Preface

The history of mathematics has undergone, in the last decades, a very fast evolution process. At first cultivated by a narrow group of researchers, from the second half of the past century it has attracted the interest of a growing number of scholars, several of them being also researchers in various areas of mathematics. A subsequent period has been marked by a growing attention towards the specific methods of the historical research, with the formation of a professional community of historians of mathematics whose scientific production has often reached an important level, both from a quantitative and from a qualitative point of view. In the present period the best products of the historians of mathematics are tending to emphasize the *cultural* dimension of their subjects of study, that is the cultural dimension of mathematics itself, through the historical approach.

Among all the historical contributions, the studies on the evolution of the so-called applied mathematics are very numerous, even if perhaps more recent than the studies on the history of the so-called pure mathematics. Nevertheless, the history of mathematical optimization can cover a long period, as optimization problems date back to antiquity: the human race, for his daily requirements of surviving has always met implicit problems of maximum or minimum. This can be summed up in the words of Leonhard Euler, who wrote: "For since the fabric of the universe is the most perfect and the work of a most wise Creator, nothing takes place in the universe in which some rule of maximum or minimum does not appear." There are also some references to optimization problems in literature and poetry; a famous example is the so-called *Dido's Problem*, contained in Virgil's "Aeneid," and which may be considered as the first literary appearance of the *isoperimetric problem* of the Calculus of Variations.

The words "minimum" and "maximum" are typical of several problems, not only of mathematics but also of physics, chemistry, engineering, economics, etc. They are central in some very general principles, such as the "principle of least action" of Maupertuis. By the mid-seventeenth century, the invention of the "Calculus" has shifted the barycenter of optimization problems from a geometrical field to a predominantly analytical field. All the analytical results subsequently obtained (from Fermat onwards) have given birth to a mathematical theory, more and more refined, both from the point of view of its theoretical formulations and from the point of view of the solution techniques.

The history of the so-called *mathematical programming* (or also *nonlinear programming,* where the word "nonlinear" means "not necessarily linear") is on the contrary more recent. If, by mathematical programming or nonlinear programming, we intend those static optimization problems (usually defined in finite-dimensional spaces), where a function of n variables ("objective function") is to be minimized or maximized, subject to a certain number of constraints, *not necessarily given as equality constraints,* we can say that its history began, roughly speaking, in the twentieth century and in particular, from the years of the Second World War. Obviously, for mathematical programming problems there is also a sort of "prehistory," with individual contributions of high scientific level.

At present mathematical programming problems (and in general optimization problems) are pervasive in several sciences, such as economics, engineering, operations research, chemistry, physics, biology, social sciences, and management sciences. Moreover, they have many important applications in these areas and promise to have even wider usage in the future.

The present book collects some papers that are, in our opinion, the first basic stepping stones in nonlinear programming (see the list in the index of the book and at the end of the introductory chapter). Here we have excluded those papers exclusively concerned with linear programming, such as the contributions of G. B. Dantzig and L. V. Kantorovich. Obviously, our choice is subjective and does not claim to be complete; however, we believe that some contributions could not have been neglected: this is the case for the Master Thesis of W. Karush (1939), here published in its full length, the papers of F. John (1948) and of H. W. Kuhn and A. W. Tucker (1951). Some basic papers of K. J. Arrow, L. Hurwicz, and H. Uzawa have been included in our list. The paper of Arrow and Hurwicz of 1956 treated, with a more general approach, the equivalence between the usual nonlinear programming problem and the saddle-point problem of the Lagrangian function, a question previously analyzed by Kuhn and Tucker in their 1951 paper. Arrow and Hurwicz (1951) devised a gradient technique for approximating saddle-points and constrained optima. This is one of the earlier gradient techniques offered for solving the constrained nonlinear optimization problem. The paper of Arrow, Hurwicz, and Uzawa (1961) on constraint qualifications is perhaps the first contribution concerned with the important problem of locating "regular" constraints and of establishing the relationships between the various constraint qualifications proposed, a problem which has not lost its importance after 50 years. Equally important is the paper of Hurwicz of 1958 (but written before), one of the first treatments of nonlinear programming problems, both for the scalar and for the vector case, in topological spaces. The papers of L. L. Pennisi (1953) and G. P. McCormick (1967) are important for their treatment of second-order optimality conditions for a general nonlinear programming problem. The papers of W. Fenchel (1949), M. L. Slater (1950), and Uzawa (1958) are important for the case of convex nonlinear programming problems; in fact, convex analysis is in itself a basic tool for the development of mathematical programming, as the works of W. Fenchel,

J. J. Moreau, and R. T. Rockafellar have shown. The paper of 1963 by R. W. Cottle is perhaps the first “translation” of the original Fritz John optimality conditions into the setting of the usual nonlinear programming problems. The papers of Bliss (1938) and Valentine (1937) have been included, due to their historical relevance in connection with the birth of optimality conditions for a nonlinear programming problem.

In the introductory chapter, entitled *A Historical View of Nonlinear Programming: Traces and Emergence,* we have attempted to recapitulate the basic facts in the history of mathematical programming: “classical” mathematical programming (i.e., with only equality constraints), linear programming, and nonlinear programming. We have also added an Appendix, containing three further contributions, useful to illuminate further the growth process of optimization theory.

The main purpose of this book is to offer researchers direct access to the original sources and classics in nonlinear programming, together with an examination of the historical context regarding the emergence and development of this field of research. We hope that this collection will be useful and stimulating for all those who are interested in deepening their knowledge of the emergence and first developments of nonlinear programming and in general of mathematical optimization.

Pavia, Italy — Giorgio Giorgi
Roskilde, Denmark — Tinne Hoff Kjeldsen

http://www.springer.com/978-3-0348-0439-4

Traces and Emergence of Nonlinear Programming
Giorgi, G.; Kjeldsen, T.H. (Eds.)
2014, XVI, 434 p.,
ISBN: 978-3-0348-0439-4
A product of Birkhäuser Basel