an arbitrary number of two-sided constraints for variables. Note that such constraints are typical for the models earlier considered, which substantially increases the accuracy of the Lagrangian estimates ψ^* for the maximum values of their objective functions. Moreover, introducing new variables by means of quadratic equalities of the form $y_{ij} = x_i x_j$ substantially expands the family of functionally redundant constraints being used and thus increases the accuracy of the estimates ψ^* .

When comparing the approaches to performing model computations presented in this and previous subsections, we should note their advantages and shortcomings.

An approach based on solving the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) without their transformation substantially reduces the problem

dimension. Since all the constraints have the form of inequalities, an exact solution of the problem can be obtained for finite values of penalty coefficients, which restricts the ravine property of the function being maximized. The set of admissible solutions of the problem is convex. In some cases, this facilitates finding the first admissible solution. At the same time, the possibilities of the analysis of the solutions obtained with this approach are bounded.

The approach stated in this subsection somewhat increases the dimension of the problem being solved. The set of admissible solutions of the problem becomes nonconvex, and nonlinear (quadratic) equality constraints needs large (theoretically infinite) values of the corresponding penalty coefficients, which increases the ravine property of the penalty function. At the same time, the transformed problem does not use incompletely defined functions, and problems because of abandoning the domain of their definition when solving the problem are excluded. The approach proposed gives ample opportunities for the analysis of solutions and considering models with fixed objectives and allows estimating the values of the objective function at the point of its global maximum.

Thus, both approaches complement each other: therefore, its simultaneous application is expedient in model computations.

2.5.3 Software Development

Software is an important component of modern information technologies. The software for decision support in planning structural and technological changes is based on the IOMSTC (Intersectoral Optimization Models of Structural and Technological Changes) software system created at the Institute of Cybernetics of the National Academy of Sciences of Ukraine. The system is implemented in the DELPHI medium, which provides its flexibility and convenience for the user.

IOMSTC consists of the user interface, base of models, and a set of program modules implementing optimization algorithms. The user interface leans upon a developed system of windows and hierarchical menus. It allows reading out initial data of the model from external files (including Excel spreadsheets and Word documents executed in a preconcerted format), forming input files of the model (the matrix A , vectors q , h , α , l , d , coefficients and the right-hand sides of resource constraints, etc.) based on these data, editing initial data of problems composed earlier and newly formulated, selecting parameters of optimization algorithms, visualizing and analyzing results of model computations, including their comparison with actual ones in order to verify the model.

A specific feature of the user interface is the presence of two types of windows in its structure: for a user being an expert in economy and for an expert in optimization methods. Windows of the first type allow forming new models based on data of external files and (or) existing models, renaming the existing models, editing initial data (including during dialogue computations considered in the previous subsections), viewing, analyzing, and saving (as a part of the problem or on external

files) the results of model computations. The user can also choose the objective function (the aim of the computations). As alternatives to be chosen, the first version of the system considers the following:

- (i) maximization of ultimate incomes of consumers (the objective function $f_1(\Delta A, \Delta q)$ corresponds to it);
- (ii) maximization of the multiplier “increase of incomes–increase of output” ($f_2(\Delta A, \Delta q)$ corresponds to it);
- (iii) finding an arbitrary point of the set of admissible solutions (arbitrary admissible solution).

Windows of the second type allow viewing the default values of the parameters of the optimization algorithm, changing the values of these parameters if necessary, obtaining information on the problem time, on the value of the objective function, and the residuals of constraints at the extremum point and comparing them with similar parameters for the initial approximation. If searching for extremum is terminated earlier than acceptable accuracy is achieved (e.g., if the objective function appeared not determined for the next current approximation), the user can find out the reason of such actions of the system. The user (an expert in optimization methods) may also change the number of initial approximations and the rule of their choice (by default, initial approximations are chosen similar to that described in Sect. 2.5.1). The structure of windows of the second type also includes a command line where the name of the program module implementing the optimization algorithm is specified. This should be an arbitrary loading module (with the .exe extension) that uses input data from files of the internal (for the system) representations of the model.

As these moduli, the first version of the system used the MULSTR program described in [37] and MULSTR1 implementing the approach to carrying out model computations stated in Sect. 2.5.2. The problems (2.39)–(2.42), (2.48), (2.50), (2.51) and (2.39)–(2.42), (2.48), (2.50), (2.52) are solved by the method of penalty functions followed by the application of the r -algorithm. The MULSTR1 program uses three penalty factors (coefficients) for different groups of constraints. The first factor is used for quadratic equality constraints (2.50). It should be considered separately since, as mentioned earlier, the value of this factor should be sufficiently large to obtain an exact solution of the problem. The second factor is used for constraints (2.39), (2.40), and (2.48), and the third for nonsmooth resource constraints (2.42). The two-sided constraints (2.41) imposed on ΔA and Δq are not included in the penalty function and are taken into account by an “even” periodic continuation of the objective function onto the domain of inadmissible values of variables [39]. This approach has proved to be efficient in searching for extremum of a wide class of nonsmooth functions. The MULSTR program intended to search for extremum under inequality constraints uses only the second and third penalty factors; therefore, if the name of this program appears in the command line, the user’s attempts to view and change the value of the first factor are blocked.

The controlled parameters of the r -algorithm which can be changed by the user for the MULSTR and MULSTR1 programs are: the initial value h_0 of the

step-by-step factor, the maximum number of iterations of the algorithm, a precision factor ε_g used in a stopping criterion based on the variation in the argument, a precision factor ε_g used in a stopping criterion based on the norm of the subgradient of the function being maximized, a space dilation coefficient α , and parameters q_1 , q_2 , and n_k of adaptive adjustment of a step-by-step factor. The user may vary all these parameters within the limits corresponding to theoretical recommendations for the r -algorithm.

A peculiar feature of the IOMSTC system is an internal base of models. Each model is a set of files of standard structure that contain information on the initial data, the method used for the computations, parameters of the computation algorithm, and upon completion of the computations—about their results (sequence of locally optimal solutions). Each model has an individual name given by the user. When a new model is created on the basis of the existing one, all the information contained in files of the base model will be copied to files of the new model. In the case of renaming model, all its files will also be renamed automatically.

The further trends in developing the IOMSTC system are as follows:

- (i) extending the list of the applied optimization algorithms due to implementing modern methods developed at the Institute of Cybernetics NASU and in other domestic and foreign scientific centers;
- (ii) enhancing service capabilities, including the recognition of handwriting, speech dialogue, etc.

Along with the National Agency on the Efficient Use of Energy Resources, potential users of the system may include divisions of the Ministry of Economic Affairs and Ministry of Industrial Policy of Ukraine involved in preparing structural reforms.

In view of the application domain of the system, the sensitivity analysis of solutions against the variation of ambiguously determined parameters of models becomes of great importance. To this end, numerical experiments have been carried out for a seven-branch model, the results being presented in Table 2.14.

As follows from the results obtained, solutions are most sensitive to variations in the parameters γ_{ij} and $\bar{\gamma}_{ij}$. The model with the criterion $f_1(\Delta A, \Delta q)$ is a little more sensitive to variations in β than to variations in the right-hand sides

Table 2.14 Variation of optimal values of objective function with the variations in model parameters

Model parameters	Range of variations	Variation of objective functions of the model (% of the largest value)	
		$f_1(\Delta A, \Delta q)$	$f_2(\Delta A, \Delta q)$
γ_{ij} and $\bar{\gamma}_{ij}$	10–50 % of initial values of coefficient	72.9–100	90.4–100
β	0.9–1.0	77.3–100	91.9–100
B_k	1.0–3.5	80.9–100	92.9–100

of the resource constraints B_k ; for the model with the criterion $f_2(\Delta A, \Delta q)$, the sensitivity to variations of parameters taken into account is approximately the same. In general, the influence of variations in parameters of the model on optimal values of the objective functions is relatively small. If all the parameters are varied simultaneously, the range of values of the objective functions is 44–100 % for $f_1(\Delta A, \Delta q)$ and 73.5–100 % for $f_2(\Delta A, \Delta q)$. Similar results have been obtained for models of greater dimensions.

Enhancing the models being used is also an important trend in further development of the IOMSTC. In the next subsection, we consider the models planned to be included in the upgraded version of the system.

2.5.4 Modified Intersectoral Models of Structural and Technological Changes

Let us consider the models being the further development of the models (2.38)–(2.43) and (2.38)–(2.42), (2.44) presented in Sect. 2.5.1. The first of these models differs from those considered earlier in the following aspects.

1. Constraints on the available fuel and energy resources are provided, and the possibility of changing unit costs of these resources for the production of branches is taken into account.
2. It is provided that structural and technological changes in some branches (first of all, in electric power industry) can be carried out by two ways:
 - (a) by changing the intensity of the use of manufacturing techniques available for the ultimate product of the branch (e.g., increasing the share of thermal engineering due to decreasing the share of atomic power engineering or vice versa);
 - (b) by changing the cost structure for available technologies due to introducing technical and technological innovations.

Introducing new technologies can be considered as a special case of the first of these ways since the intensity of using the corresponding technology was equal to zero at the beginning of its introduction and became a positive number after the introduction.

3. The model takes into account ecological constraints, including constraints for the emission of greenhouse gases.

The model is static; however, its dynamic analogs can be developed in the future. Let us consider the basic assumptions and introduce the notation of the model.

Let the economy consist of n pure branches where structural and technological changes are possible. For a branch j_0 (if this is electric power industry, $j_0 = 17$ by the existing nomenclature of intersectoral balance), these changes can be detailed as variations in the intensity of the use of technologies and as variations in the cost structure of forms of production for technologies. Denote by K_{j_0} the number of

technologies used in the branch j_0 and number these technologies by $k, k = \overline{1, K_{j_0}}$. Let $W_k^{(j_0)}$ be the intensity of using the technology k in the branch j_0 . Then assume that

$$\sum_{k=1}^{K_{j_0}} W_k^{(j_0)} = 1.$$

Denote by a_{ij} the coefficients of direct expenses of production of the branch i for manufacturing a unit product of the branch j (coefficients of the Leontief matrix), $i = \overline{1, n}, j = \overline{1, n}, j \neq j_0$. Denote by $\bar{a}_{ij_0}^k$ specific expenses of production of the branch i for manufacturing a unit product of the branch j_0 using the technology k . The corresponding elements of the column j_0 of the Leontief matrix are defined as follows:

$$a_{ij_0} = \sum_{k=1}^{K_{j_0}} \bar{a}_{ij_0}^k W_k^{(j_0)}. \quad (2.59)$$

Denote by q_j the share of ultimate incomes in the price of the product of the branch $j, j = \overline{1, n}$. Assume that the share of added value $\bar{q}_j$ in the price of the product of the branch j linearly depends on q_j :

$$\bar{q}_j = l_j q_j + d_j, \quad j = \overline{1, n}. \quad (2.60)$$

The known coefficients l_j and d_j reflect the value of the fiscal multiplier of ultimate incomes and the share of other components of the added value in the price of the product of the branch j . Assume that ultimate consumption y_i of each branch linearly depend on the income of ultimate users inside the system being modeled

$$y_i = \alpha_i D + h_i, \quad i = \overline{1, n}, \quad (2.61)$$

where α_i and h_i are some known coefficients.

The purpose of modeling, similar to the problem (2.38)–(2.43), is to determine the ways of structural and technological changes that increase the efficiency of the economy. In view of this, the model has several objective functions; performing computations can deal with one-criterion optimization, where extremum of one objective function selected from all others is determined, or with a multicriterion problem, where all the objective functions are taken into account simultaneously. To solve such a problem, a combination of the methods of convolutions and constraints considered in [27] can be used.

Denote by g_j the vector of direct unit costs of bounded resources in the branch $j, j = \overline{1, n}$, the value of g_{j_0} being determined from unit costs of resources for separate technologies $g_{j_0}^k$ and the intensity of use of technologies $W_k^{(j_0)}$ similar to (2.59):

$$g_{j_0} = \sum_{k=1}^{K_{j_0}} g_{j_0}^k W_k^{(j_0)}. \quad (2.62)$$

Let G be the vector of available resources. In what follows, by g we will denote the matrix consisting of columns g_j , $j = \overline{1, n}$.

Denote by $\tilde{g}_j$ the vector of specific emissions of polluting substances for the branch j . The vector $\tilde{g}_{j_0}$ is computed similar to (2.62) based on specific emissions of polluting substances for separate technologies $\hat{g}_{j_0}^k$ and the intensity of use of these technologies $W_k^{(j_0)}$:

$$\tilde{g}_{j_0} = \sum_{k=1}^{K_{j_0}} \hat{g}_{j_0}^k W_k^{(j_0)}. \quad (2.63)$$

Denote by $\tilde{G}$ the vector of maximum admissible emissions of polluting substances and by $\tilde{g}$ the matrix created by the column vectors $\tilde{g}_j$.

Along with the resources used during the production, the model takes into account the resources used to change the cost structure of the branch. Such resources are assumed to be spent only to reduce the coefficients of costs of product a_{ij} , $\bar{a}_{ij_0}^k$ and costs of resources g_j , $\bar{g}_{ij_0}^k$. The relationship between the decrease in the above-mentioned parameters and costs of resources is linear proportionality with coefficients b_{vij} , b_{vik} , $\bar{b}_{vj}$, and $\bar{b}_{vk}$, where v is the resource number. Denote by B_v the amount of the resource v .

The task is to determine the changes of the coefficients of direct costs Δa_{ij} , $i = \overline{1, n}$, $j = \overline{1, n}$, Δ_{ij} $j \neq j_0$, the value $\Delta \bar{a}_{ij_0}^k$ of unit costs for separate technologies that constitute the branch j_0 , the intensity of use of technologies $\Delta W_k^{(j_0)}$, $k = \overline{1, K_{j_0}}$, the share of ultimate incomes in the price of the product Δq_j and unit costs of the resources Δg_j , $\Delta \bar{g}_{j_0}^k$, $j = \overline{1, n}$, $j \neq j_0$, $k = \overline{1, K_{j_0}}$, satisfying the following conditions.

1. The equalities that determine Δa_{ij_0} and Δg_{j_0} and follow from (2.59) and (2.62):

$$\begin{aligned} \Delta a_{ij_0} &= \sum_{k=1}^{K_{j_0}} \left(\left(\bar{a}_{ij_0}^k + \Delta \bar{a}_{ij_0}^k \right) \left(W_k^{(j_0)} + \Delta W_k^{(j_0)} \right) - \bar{a}_{ij_0}^k W_k^{(j_0)} \right), \\ \Delta g_{j_0} &= \sum_{k=1}^{K_{j_0}} \left(\left(\bar{g}_{j_0}^k + \Delta \bar{g}_{j_0}^k \right) \left(W_k^{(j_0)} + \Delta W_k^{(j_0)} \right) - \bar{g}_{j_0}^k W_k^{(j_0)} \right), \\ \tilde{g}_{j_0} &= \sum_{k=1}^{K_{j_0}} \Delta \hat{g}_{j_0}^k \left(W_k^{(j_0)} + \Delta W_k^{(j_0)} \right), \quad j = \overline{1, n}. \end{aligned} \quad (2.64)$$

2. Constraints for the range of variation of variables of the model determined from technological considerations:

$$\begin{aligned}
 \gamma_{ij} \Rightarrow \leq \Delta a_{ij} \leq \bar{\gamma}_{ij}, \quad \delta_j \leq \Delta g_j \leq \bar{\delta}_j, \quad i, j = \overline{1, n}, \\
 \underline{W}_k^{(j_0)} \leq \Delta W_k^{(j_0)} \leq \overline{W}_k^{(j_0)}, \quad k = \overline{1, K_{j_0}}, \\
 \tilde{\gamma}_{ik} \leq \Delta \bar{a}_{ij_0}^k \leq \bar{\tilde{\gamma}}_{ik}, \quad i = \overline{1, n}, \quad k = \overline{1, K_{j_0}}, \\
 \tilde{\delta}_k \leq \Delta \bar{g}_{j_0}^k \leq \bar{\tilde{\delta}}_k, \quad k = \overline{1, K_{j_0}}, \\
 0 \leq q_j + \Delta q_j \leq 1, \quad i = \overline{1, n}, \\
 0 \leq a_{ij} + \Delta a_{ij} \leq 1, \quad i, j = \overline{1, n},
 \end{aligned} \tag{2.65}$$

where $\gamma_{ij} \Rightarrow$, $\bar{\gamma}_{ij}$, δ_j , $\bar{\delta}_j$, $\underline{W}_k^{(j_0)}$, $\overline{W}_k^{(j_0)}$, $\tilde{\gamma}_{ik}$, $\bar{\tilde{\gamma}}_{ik}$, $\tilde{\delta}_k$, and $\bar{\tilde{\delta}}_k$ are some predetermined quantities.

3. The constraints following from the definition of the concept “the share of added cost in the price of a product”:

$$0 \leq a_{jj} + \Delta a_{jj} + l_j (q_i + \Delta q_j) + d_j \leq 1, \quad j = \overline{1, n}. \tag{2.66}$$

4. The constraints that exclude intensification of the inflation of costs and are similar to those considered in Sect. 2.5.1:

$$\sum_{i=1, i \neq j}^n \frac{a_{ij} + \Delta a_{ij}}{1 - (a_{jj} + \Delta a_{jj}) - (l_j (q_j + \Delta q_j) + d_j)} \leq \beta, \quad j = \overline{1, n}, \tag{2.67}$$

where $\beta \leq 1$ is a predetermined quantity.

5. The resource constraints for structural and technological changes

$$\begin{aligned}
 \sum_{i=1}^n \sum_{j=1}^n b_{vij} \max(0, -\Delta a_{ij}) + \sum_{i=1}^n \sum_{k=1}^{K_{j_0}} \tilde{b}_{vik} \max(0, -\Delta \bar{a}_{ij_0}^k) \\
 + \sum_{j=1, j \neq j_0}^n \bar{b}_{vj} \max(0, -\Delta g_j) + \sum_{k=1}^{K_{j_0}} \bar{b}_{vk} \max(0, -\Delta g_{j_0}^k) \leq B_v.
 \end{aligned} \tag{2.68}$$

6. The equality that determines the profit of ultimate users D after structural and technological changes:

$$D = \frac{(q + \Delta q)^T (E - (A + \Delta A))^{-1} h}{1 - (q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha}, \quad (2.69)$$

where $A = \{a_{ij}\}$ is the Leontief matrix, $i, j = \overline{1, n}$; $\Delta A = \{\Delta a_{ij}\}$ is the matrix of changes for A , $i, j = \overline{1, n}$; E is a unit matrix; $q = (q_1, \dots, q_n)$; $\Delta q = (\Delta q_1, \dots, \Delta q_n)$; $h = (h_1, \dots, h_n)$, $\alpha = (\alpha_1, \dots, \alpha_n)$; T is the operation of transposition.

7. The constraints for resources used during the production

$$(g + \Delta g)^T (E - (A + \Delta A))^{-1} (\alpha D + h) \leq G, \quad (2.70)$$

where Δg is a matrix of column vectors Δg_j , $j = \overline{1, n}$.

8. Constraints for the emission of polluting substances

$$\tilde{g}^T (E - (A + \Delta A))^{-1} (\alpha D + h) \leq \tilde{G}. \quad (2.71)$$

9. The sum of new values of the intensities of use of technologies should be equal to unity (gross variations of these intensities are zero)

$$\sum_{k=1}^{K_{j_0}} \Delta W_k^{(j_0)} = 0. \quad (2.72)$$

Among all the admissible solutions satisfying relationships (2.64)–(2.72), it is necessary to choose such that

(a) maximize ultimate incomes of consumers

$$F_1 = D \rightarrow \max; \quad (2.73)$$

(b) maximize the value of the multiplier “increase of incomes–increase of output”

$$F_2 = (q + \Delta q)^T (E - (A + \Delta A))^{-1} \alpha^{(1)} \rightarrow \max, \quad (2.74)$$

where $\alpha^{(1)} = (\alpha_1^{(1)}, \dots, \alpha_n^{(1)})$ is a predetermined numerical vector;

(c) minimize the costs of especially scarce resource with industrial purpose

$$F_3 = (g^{(1)} + \Delta g^{(1)})^T (E - (A + \Delta A))^{-1} (\alpha D + h) \rightarrow \min, \quad (2.75)$$

where $g^{(1)}$ is a row of the matrix g containing coefficients of unit costs of this resource, $\Delta g^{(1)}$ is the corresponding row of the matrix Δg .

Note that the resultant model (2.64)–(2.75) is a rather complex optimization problem. The objective functions F_2 and F_3 and constraints (2.64), (2.67)–(2.71) are nonlinear. A part of these constraints are equalities, which makes the set of admissible solutions nonconvex and additionally complicates the problem; the functions F_2 and F_3 are nonconvex too. Therefore, it is necessary to develop specialized algorithms to solve such a problem.

Note that the optimization problem (2.64)–(2.75) belongs to the same class as the problems (2.38)–(2.43) and (2.38)–(2.42), (2.44) do. Therefore, the above-mentioned algorithms can be developed based on the approaches stated in the previous subsections.

In the second of the considered models, the prices for the product manufactured are considered to be a tool of varying the coefficients of direct costs.

In spite of the fact that the capabilities of a state to control prices are limited under conditions of transitive (and the more so market) economy, the price regulation remains to be an efficient tool of economic policy. In particular, it can be used for technological and structural changes, stimulation of energy- and resource-saving. In this connection, of interest is to develop models similar to (2.38)–(2.43) and (2.38)–(2.42), (2.44), in which the controlled parameters influencing the coefficients of direct costs are the prices for production of branches. Below, we consider one of such models.

Like in the problems considered earlier, assume that the national economy consists of n branches. Denote by a_{ij}^0 , $i, j = \overline{1, n}$, the coefficients of direct costs in value terms, computed for the prices valid at the moment of calculating the intersectoral balance. It is necessary to determine expedient changes of these prices that maximize the ultimate incomes of consumers or the multiplier.

Taking into account the formulation of the problem, denote by p_i the changed price index for the production of the branch i ($i = \overline{1, n}$) related to the price valid at the moment of calculating the balance, and by $\tilde{q}_i$ the share of incomes of consumers in the price of the production of the branch i . The last quantity is expressed in terms of the original price of this production. Similar to the previous models, assume that the value q_i of the added cost in the cost of a unit of production of the branch i linearly depends on q_i :

$$\tilde{q}_i = l_i q_i + d_i, \quad i = \overline{1, n},$$

where $l_i > 0$ and $d_i > 0$ are some given coefficients having the same meaning as for the previous models.

The equations of intersectoral balance for prices [31]

$$p_j = \sum_{i=1}^n a_{ij}^0 p_i + \tilde{q}_j, \quad j = \overline{1, n},$$

yield the relationships

$$p_j = \sum_{i=1}^n a_{ij}^0 p_i + l_j q_j + d_j, \quad j = \overline{1, n}, \quad (2.76)$$

that form the first group of constraints of the model under study.

The coefficients of direct costs $a_{ij}(p)$ for the established prices, depending on the price indices $p = (p_1, \dots, p_n)$, are determined from a_{ij}^0 as follows:

$$a_{ij}(p) = \frac{a_{ij}^0 p_i}{p_j}, \quad i, j = \overline{1, n}. \quad (2.77)$$

By virtue of the law of value, price variation should not change the sum of price indices. Since the equality $p_i = 1, i = \overline{1, n}$, was true for prices of the balance, the current values of p_i should satisfy the constraint

$$\sum_{i=1}^n p_i = n. \quad (2.78)$$

The values of the model variables p_i and q_i should be non-negative:

$$p_i \geq 0, \quad q_i \geq 0, \quad i = \overline{1, n}. \quad (2.79)$$

Among $p = (p_1, \dots, p_n)$ and $q = (q_1, \dots, q_n)$ satisfying conditions (2.76)–(2.79), it is required to find those maximizing the ultimate income of consumers

$$\tilde{f}_1(p, q) = \frac{q^T (E - A(p))^{-1} h}{1 - q^T (E - A(p))^{-1} \alpha} \rightarrow \max, \quad (2.80)$$

or the value of the multiplier “increase of incomes–increase of output”

$$\tilde{f}_2(p, q) = q^T (E - A(p))^{-1} \alpha^1 \rightarrow \max. \quad (2.81)$$

In (2.80), (2.81), we use the same notation as in (2.43), (2.44), $A(p) = \{a_{ij}(p)\}$.

For the problems (2.76)–(2.80) and (2.76)–(2.79), (2.81) to be formulated correctly, it is required that the matrix $(E - A(p))^{-1}$ exists at any point of the set of admissible solutions. To this end, it is possible to use the necessary and sufficient existence condition for inverse matrix [31]

$$1 > \lambda(A(p)), \quad (2.82)$$

where $\lambda(A(p))$ is the Frobenius number of the matrix $A(p)$ (a real positive eigenvalue of this matrix, whose absolute value is maximum among all the eigenvalues).

Taking into account inevitable discrepancy of the initial data and the necessity of creating reserves during management decision-making, it is expedient to consider the following condition instead of (2.82) as the model constraint:

$$\lambda(A(p)) \leq \beta_1, \quad (2.83)$$

where $0 < \beta_1 < 1$ is a preset number sufficiently close to unity.

Along with condition (2.83), it is possible to use the sufficient existence conditions for the matrix $(E - A(p))^{-1}$, for example,

$$\sum_{i=1}^n a_{ij}(p) < 1, \quad j = \overline{1, n}$$

or

$$\sum_{i=1}^n a_{ij}(p) < \beta_1 < 1, \quad j = \overline{1, n}. \quad (2.84)$$

Conditions (2.84) specify a narrower domain of solutions compared with (2.83); however, being linear inequalities, they simplify the problem.

The model may also include two-sided constraints for possible ranges of variation of p_i and q_i :

$$\underline{p}_i \leq p_i \leq \overline{p}_i, \quad \underline{q}_i \leq q_i \leq \overline{q}_i, \quad i = \overline{1, n}. \quad (2.85)$$

The optimization problems obtained in this case have nonconvex objective functions, and if condition (2.83) is taken into account, a nonlinear constraint as well. To solve them, the approaches considered in Sects. 2.5.1 and 2.5.2 can be applied. Because of the presence of equality constraints, of interest is the compatibility of constraints (2.76) and (2.78). Basically, these constraints should be satisfied for $p_i = 1, i = \overline{1, n}$, and for q_i that correspond to the share of incomes of consumers in the price of production of the branches at the moment of calculating the balance. However, since model parameters, for example, l_i and $d_i, i = \overline{1, n}$, vary depending on the fiscal policy, this satisfaction may be violated and there may be necessity to check quickly the constraints for compatibility. This can be executed as follows. As follows from (2.76),

$$p = (E - A^T)^{-1} (l, q) + d, \quad (2.86)$$

where $A = \{a_{ij}^0\}$, $\langle l, q \rangle$ is the componentwise product of the vectors $l = (l_1, \dots, l_n)$ and $q = (q_1, \dots, q_n)$, $d = (d_1, \dots, d_n)$. According to (2.86), the dependence of p on q is continuous. Components of the matrix $(E - A^T)^{-1}$ (total expenditure coefficients), all the l_i and d_i are non-negative. Therefore, for q such that the indices $p = (p_1, \dots, p_n)$ defined by (2.86) satisfy (2.78) to exist, it is necessary and sufficient that the inequality

$$\tilde{e}^T (E - A^T)^{-1} d \leq n$$

holds and there exist admissible values of $q = q^1$ such that the following inequality holds:

$$\tilde{e}^T (E - A^T)^{-1} (\langle l, q \rangle + d) \leq n,$$

where $\tilde{e}$ is an n -dimensional vector consisting of unities.

The models (2.64)–(2.75) and (2.76)–(2.81), (2.83)–(2.85) are also planned to be included in the next versions of the IOMSTC system considered in Sect. 2.5.3.

2.5.5 Conclusions and Fields of Further Studies

The obtained results allow making the following conclusions.

1. Intersectoral optimization models with variable coefficients of direct costs are an effective decision support tool for planning structural reforms. They allow determining basic trends of structural and technological changes to increase the efficiency of the economy, to reduce the industrial consumption of energy resources, and to increase the real incomes of ultimate users without significant inflationary effects.
2. Performing computations with the use of the above-mentioned models necessitates searching for a conditional extremum of nonconvex functions defined with the use of inversion of the matrix of variables. Two approaches are proposed to develop solution algorithms for such problems. The first of them is based on using a differentiation formula for implicitly defined functions, the second—on the problem transformation by introducing additional variables. In both cases, the problem reduces to maximizing a nonsmooth penalty function by the r -algorithm. Since the resultant penalty functions are nonconvex, it is proposed to search for points of their optimum from several initial approximations. The search can easily be parallelized and implemented with a modern multiprocessor equipment, in particular, on the SKIT-2 and SKIT-3 clusters developed at the Institute of Cybernetics of NASU.

3. Both approaches to the solution of optimization problems have advantages and shortcomings. In particular, within the framework of the first approach, the problem dimension can be substantially reduced due to eliminating “insignificant” components of the matrix of direct cost coefficients, at the same time, the second approach increases the dimension due to additional variables. However, since these variables can be interpreted as parameters of the branch structure of ultimate incomes, the latter approach provides wider opportunities for the analysis of the solution, estimation of the influence of the developed or desirable structure of incomes (i.e., interests of separate economic entities) on the choice of trends of structural and technological changes and revealing the contradictions arising in this case. It is proved for models that for the specified structure of incomes, the set of admissible solutions is convex, which simplifies such an analysis.
4. The models proposed together with the necessary software and dataware and a worked-out application form a basis of the information technologies of management decision support implemented in a specialized DSS. The IOMSTC system has been developed at the Institute of Cybernetics of NASU as a prototype version of such a DSS.
5. An upgraded version of the IOMSTC system is planned to include intersectoral optimization models in view of intrabranh technological structure and the model of influence of price factors. This leads to solving more complex optimization problems, but the approaches considered earlier can be applied to construct computation algorithms for them. In our opinion, further studies in this field will be carried out in the following directions:
 - developing computation algorithms based on advanced models;
 - creating components of software and dataware for these computations;
 - expanding service capabilities of the IOMSTC system and means of its switching to multiprocessor computers;
 - generalizing the experience of applying the above-mentioned models for management decision support.

2.6 Expert Assessments Processing by Ordinal Regression Methods

In order to build the fuzzy set it is necessary to determine the universe of elements X and the membership function $\mu(x) \in [0; 1]$. For any $x \in X$ the value $\mu(x)$ determines the degree of element's x membership in the fuzzy set. The higher values of the membership function correspond with the elements which possess a higher degree of membership with this set. The building of $\mu(x)$ can be based on the expert assessments of the membership degree of separate elements. Generally, these are ordinal assessments of the following type: “the element $x^{(1)}$ has a higher (smaller) degree of membership with the set than the element $x^{(2)}$,” that is why the methods

of their processing with the purpose of building the function $\mu(x)$ in its analytical form are called ordinal regression methods [24].

The building of $\mu(x)$ by these methods consists of two stages.

1. We build an arbitrary, bounded on X function $F(x)$ which is the membership degree function.
2. $F(x)$ is transformed in such a way that its values belong to the interval $[0; 1]$.

To define the function $F(x)$ we first need to assign a certain analytical form $\overline{F}(x, \gamma)$ for it, which is dependent on some unknown parameters γ . For example, if x is an n -dimensioned numerical vector, this could also be a linear function $\overline{F}(x, \gamma) = \sum_{i=1}^n \gamma_i x_i$, where γ_i are unknown coefficients. More complex analytical forms are also possible. To identify γ we use a trial sample – a certain set of elements $(x^{(1)}, \dots, x^{(N)})$ and relations between them of the following type: “the element $x^{(i)}$ has a higher (smaller) degree of membership with the set than element $x^{(j)}$.” The experts determine such relations. These relations can be depicted in the form of a matrix $t = \{t_{ij}\}$, where

$$\begin{cases} 1, & \text{if the degree of membership with the set is higher for element } x^{(i)} \\ & \text{than for element } x^{(j)}, \\ t_{ij} = -1 & \text{if the degree of membership for } x^{(i)} \text{ is higher than for } x^{(j)}, \\ 0, & \text{if the expert cannot compare the elements among themselves} \\ & i, j = \overline{1, N}. \end{cases}$$

It is reasonable to assume that

$$\begin{aligned} \overline{F}(x^{(i)}, \gamma) &\geq \overline{F}(x^{(j)}, \gamma), \text{ if } t_{ij} = 1, \\ \overline{F}(x^{(i)}, \gamma) &\leq \overline{F}(x^{(j)}, \gamma), \text{ if } t_{ij} = -1. \end{aligned}$$

Thus, the values of γ can be estimated as the solution of the system of inequalities:

$$\left(\overline{F}(x^{(i)}, \gamma) - \overline{F}(x^{(j)}, \gamma) \right) t_{ij} \geq 0, \quad i, j = \overline{1, N}. \quad (2.87)$$

If the system (2.87) has at least one solution, then the set of its solutions will coincide with the set of points of the function maximum:

$$G(\gamma) = \min \left(0, \min_{i, j = \overline{1, N}} \left(\left(\overline{F}(x^{(i)}, \gamma) - \overline{F}(x^{(j)}, \gamma) \right) t_{ij} \right) \right),$$

which can be found applying numerical methods of non-differential optimization [39]. Under the condition that the solutions of the system (2.87) do not exist, the

point of the function $G(\gamma)$ maximum can be viewed as the pseudo-solution of this system.

Optimization algorithms can also be applied for γ estimation under more complex structures of expert assessments. For instance, if the degree of membership of elements $(x^{(1)}, \dots, x^{(L)})$ with the fuzzy set is estimated by L experts, their estimates form matrixes $t^{(1)}, \dots, T^{(L)}$, similar to t , and the sets $M(T^{(k)})$ of the solutions of the system of the type (2.87) provided $t = T^{(k)}$, $k = \overline{1, L}$, are not empty, the value γ can be estimated having solved the following problem:

$$G_1(\gamma) = \max_{k=\overline{1, L}} d\left(\gamma, M(T^{(k)})\right) \rightarrow \min, \quad (2.88)$$

where

$$d\left(\gamma, M(T^{(k)})\right) = \min_{\bar{\gamma} \in M(T^{(k)})} \|\gamma - \bar{\gamma}\|^2.$$

This is also a non-differential optimization problem.

Having solved the system (2.87) or the optimization problem (2.88), we derive the estimator $\hat{\gamma}$ for parameters γ . Let us consider $F(x) = \overline{F}(x, \hat{\gamma})$. After $F(x)$ is built, we define $\mu(x)$ as:

$$\mu(x) = \frac{F(x) - F^{\min}}{F^{\max} - F^{\min}},$$

where

$$F^{\max} = \max_{x \in X} F(x), \quad F^{\min} = \min_{x \in X} F(x).$$

Similar to simply ordered sets, elementary set-theoretical operations, in particular those of unification, intersection, and complement were defined for fuzzy sets. At the same time, it is considered that the universe X is identical for all sets. Then the membership function of sets A and B intersection equals:

$$\mu(x) = \min(\mu_A(x), \mu_B(x)),$$

where $\mu_A(x)$, $\mu_B(x)$ are the membership functions for sets A and B .

By analogy, the membership function for A and B sets unification is $\mu(x) = \max(\mu_A(x), \mu_B(x))$, and the complement function for the set A is $\mu(x) = 1 - \mu_A(x)$. This allows identifying the fuzzy set membership function by the membership functions of the sets that generate it and by all the operations with these sets. The sequence of operations can be shown as a graph, the final nodes of which ("tree leaves") are the initial fuzzy sets, transient nodes are operations performed in a certain sequence. The arches of the graph define those results of the former

operations that are being used in the next operation. The initial node of the graph (“tree root”) is the result of the performance of a certain operations sequence. After we identify the membership functions $\mu_i(x)$ for the final nodes (these functions can be built by the above-considered methods of ordinal regression), we successively build the membership function $\mu_j(x)$ for the intermediate nodes j according to the following formula:

$$\mu_j(x) = \begin{cases} \min_{i \in I(j)} (\mu_i(x)), & \text{if the node } j \text{ corresponds with the intersection operation,} \\ \max_{i \in I(j)} (\mu_i(x)), & \text{if the node } j \text{ corresponds with the unification operation,} \\ 1 - \mu_{I(j)}(x), & \text{if the node } j \text{ corresponds with the complement operation} \end{cases}$$

In relation (2.88) $I(j)$ is the set of nodes adjacent with the node j . In case of the complement operation we will consider that this set is always comprised of one node.

The building of the membership function ends in the initial node, which corresponds with the membership function $\mu_0(x)$ of the set comprised of the sequence of operations.

Notably, such approach could be applied for comparing alternatives using complex-hierarchy structure [26] criteria aggregates. In contrast with some other methods designed for such problems solving, as AHP [22], for example, this method has a more strict theoretical validation, as well as allows to explain such phenomena as rank reversion [26] and to interpret the results obtained more distinctly. The above properties of this approach determine its application for solving the problems of relations harmonization around the suburban area.

2.7 Example of Ordinal Regression Methods Application for Defining the Limits and Status of the Suburban Area

To define the limits of the suburban area let us make use of the above-considered fuzzy sets theory methods and those of ordinal regression algorithms. Obviously, the suburban area will include the highly urbanized areas, which gravitate toward the city – the center of agglomeration. Thus, if we consider the suburban area as a fuzzy set, it will be the intersection of two fuzzy sets:

1. The area with a sufficient level of urbanization.
2. The area (or a list of towns and other settlements) with strong gravitation toward the agglomeration center.

The membership functions for these sets were constructed by ordinal regression methods.

We have built the membership functions for the areas with a sufficient degree of urbanization for individual regions of Kyiv oblast (for administrative units of the lower level). The following indicators were taken into account:

$f_1^{(1)}$ is share of urban population in the total population of the area (%);

$f_2^{(1)}$ is share of industry products and services in the gross regional product (%);

$f_3^{(1)}$ is share of those employed in non-agricultural sector in the total number of the region's employees (%).

The data for these indicators for a range of regions in Kyiv oblast is provided in Table 2.15. Along with the areas neighboring Kyiv, the table presents data for some distant regions. This was done with the purpose of trial sample analysis simplification. The trial sample is comprised of the data for Ivankivsky, Kaharlytsky, Baryshevsky, Yahotynsky, and Vasylkivsky regions. Based on formal and normalized arguments, the experts in economical geography placed these regions according to the level of urbanization (from the least urbanized to the most urbanized) in the sequence given above. Having solved the system of the type (2.18) comprised of the indicators for these regions, we derived the following function:

$$F_1(f) = f_1^{(1)} + 1.4f_2^{(1)} + 1.9f_3^{(1)},$$

which numerically depicts greater or smaller degree of the object membership in the fuzzy set being considered. Taking into account that the maximum possible value of this function, which can be achieved at 100 % values of indicators, is $F^{\max} = 430$, and the smallest (which is obtained at the zero values of the indicators) is $F^{\min} = 0$, we will define the membership function as

$$\mu_1(f) = \frac{F_1(f)}{430}.$$

The values of the functions $F_1(f)$ and $\mu_1(f)$ are also provided in Table 2.15. It should be emphasized that all the regions adjacent to Kyiv have sufficiently high values of the membership function: for them $\mu_1(f) > 0.6$. Thus, these areas could be considered as sufficiently urbanized. At the same time, the Kaharlytsky region, which is quite close to Kyiv, is not such an area and it is inappropriate to consider it as part of the suburban area.

The model that defines the area's degree of gravitation toward the big city could be considered as the generalization of a well-known "gravitation" model, which is broadly applied for preliminary macroeconomic analysis. The variables of the suggested model are as follows:

- $f_1^{(2)} = d_{cc}$ the distance (in km) between the centers of the populated area being considered and the big city;
- $f_2^{(2)} = d$ the distance (in km) between the closest points of the populated area and of the big city area;
- $f_3^{(2)}$ distance (in km) between the closest points of the populated area considered and of the “compact” agglomeration. It is considered that the “compact” agglomeration around a big city is comprised of the populated areas located at the distance d from each other which is not more than 1 km;
- $f_4^{(2)}$ the share, % of industrial products produced in this populated area that are being sold in a big city – the center of agglomeration. This indicator shows the degree of local production orientation toward the big city market;
- $f_5^{(2)}$ the share of work-capable population which is employed in a big city. This indicator reflects the participation of local population in pendulum migration.

The values for these indicators calculated for some towns around Kyiv are presented in Table 2.16.

According to these methods of ordinal regression, and utilizing the trial sample consistent of: Irpin, Glevaha, Boryspil, Vasylkiv, Dymir, Yahotyn (these towns are placed in the order of their decreasing level of gravitation toward Kyiv) we built the following function, which depicts this degree:

$$F_2(f) = -f_1^{(2)} - 1.5f_2^{(2)} - 1.87f_3^{(2)} + 2.203f_4^{(2)} + 2.597f_5^{(2)}.$$

The negative coefficients for $f_1^{(2)}$, $f_2^{(2)}$, $f_3^{(2)}$ indicate that an increase in distance has a negative influence on the degree of gravitation toward the agglomeration center (Tables 2.15 and 2.16).

Taking into account that $F_2^{\max} = 190$, $F_2^{\min} = -270$, we derive the following expression for the fuzzy set membership function for the towns that gravitate toward Kyiv:

$$\mu_2(f) = \frac{F_2(f) + 270}{460}.$$

The values of the functions $F_2(f)$ and $\mu_2(f)$ are also presented in Table 2.16.

It was mentioned earlier that the suburban area could be considered as a fuzzy set of populated areas – the intersection of the two previously built sets. Thus, the membership function $\mu(f)$ for this set equals:

Table 2.15 Membership indicators and membership function values for a fuzzy set of an urbanized area

Regions	Adjacent to a big city	Indicators			F_1	μ_1
		$f_1^{(1)}$	$f_2^{(1)}$	$f_3^{(1)}$		
Vyshgorodsky	Yes	53.7	67.3	77.3	294.79	0.686
Ivankivsky	No	31.2	49.05	45.0	186.0	0.433
Borodyansky	No	56.3	69.9	76.9	300.27	0.698
Irpın (town and settlements)	Yes	100.0	93.6	96.8	414.98	0.965
Makarivsky	No	26.2	62.4	76.7	259.39	0.603
KyivNoSvyatoshynsky	Yes	47.6	74.7	86.3	316.15	0.735
Fastivsky	No	66.7	79.7	73.1	317.17	0.738
Vasylkivsky	No	56.2	70.4	79.2	305.24	0.710
Obuhivsky	Yes	60.4	79.2	81.4	325.94	0.758
Kaharlytsky	No	32.5	48.7	49.1	193.97	0.451
Brovarsky	Yes	59.1	82.9	83.2	333.24	0.775
Boryspilsky	Yes	46.8	78.1	80.9	309.85	0.722
Baryshevsky	No	42.7	71.5	75.3	285.87	0.665
Pereyaslav-Khmelnytsky	No	42.9	66.1	69.1	266.73	0.620
Yahotynsky	No	56.7	69.7	71.1	289.37	0.673
Rokytnyansky	No	36.6	49.1	50.1	200.53	0.466

$$\mu(f) = \min(\mu_1(f), \mu_2(f)),$$

where f is the vector of indicators for a corresponding populated area (for the $\mu_2(f)$ function) or for the region where it is located (for $\mu_1(f)$ function). The values of $\mu(f)$ are given in Table 2.17. This data indicate that such towns as Pereyaslav-Khmelnytsky, Berezan, Fastiv and Yahotyn (as well as the corresponding regions) should not be considered as part of a suburban area if the membership criteria is set as the satisfaction of inequality $\mu(f) > 0,6$ (i.e., if the level of trust exceeds 60 %).

Thus, we can assert that the Kyiv suburban area is comprised of Baryshevsky (excluding the eastern part around Berezan town), Boryspilsky, Borodyansky, Brovarsky, Vasylkivsky (with the exception of its southern part), Vyshgorodsky, Kyiv-Svyatoshynsky, Makarivsky, Obuhivsky (with the exception of the southern part) regions and the following towns of oblast subordination: Boryspil, Brovary, Vasylkiv, Irpin (together with the subordinated settlements) and Rzhyschiv. The suburban area occupies around 5.7 thousand square km. The population is around 820 thousand people.

After we at least approximately define the limits of the suburban area we encounter the problem of appropriate legislation formulation which would regulate the relations between the local and oblast authorities in the area. The various relevant proposals appearing in the press define the main three possible alternative concepts of such legislation:

Table 2.16 Membership indicators and membership function values for “gravity” fuzzy set

Towns and settlements	Indicators					F_2	μ_2
	$f_1^{(2)}$	$f_2^{(2)}$	$f_3^{(2)}$	$f_4^{(2)}$	$f_5^{(2)}$		
Vyshgorod	15.5	0	0	45.0	32.0	166.74	0.949
Irpin (town)	21.5	0.5	0	51.0	31.0	170.61	0.958
Bucha	25	3.5	0	49.0	31.0	158.2	0.931
Vorzel	29	6.5	0	50.0	34.0	159.7	0.933
Chabany	15	1.8	1.8	43.0	41.0	180.1	0.978
Boryspil	35	8	8	47.0	35.0	135.44	0.882
Glevaha	25.5	10.5	7.5	49.0	38.0	151.36	0.916
Vasylkiv	34.5	16	13	48.0	33.0	108.64	0.823
Obuhiv	42	11	7	53.0	35.0	136.1	0.883
Borodyanka	46	29	14	55.0	36.0	98.99	0.802
Rzhyshev	65	38	30	54.0	37.0	36.951	0.667
Baryshevka	56	34	29	47.0	32.0	25.416	0.642
Berezan	66	44	38	44.0	33.0	−20.4	0.543
Pereyaslav-Khmelnytsky	79	54	49	49.0	27.0	−73.56	0.427
Makariv	43	28	21.5	53.0	42.0	100.63	0.806
Yahotyn	92	56	53	46.0	24.0	−111.4	0.345
Fastiv	79	67	62	44.0	27.0	−128.39	0.308
Dymer	34	22	20	47.0	39.0	100.4	0.805
Piskivka	65	48	30	52.0	32.0	22.74	0.636
Velyka Dymerka	33	5	5	59.0	37.0	176.2	0.970

1. The subordination of suburban populated areas to the Kyiv city authorities. At the same time, the northern regions of Kyiv oblast which suffered from Chernobyl Nuclear Power Plant Accident the most, as well as some of the regions of the neighboring Zhytomyr oblast, which are in the same condition, should form a separate centrally controlled administrative unit. The Kyiv oblast will include only southern regions to which some regions of the closest oblasts could be added. The oblast authorities will be placed in a local sub-regional center – a city with a 200 thousand population (this is a typical size of an oblast center for agricultural regions of Ukraine).
2. The subordination of Kyiv city authorities to the oblast authorities that is to convert Kyiv to the status of other big cities in Ukraine.
3. The establishment of consultative-advisory body, which would coordinate the activities of city and oblast authorities, related to the suburban area, and would take into account the interest of local governing institutions of the suburban area.

These alternatives were analyzed by means of fuzzy sets theory, taking into consideration various criteria. The hierarchy of criteria was defined in a form of a graph. The various interests of individual participants were reflected as different

Table 2.17 Values of suburban area membership function for towns and settlements

Towns and settlements	μ_1	μ_2	μ	Including into suburban area (significance level $\mu_0 = 0.6$)
Vyshgorod	0.686	0.949	0.686	Yes
Irpın (town)	0.965	0.958	0.958	Yes
Bucha	0.965	0.913	0.931	Yes
Vorzel	0.965	0.933	0.933	Yes
Chabany	0.735	0.978	0.735	Yes
Boryspil	0.722	0.882	0.722	Yes
Glevaha	0.710	0.916	0.710	Yes
Vasylkiv	0.710	0.823	0.710	Yes
Obuhiv	0.758	0.883	0.758	Yes
Borodyanka	0.698	0.802	0.698	Yes
Rzhyshevchiv	0.758	0.667	0.667	Yes
Baryshevka	0.665	0.642	0.642	Yes
Berezan	0.665	0.543	0.543	No
Pereyaslav-Khmelnytsky	0.62	0.427	0.427	No
Makariv	0.603	0.806	0.603	Yes
Yahotyn	0.673	0.345	0.345	No
Fastiv	0.738	0.308	0.308	No
Dymer	0.686	0.805	0.686	Yes
Piskivka	0.698	0.636	0.636	Yes
Velyka Dymerka	0.775	0.970	0.775	Yes

values of the weight coefficients which were multiplied by membership functions of the previous sets (“the decision is good under a separate criterion”) while building the resultive fuzzy set (“the decision is good under the totality of criteria).

The performed analysis demonstrates that the first alternative will be the best, and the second – the worst taking into consideration the interests of Kyiv city authorities and those of local governing institutions of suburban areas. Taking into consideration the interests of oblast authorities – the first alternative is the worst and the second alternative is the best. For the state authorities the third alternative is the best, the second alternative is the worst. Thus, the third alternative should be considered as a reasonable compromise between multi-directed interests of the participants of the processes we studied.

Conclusions

1. In a transition economy, the processes of interaction between a big city and a suburban area are very complex, unsteady as well as multifaceted. A range of interrelated economic, ecological, social, technical as well as legal issues arise during the course of these processes. All this stipulates the rational for Compram method application to research the processes mentioned.
2. Within the framework of the above-mentioned method, a detailed problem description has been made; the major concepts for its solution have been identified. Also, we defined the phenomena which require top-priority attention. In particular, the development of legislation, which would coordinate the actions of city and regional authorities in the suburban area, is considered in this paper as a mental idea for solving the existing problems.
3. The definition of suburban area geographical limits and the comparison of alternatives for legislation development are important tasks, which need to be accomplished in order to implement the above-mentioned idea.
4. While solving these problems we should consider various weakly structured factors. It is recommended to implement the methods of fuzzy sets theory and ordinal regression.
5. On the basis of the suggested approach, a multi-criteria analog of a gravitation model was developed, where the gravitation degree of separate localities toward the big city as well as the degree of urbanization of the surrounding area were taken into account, a comparison procedure for legislation development alternatives was developed. Together, all this allowed to formulate some “knowledge islands” regarding some issues considered. In particular, this allowed defining the geographical limits of the suburban area and to formulate a hypothesis regarding the participants’ interests in legislative changes implemented in a particular direction. With regard to the considered example of Kyiv and its interaction with the suburban area, the crucial statement is that about a considerable size of the territory which is occupied by such an area (for a range of populated centers their borders are more than 60–70 km away from the city center), about the limited interest of the procedures participants in the gradualist approaches toward legislation changes, and about the size of the interests conflict case of the radical changes.
6. The results achieved allow passing in the nearest future to the development of semantic and casual models of interaction issues of a big city and a suburban area.
7. Fuzzy sets theory methods, as well as the ordinal regression methods could be applied within the Compram method scheme as the means of individual hypothesis, theories and islands of knowledge formulation. An important advantage of these methods is the possibility to take into account imperfect, weakly structured information, in particular the ordinal expert estimations.

8. The approach suggested is universal enough. It could be applied toward the research of interaction processes of a big city and a suburban area for both transition and developed market economy countries.

The proposed decision support procedure is based on the similar AHP approach, but it has some advantages in comparison with the latter. The procedure has stronger theoretical background from the fuzzy set theory viewpoint; it explains the possible rank reversion as the description of the efficient ranking set. Implementation of different logical operations allows to present different hierarchical relations between criteria and alternatives.

The membership function calculation algorithm allows not only to compare existing alternatives, but also to explain the comparison result and to determine the “weak point” for each alternative. The procedure creates the possibilities to analyze the consequences of the changes in the criteria values and preferences.

The minmax operators are used in the mentioned algorithm. It means that the “better” values of some criterion (or of a criteria group) cannot compensate the “worst” value of other criteria. Such an assumption seems to be quite natural if we deal with incomparable high-level criteria. However, at the lowest hierarchy level, where criteria are interchangeable, the other operators (for instance, weighed sum) are also available.

The features of the hierarchical human expertise procedure allow to use it in applied DSSs.

To develop such systems is also an integral part of our further research.

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