

Contents

1	Introduction	1
	Appendix: SAGE	6
2	Primer on k-Schur Functions	9
1	Background and Notation	10
1.1	Partitions and Cores	10
1.2	Bounded Partitions, Cores, and Affine Grassmannian Elements	14
1.3	Weak Order and Horizontal Chains	22
1.4	Cores and the Strong Order of the Affine Symmetric Group	26
1.5	Symmetric Functions	31
1.6	Schur Functions	34
1.7	Hall–Littlewood Symmetric Functions	37
1.8	Macdonald Symmetric Functions	39
1.9	Empirical Approach to k -Schur Functions	43
1.10	Notes on References	47
2	From Pieri Rules to k -Schur Functions at $t = 1$	47
2.1	Semi-standard Tableaux and a Monomial Expansion of Schur Functions	48
2.2	Weak Tableaux and a Monomial Expansion of Dual k -Schur Functions	51
2.3	Other Realizations	57
2.4	Strong Marked Tableaux and a Monomial Expansion of k -Schur Functions	60
2.5	k -Littlewood–Richardson Coefficients	66
2.6	Notes on References	70
3	Definitions of k -Schur Functions	71
3.1	Atoms as Tableaux	71
3.2	A Symmetric Function Operator Definition	77
3.3	Weak Tableaux II	79

3.4	Strong Tableaux II	83
3.5	Notes on References	86
4	Properties of k -Schur Functions and Their Duals	87
4.1	k -Schur Functions Are Schur Functions When $k \geq \lambda $ and When $t = 0$	87
4.2	The k -Schur Function Is Schur Positive	88
4.3	At $t = 1$, the k -Schur Functions Satisfy the k -Pieri Rule	89
4.4	k -Conjugation	91
4.5	The k -Schur Functions Form a Basis for $\Lambda_{(k)}^t$	92
4.6	The k -Rectangle Property	93
4.7	When $t = 1$, the Product of k -Schur Functions Is k -Schur Positive	94
4.8	Positively Closed Under Coproduct	94
4.9	The Product of a k -Schur and ℓ -Schur Function Is $(k + \ell)$ -Schur Positive	96
4.10	Branching Property from k to $k + 1$	96
4.11	k -Schur Positivity of Macdonald Symmetric Functions	98
5	Directions of Research and Open Problems	99
5.1	A k -Murnaghan-Nakayama Rule	99
5.2	A Rectangle Generalization at t a Root of Unity	101
5.3	A Dual-Basis to $s_{\lambda}^{(k)}[X; t]$	102
5.4	A Product on $\Lambda_{(k)}^t$	105
5.5	A Representation Theoretic Model of k -Schur Functions	107
5.6	From Pieri to K -Theoretic k -Schur Functions	109
6	Duality Between the Weak and Strong Orders	112
6.1	k -Analogue of the Cauchy Identity	113
6.2	A Brief Introduction to Fomin's Growth Diagrams	115
6.3	Affine Insertion	118
6.4	The t -Compatible Affine Insertion Algorithm	125
7	The k -Shape Poset and a Branching Rule for Expressing k -Schur in $(k + 1)$ -Schur Functions	126
3	Stanley Symmetric Functions and Peterson Algebras	133
1	Stanley Symmetric Functions and Reduced Words	134
1.1	Young Tableaux and Schur Functions	135
1.2	Permutations and Reduced Words	135
1.3	Reduced Words for the Longest Permutation	136
1.4	The Stanley Symmetric Function	136
1.5	The Code of a Permutation	137
1.6	Fundamental Quasi-symmetric Functions	138
1.7	Exercises	138
2	Edelman-Greene Insertion	139
2.1	Insertion for Reduced Words	139
2.2	Coxeter-Knuth Relations	141
2.3	Exercises and Problems	141

3	Affine Stanley Symmetric Functions	144
3.1	Affine Symmetric Group	144
3.2	Definition	144
3.3	Codes	145
3.4	$\Lambda_{(n)}$ and $\Lambda^{(n)}$	146
3.5	Affine Schur Functions	146
3.6	Example: The Case of \tilde{S}_3	147
3.7	Exercises and Problems	147
4	Root Systems and Weyl Groups	149
4.1	Notation for Root Systems and Weyl Groups	149
4.2	Affine Weyl Group and Translations	150
5	NilCoxeter Algebra and Fomin-Stanley Construction	151
5.1	The NilCoxeter Algebra	151
5.2	Fomin and Stanley's Construction	152
5.3	A Conjecture	153
5.4	Exercises and Problems	155
6	The Affine NilHecke Ring	155
6.1	Definition of Affine NilHecke Ring	156
6.2	Coproduct	157
6.3	Exercises and Problems	158
7	Peterson's Centralizer Algebras	158
7.1	Peterson Algebra and j -Basis	158
7.2	Sketch Proof of Theorem 7.3	159
7.3	Exercises and Problems	160
8	(Affine) Fomin-Stanley Algebras	161
8.1	Commutation Definition of Affine Fomin-Stanley Algebra	161
8.2	Noncommutative k -Schur Functions	162
8.3	Cyclically Decreasing Elements	163
8.4	Coproduct	164
8.5	Exercises and Problems	165
9	Finite Fomin-Stanley Subalgebra	165
9.1	Problems	167
10	Geometric Interpretations	167
4	Affine Schubert Calculus	169
1	Introduction	169
2	Root Data	171
2.1	Cartan Data and the Weyl Group	171
2.2	Root Data	173
2.3	Affine Root Data	176
3	NilHecke Ring and Schubert Calculus	179
3.1	NilHecke Ring	180
3.2	Coproduct on \mathbb{A}	184
3.3	Duality and the GKM Ring	187
3.4	Multiplication in Λ and Coproduct in \mathbb{A}	191

3.5	Forgetting Equivariance	192
3.6	Parabolic Case	193
3.7	Geometric Interpretations	194
4	Affine Grassmannian	196
4.1	Affine Grassmannian as Partial Affine Flags	197
4.2	Small Torus Version of Λ_{af}	198
4.3	Homology of the Affine Grassmannian	199
4.4	Small Torus Affine NilHecke Ring and Peterson Subalgebra ..	200
4.5	The j -Basis	203
4.6	Homology Structure Constants	205
4.7	Peterson's "Quantum Equals Affine" Theorems	205
A	Appendix: Proof of Coalgebra Properties	206
B	Appendix: Small Torus GKM Proofs	208
B.1	Small Torus GKM Condition for $\widehat{\mathfrak{sl}}_2$	210
C	Appendix: Homology of Gr	212
Bibliography		213

k-Schur Functions and Affine Schubert Calculus

Lam, Th.; Lapointe, L.; Morse, J.; Schilling, A.;

Shimozono, M.; Zabrocki, M.

2014, VIII, 219 p. 126 illus., Hardcover

ISBN: 978-1-4939-0681-9