Preface

The classical collective risk model was introduced by Lundberg [46] in 1903 and developed by Crámer [20] in 1930 to describe the free surplus process of an insurance company. In this model, called Cramér–Lundberg, the premiums are assumed to be collected continuously over time with constant intensity and the total claim amount at a given time is given by a compound Poisson process. Initially, the main problem of classical risk theory was to calculate the probability of ruin, but De Finetti [24] proposed in 1957 a more realistic and economically motivated stability criterion: the management of the company should look for maximizing the expectation of the present value of all dividends paid to the shareholders up to ruin time.

Nowadays, the main problems of stochastic control in insurance are to minimize the ruin probability and to maximize cumulative expected discounted dividend payouts, where the insurer can control the risk in several ways. One possibility is to invest dynamically part of the surplus on financial assets. Another possibility is to pass part of the premium to the reinsurer, which in return covers certain fraction of the claims.

The usual approach to deal with these kinds of problems is the method of dynamic programming. This approach was introduced by Bellman [14] in 1954 for optimal deterministic control problems. The basic idea is to relate the optimal problem with a certain differential equation called the Hamilton–Jacobi–Bellman equation (HJB). If the solution of the HJB equation exists, then this solution would be the optimal value function of the original control problem. However, there could be a trouble in the classical dynamic programming approach: there has to be a classical solution to the HJB equation, i.e., the solution has to be smooth to the order of derivatives involved in the equation.

In the problems of maximizing the survival probability, the corresponding HJB equations have classical solutions when the claim-size distribution has bounded density, but this is not generally the case in the problems of maximizing dividends. However, this holds under certain conditions on the claim-size distributions; see the comments and remarks of Chap. 5. In the other cases the optimization problem cannot be solved in this framework. One of the reasons that makes these problems harder for a general claim-size distribution is that the associated HJB equations involve integrodifferential operators due to the jumps in the free surplus process. In order to overcome this issue, some authors studied the diffusion approximation to the Cramér–Lundberg model; this approximation simplifies the HJB equations: they are ordinary differential equations with classical solutions.

In order to solve these problems in the general setting, it is natural to consider a weaker definition of solutions of the HJB equation; the notion of viscosity solutions introduced by Crandall and Lions [21] in 1983 is especially well suited for this task. This tool enables to find solutions of first-order integrodifferential equations or degenerate second-order integrodifferential equations. This approach has been widely used in finance theory for a long time; however it was not used in insurance control until quite recently.

The aim of this brief is to address the problem of maximization of survival probability as well as the maximization of dividends in the classical collective risk model using the viscosity approach. First we will show that the optimal value function can be characterized as either the unique or the smallest viscosity solution of the associated HJB equation, and then we found a strategy (the optimal) whose value function coincides with the optimal value function. In the problem of maximizing the survival probability, we will show that both the optimal reinsurance and the optimal investment controls depend only on the current surplus. The same holds for the problem of optimal dividend payments and besides this, the optimal dividend strategy has a band structure; roughly speaking, this means that the payment of dividends depends only on the current surplus and it is characterized by three sets $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ which partitioned the state space of the surplus process. Each of these sets is associated with a certain dividend payments action. The concept of band strategy was introduced by Gerber [29].

This work is organized as follows: In Chap. 1, we present the classical collective risk model for an insurance company and introduce the notion of the survival probability and the optimal expectation of the discounted dividend payments as functions of the initial surplus. We also study the basic properties of these value functions and derive the associated HJB equations. In Chap. 2, we introduce two ways to control the risk: reinsurance and investment. We study the basic properties of the survival probability functions as well as the optimal dividend payments with reinsurance and investment. We also derive the associated HJB equations in all these cases. In Chap. 3, we introduce the notion of viscosity solutions and show that the value functions are indeed viscosity solutions of the corresponding HJB equations. In Chap. 4, we characterize the optimal value functions among the viscosity solutions of the corresponding HJB equations. In Chap. 5, we show the existence of optimal stationary strategies and describe their structure. In Chap. 6, we present a method to construct systematically the optimal value functions and the optimal strategies in a quite general setting and show some numerical examples.

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