# Chapter 8
# Not Only in Latin, but also in Dutch, Chinese, Italian and German

*But they most despise the low, grovelling populace when they bring out their mathematics of triangles, squares, circles, and other similar geometric figures, which then they lay the one on the other, and mix up as in a labyrinth: at last they astonish the idiots with several letters, arranged like an army in order of battle, and subdivided in different companies.*
*But at last the preacher brought it thus far, that he could demonstrate the whole Trinity to be represented by these first rudiments of grammar, as clearly and plainly as it was barely possible for a mathematician to draw a proof in the sand.*
Erasmus from Rotterdam, *In praise of folly*, 204, 239.

*Mark how one string, sweet husband to another, Strikes each in each by mutual ordering; Resembling sire and child and happy mother, Who, all in one, one pleasing note to sing.*
Shakespeare, *Sonnet*, VIII.

## 8.1 Aristoxenus with Numbers, or Simon Stevin and Zhu Zaiyu

The rules for music presented by Maurolico were predominant in texts written during the second half of the sixteenth century, and we have to consider them to represent orthodoxy in that period. Aristoxenus[1] had sustained, on the contrary, that the tone could be divided into equal parts. We can easily imagine that in playing his stringed instrument, a musician would not find any particular difficulty in placing his finger at the point where his ear, or the requirements of music, guided him, and thus

[1] Part I, Sect. 2.5.

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**Table 8.1** Stevin's equal temperament

| 1 | Selftoon [Selftone] | Eerste [First (prime)] |
|---|---|---|
| $\sqrt[12]{\frac{1}{2}}$ | Halftoon [Semitone] | Cleen tweede [Minor second] |
| $\sqrt[6]{\frac{1}{2}}$ | Toon [Whole tone] | Groote tweede [Major second] |
| $\sqrt[4]{\frac{1}{2}}$ | Onderhalftoon [One and a half tones] | Cleen derde [Minor third] |
| $\sqrt[3]{\frac{1}{2}}$ | Tweetoon [Ditone] | Groote derde [Major third] |
| $\sqrt[12]{\frac{1}{32}}$ | Twee en halftoon [Two and a half tones] | Vierde [Fourth] |
| $\sqrt[2]{\frac{1}{2}}$ | Drietoon [Tritone] | Qua groote vierde [Bad major fourth] of qua cleene vijfde [or bad minor fifth] |
| $\sqrt[12]{\frac{1}{128}}$ | Drie en halftoon [Three-tone-and-half] | Vijfde [Fifth] |
| $\sqrt[3]{\frac{1}{4}}$ | Viertoon [Four tones] | Cleen seste [Minor sixth] |
| $\sqrt[4]{\frac{1}{8}}$ | Vier en halftoon [Four and a half tones] | Groote seste [Major sixth] |
| $\sqrt[6]{\frac{1}{32}}$ | Vijftoon [Five tones] | Cleen sevende [Minor seventh] |
| $\sqrt[12]{\frac{1}{2048}}$ | Vijf en halftoon [Five and a half tones] | Groote sevende [Major seventh] |
| $\frac{1}{2}$ | Sestoon [Six tones] | Dobbeleerste, achtste [Double-first (octave)] |

also at the point corresponding to $\sqrt{72}$. Now the discussion about the division of the tone assumed the characteristics of controversies between continuous or discrete conceptions of the world, becoming its musical representation. We have followed these in their manifold variations, found in the cultures that were discussed in Part I.

As we have seen, natural philosophers confuted Aristoxenus using arguments based on Pythagorean concepts of numbers, and even invoking God, thus revealing a greater interest in the mathematical theory of music than in composing or playing actual notes. The position of Aristoxenus remained, at most, the unorthodox opinion of a heretical minority, though not totally silent. We have seen that it was also defended by Vincenzio Galilei, in contrast with the orthodox variant of Zarlino.[2]

But now that we arrive at the seventeenth century, it was Simon Stevin (1548–1620) who challenged the prevailing orthodoxy of the Pythagoreans most clearly and decidedly. And he did so on their own ground: that of numbers.

He refused the Greeks' way of dividing the octave 6:3, by means of the harmonic mean $4 = 2 \times \frac{6 \times 3}{6+3}$, into a fifth and a fourth. From this, he wrote, "oneindelicke ydelheden volghen" ["Endless vanities follow"].[3] He had decided that all the tones and all the semitones should be equal to one another, and consequently he calculated them as follows (Table 8.1).[4]

[2]Part I, Sect. 6.7.

[3]Stevin 1966, pp. 428–429.

[4]Stevin 1966, pp. 426–427 and 440–441. Tonietti 2003b, pp. 226–228.

**Table 8.2** Stevin's calculation of the roots for the equal temperament

| | |
|---|---|
| First | 10,000 |
| Minor second | 9,438 |
| Major second | 8,908 |
| Minor third | 8,409 |
| Major third | 7,936 |
| Good fourth | 7,491 |
| Bad fourth | 7,071 |
| Fifth | 6,674 |
| Minor sixth | 6,298 |
| Major sixth | 5,944 |
| Minor seventh | 5,611 |
| Major seventh | 5,296 |
| Double-first [octave] | 5,000 |
| Double minor second | 4,719 |
| Double major second | 4,454 |

Stevin (1966, pp. 442–449)

He expected the objection that such a sweet sound as the fifth could not derive from a number like $\sqrt[12]{\frac{1}{128}}$, which he described as "onuijtsprekelick, onredelick, ongeschickt" ["unspeakable, irrational, and inappropriate"],[5] but he rebutted that. He had demonstrated elsewhere that also numbers like these possessed an equally natural perfection, like all the others, and that what was irrational, rather, was the lack of understanding that they had met with. In another table (Table 8.2), he calculated approximations for the above square roots, and then compared them with the "ghedwaelde" ["erroneous"][6] divisions of Pythagoras, Ptolemy, Boethius and Zarlino.

In his *Arithmétique [Arithmetic]* (1585), Stevin had sustained that numbers like $\sqrt{8}$ were not "absurds, irrationels, irreguliers, inexplicables, sourds" ["absurd, irrational, irregular, inexplicable, surd"][7]. Because incommensurability could not determine the absurdity of incommensurable terms which were numerical, if it did not cause the same with geometrical terms, like lines and surfaces. He repeated the concept in the third of his *Thèses mathématiques* [mathematical theses]: "Que racine quelconque est nombre" ["Any root is a number"]. And again in the fourth: "Qu'il n'y a aucuns nombres absurds, irrationels, irreguliers, inexplicables, ou sourds." ["No numbers are absurd, irrational, irregular, inexplicable or surd".][8]

Many of the characters that we have met in our story also knew of some procedures to extract the square root or cubic root of a number. But those who had remained prisoners of the Pythagorean tradition could not consider the result as

[5]Stevin 1966, p. 440.

[6]Stevin 1966, pp. 446–447.

[7]Stevin 1958, pp. 532–533.

[8]Stevin 1958, p. 738.

an appropriate solution to any problem. Now, however, Stevin used roots like any other number, and consistently did not separate numbers from geometry and did not consider them to be discontinuous magnitudes. He had been able to conceive of the equable temperament in 12 equal semitones because he no longer believed in the Pythagorean principles of numbers. $\sqrt[12]{\frac{1}{128}}$ was not to be considered different in quality from $\frac{2}{3}$, because they were both points in the geometrical *continuum* of numbers.

However, this Dutchman did not succeed in gaining consensus for his numerical equable temperament. His incomplete manuscript *Vande Spiegheling der Singconst [On the theory of the art of singing]* was to be discovered only during the nineteenth century in the library of the musician Constantijn Huygens, secretary to the Prince of Orange, and the father of Christiaan Huygens, who we shall meet later on in Sect. 9.3.

Stevin expounded his theory by giving first the definitions, and then the postulates and the theorems, as if he took his inspiration from Euclid. He postulated that the depth of a note depended on the part [length] of the string considered, because this did not require any demonstration, but derived from experience.[9] His second postulate stated that "Heele toonen al even groot te wesen, dat sgelijcx oock halve toonen al even groot sijn." ["All whole tones are equal and likewise all semitones are equal".][10] This he explained as it were the choice to consider equal the intervals $ut[do] - re, re - mi, fa - sol, sol - la$. The explanation was followed by the values of the roots (Table 8.1) called "*Vertooch*" ["Theorem"?] with a "*Bewijs*" ["Proof"], which however is missing.

Our Dutchman made his division of the octave into 12 equal parts depend on the term "*everedenheijt*" ["Equirationality"]. He sustained that it was now easy for him to consider all the semitones as having the same ratio because the word existed in the "*duytsche*" ["Dutch"] language. On the contrary, the Greeks could not divide it in the same way (and 6:4 [Fifth] did not have the same ratio as 4:3 [Fourth]) because they would not have had any way of writing it in Greek. "The cause of these errors is that the Greek language could not interpret this term together with all other mathematical terms as properly as the Dutch language does." Stevin went on here to discuss the idea that Greek had been "useful in order to bring to light various discoveries of the Greeks, Latin (because it happened to become the common language of the world) serves to be understood in all countries, and also to study all sciences about all subjects, which have been described in it by various kinds of people, French, Italian, Spanish, Polish, etc. It serves to carry on trade, everybody according to his situation. But DUTCH[11] serves to teach the liberal arts, to fathom the hidden secrets of nature, and to prove that miracle is not a miracle."[12]

[9]Stevin 1966, p. 425.

[10]Stevin 1966, pp. 426–427.

[11]Emphasis in the text.

[12]Stevin 1966, pp. 426–429.

Simon Stevin took this motto, that a miracle was not really a miracle, as his own, placing it on the title page of his book dedicated to static mechanics *De Beghinselen der Weeghconst [Elements for the art of weighing]* (1586). Here, among other things, he gave the rules for the inclined plane.[13] Beyond the love for his native tongue, Stevin's considerations about Greek and Latin are particularly interesting. They reveal that Latin was starting, at least in certain contexts, to lose its role of lingua franca for communication between natural philosophers; it was gradually to be eclipsed by the various national languages, as the single States strengthened their claims for prerogatives. The progressive weakening of Latin was to stimulate the search for another common language, at least for mathematical sciences. We shall see that the problem was to lead to the invention of a special symbolism for these sciences.

Just as the Dutchman made use of geometry to evaluate the relationships between figures that were not easy to recognise purely by sight, so his hearing would not be sufficient to judge relationships between the depth of sounds. The ancients knew, as was clear by listening, that the semitone was greater than the Pythagorean division of the tone, 256:243. And yet they decided to take it as correct, because they would have considered that "… the defect was in our singing (as if one should say: the sun may lie, but the clock cannot)". The Greeks had also rejected the pleasures of minor and major thirds, because they had taken the wrong numbers. Nor were the attempts of Zarlino to fare any better. And he repeated that all these errors derived from his persuasion: it was not possible in these languages to understand the nature of "… equirationality … tool …", which on the contrary existed in Dutch, "… the most perfect language of languages." It is important to note that in his text, Stevin wrote $\frac{3}{2}$ or $\frac{9}{8}$, in other words, *fractions*, where other theoreticians had used ratios, 3:2 or 9:8.[14]

He now took the keyboard of an organ or a harpsichord, and he explained how to tune instruments by means of the fifths, fourths, tones and semitones that he had calculated (Table 8.1). He wrote for musicians, because he knew perfectly well that they were the ones who would receive most benefit from the equable temperament.[15]

Therefore the notes were to be arranged in a scale, calculated in accordance with the proportional means of the relative numbers, which, as we have seen above, were no longer to be called irrational. To "… obtain the true perfect sounds of natural singing", he then divided the classic monochord in accordance with his own geometrical progression based on $\sqrt[12]{\frac{1}{2}}$. Even he invoked Euclid, and drew a line, but the distances on this line were calculated by extracting the necessary numerical roots, accepting only the octave as a ratio between whole numbers. And assuming this to be $\frac{10{,}000}{5{,}000}$, he obtained the following table, with suitable approximations (Table 8.2).

[13] Dijksterhuis 1971, pp. 432–440. Minnaert 1970, pp. 48–49.

[14] Stevin 1966, pp. 429–433.

[15] Stevin 1966, pp. 436–437.

In any case, the ear would not be sufficient to tune all the intervals of the fifth correctly on organs and harpsichords.[16] But an organist made the objection that [on his instrument which had not been well tempered] three major thirds did not make an octave.[17] And yet it would have been the musicians and composers that transformed his heretical position into the common modern-day practice of the equable temperament, now used everywhere. They only had to start to prefer moving freely with their modulations between one tonality and another, in a desire to play them on their fixed-tuning instruments. This happened above all with Johann Sebastian Bach (1685–1750) and Jean Philippe Rameau (1683–1764), while the composers of the sixteenth century continued to write polyphonic vocal compositions with several voices, which gave rather a sense of eternal immobility, like those of a certain Roland de Lassus (1530–1594).

Stevin had been one of the first to adopt also fractions and decimal figures in calculations. His many interests led him above all to solve the most varied kinds of practical problems: conical gears for the transmission of movement, hydraulics for floodgates, canals, drainage, windmills, navigation, astronomy and wartime fortifications. These activities had been requested to him by the historical context. The Orange dynasty, for whom he worked, was seeking its independence from the Spaniards by fair means or foul, and was expanding overseas.

His ideas of the mathematical sciences were the most appropriate for these purposes: fewer theorems and more calculations. Thus with him, the long journey undertaken by numbers, starting from the Orient, with the Chinese, the Indians and the Arabs, and passing through the abacus schools and Renaissance workshops, arrived in seventeenth century Europe. When the orthodox European tradition of the *quadrivium* had placed numbers among the discontinuous magnitudes, it was this Dutchman who now moved them among the continuous ones. "Que nombre n'est poinct quantité discontinue." ["A number is not a discontinuous quantity at all"]. "... à une continue grandeur, correspond le continue nombre qu'on lui attribue ... le nombre est quelque chose telle en grandeur, comme l'humidité en l'eau, car comme ceste ci s'estende par tout & en chasque partie de l'eau; ..." ["... a continuous magnitude corresponds to the continuous number that is attributed to it ... number in magnitude is something like humidity in water, because it extends everywhere, in every part of the water; ..."]. Instead of being the general generating principle, the number 1 was put together with the others.[18]

[16]Stevin 1966, pp. 456–459.

[17]Stevin 1966, p. 420. Even in the twentieth century, the editor of the manuscript, Adriaan D.Fokker, displayed a far greater tendency to censure towards Stevin's avoidance of Pythagorean whole numbers, judging that he had acted "... *haughtily and aggressively* ..."; Stevin 1966, p. 419. On the contrary, the Dutchman, Rudolf Rasch has recently provided us with more interesting details about the original pages of his earlier fellow-countryman, duly pointing out the influence that he had, more often than not in a negative sense, on Beeckman, Mersenne, Descartes and the two Huygens; Rasch 2008. See below, Sects. 9.1–9.3. Cf. Bos 2004.

[18]Stevin 1958, pp. 501–503 and 738.

Following these ideas, Stevin developed the method of exhaustion used by Archimedes. He substituted the latter's proofs with the argument that if the difference between two magnitudes could be made smaller than any quantity assigned, then the two magnitudes would be equal. This was the case for the difference between the polygon with an increasing number of sides and the circle. For this reason, he should be included in the history of infinitesimal calculus, which would spoken in the same way of a 'limit'.

Our Dutchman used procedures of the same kind to calculate curves on a sphere that intersected the meridians at constant angles. This problem had become important for navigation, where Stevin determined a ship's route along a maximum circle or along a line of the above kind. For sailors, over distances that are not too great, it is much more convenient to follow this second method, to which Willebrord Snellius (1580–1626), in translating Stevin's writings into Latin, gave the Greek name of 'loxodrome' [an oblique course]. Using the compass, it is easier to keep the prow of the ship at a constant angle with respect to North (the meridian), tracing the route on Mercator's nautical maps. This famous mapmaker, whose real name was Gerhard Kremer (1512–1594), was another Dutchman who, thanks to the relative projection of the sphere on to the cylinder tangent to the equator, had drawn nautical maps where meridians and parallels intersect at right angles, and the loxodromes are straight.[19]

In a word, Stevin proposed putting an end to that millennial separation between numbers and geometry to which the Pythagoreans, Plato and Euclid had accustomed us. Furthermore, his numbers became effective to solve the problems of the world, because they had become a part of it, as humidity in water. But even in this field, he did not immediately find many followers. Borrowing from others, our Dutchman also tried to introduce a 'little' novelty into symbolic notation, writing algebraic equations. Instead of the word 'thing' for the unknown, he indicated the power to which a number is raised **1**, **2**, **3** in a small circle for the square, or the cube, where nowadays we would put $x, x^2, x^3$. He too, as everybody did, anyway continued to feel the need to anchor these numbers and reasonings to the solid propositions of Euclid; after all, the Arabs had done the same. And he interpreted an equation, rewriting it in the form of a ratio, even if elsewhere he regularly used fractions.[20]

We can be almost certain that Stevin had read some Arabic texts, because he quoted "Mahomet", together with Pacioli, Cardano, Tartaglia, Michael Stifel (1487–1567) and others.[21] In astronomy, he wrote a book to sustain the model of Copernicus. From this Dutchman, we can arrive directly, passing through the intermediation of only one real person, on the one hand at Christiaan Huygens, and on the other at René Descartes.[22]

[19] Struik 1958, pp. 7–9.

[20] Struik 1958, pp. 463, 471 and 474.

[21] Struik 1958, p. 474.

[22] Struik 1958, p. 8.

Stevin served as a general commissioner in the army of the newly-formed Holland, dealing with various matters: from training for soldiers to siege tactics, from fortifications to camps. He actually took part in some battles.[23]

Had it not been for all these warlike activities, so far removed from the world of Confucius, our Dutchman would have appeared to be at the same time the European natural scholar farthest, with his novelties, from his orthodox colleagues, and closest to the style that was common in China. Among the characteristics of his, which were seen before in the Country-at-the-Centre,[24] we may indicate the interest above all in practical problems, and the idea of fractional and decimal numbers to be used for measuring any magnitude, far from the Pythagorean prohibitions of irrationals, and thus seen as a *continuum*. Last but not least, was a certain indifference to justifications by means of philosophies of transcendence. This led him not to take sides with one religion or another, in a period of schisms and heresies when this was almost compulsory. Rather, he accepted the authority of political power, which was to be obeyed in any case.[25] Living in the land of the windmills, he had also invented a cart propelled by sail power over land, like boat over water, another device that was already present in China.[26]

For all these reasons, we may entertain the hypothesis that, in the crowded, industrious ports of his country, this Dutchman might have profited from contact with the Chinese culture, and might have taken some inspiration from this. However, there is no confirmation or proof of this. Thus, putting aside the model of the single origin for scientific and technical inventions, we will continue to follow the idea of their independence. And yet Joseph Needham indicated another case by means of which he tried to demonstrate the possibility of hypothesising the transmission of a not negligible contribution from China to Europe. In those same years sure, at the end of the sixteenth century, also Zhu Zaiyu, a prince of the Ming dynasty, proposed, and calculated numerically a division of the musical octave into 12 equal semitones.[27]

The *lülü* tuning system, which we have already seen above, as described in the text of Cheng Dawei,[28] was not the only one that existed in the late Ming empire of the sixteenth century. In 1584, Zhu Zaiyu (1536–1611) had already calculated and proposed a new system.

Xinzhi lüzhun
[Exact standard for *lü* based on a new system].[29]

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[23]Minnaert 1970, p. 50.

[24]Part I Chap. 3.

[25]Minnaert 1970, pp. 47 and 50–51.

[26]Needham, Wang & Robinson 1962, p. 228.

[27]Needham & Wang & Robinson 1962, pp. 220–228. Tonietti 2003b, pp. 234–237.

[28]Part I Sect. 3.2.

[29]Zhu 1584, p. 20. Needham, Wang & Robinson 1962, p. 222. Chinese characters in Appendix D.

**Table 8.3** Zhu's calculation of equal temperament

| Lülü | Length (chi) | Diameter (cun) |
|---|---|---|
| 1. Huáng Zhong | 2 | 0.500000 |
| 2. Dà Lǔ | 1.88774862 | 0.4857659 |
| 3. Dà Cù | 1.78179743 | 0.4719371 |
| 4. Jiá Zhong | 1.68179283 | 0.4585020 |
| 5. Gu Xǐ | 1.58740105 | 0.4454493 |
| 6. Zhòng Lǚ | 1.49830707 | 0.4327682 |
| 7. Ruí Bin | 1.41421356 | 0.4204482 |
| 8. Lín Zhong | 1.33483985 | 0.4084788 |
| 9. Yí Zé | 1.25992104 | 0.3968502 |
| 10. Nán Lǚ | 1.18920711 | 0.3855527 |
| 11. Wú Shè | 1.12246204 | 0.3745767 |
| 12. Yìng Zhong | 1.05946309 | 0.3639132 |
| 13. Huáng Zhong | 1 | 0.3535533 |

This famous prince constructed an instrument with 12 strings which was to serve as a standard to tune the others. He fixed studs in it, to indicate where to place the fingers in order to obtain the desired note (Fig. 8.1).

He, too, used numbers to calculate the relative length of the strings, but he did this: "… for seeking harmony in the notes, and did not make the notes submit to numbers."[30]

In studying the texts of the "decreasing, increasing" tradition, he noticed differences of tuning between the *lülü* fixed by this method and the stringed instruments constructed by artisans and used by musicians. Thus he proposed a new method.

> Chuangli xinfa. Zhiyi chiwei shiyi milü chuzhifan shier biansuoqiu lülü zhenshu.
> [I give origin to a new rule. As the dividend, let one *chi* [about 33 cm] be fixed; to [obtain] the right ratio, divide it [extract the square root[31]]. In brief, find the right numbers for the twelve *lülü* from one place to another].[32]

In another book of 1596, Zhu explicitly gave his own procedure for calculating the length of the strings. These also included the octave, because he also considered the length twice that of the basic *chi*. Therefore the octave was divided into 12 equal semitones, using the ratio established by $\sqrt[12]{2}$ (Table 8.3).[33]

Thus Zhu proposed what is nowadays called the equable temperament in Europe and all over the world; in order to achieve it, he calculated the length of strings and pipes, using the appropriate square or cubic roots of two, or rather, the appropriate geometrical means.

[30]Needham, Wang & Robinson 1962, pp. 220–221.

[31]In China, where no Pythagorean prohibition of irrational numbers existed, the extraction of a root was seen as a kind of division; Tonietti 2006a; see above, Part I, Chap. 3.

[32]Zhu 1584, p. 8. Needham, Wang & Robinson 1962, p. 223. Chinese characters in Appendix D.

[33]Zhu 1596.

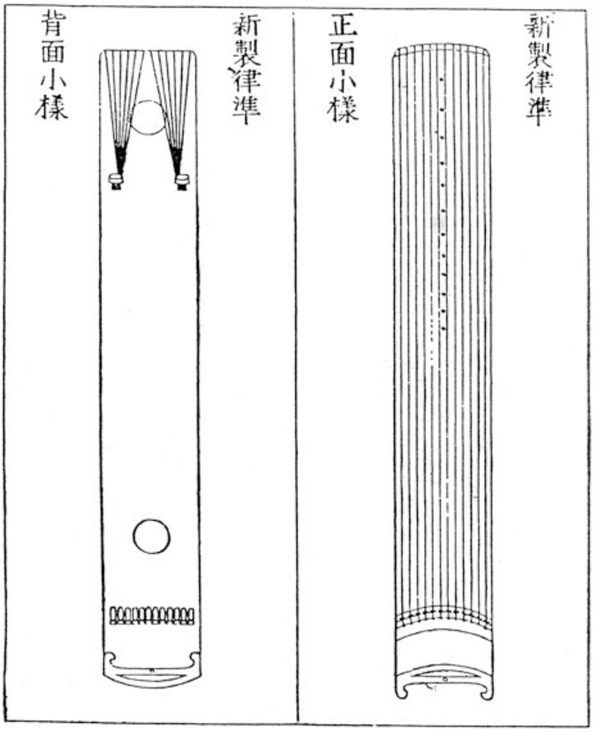


**Fig. 8.1** The instrument on which Zhu Zaiyu tempered the twelve lülü in an equable manner Zhu Zaiyu ("Picture of the stringed instrument", 1584)

Our prince took a string and a Huang Zhong pipe 2 *chi* long, that is to say, tuned to the low octave of the basic note, and calculated the length of the Ying Zhong to be 1.05946 *chi*. This consequently represented a tempered semitone, seeing that it was the interval between the 12th note and the octave, or (which is the same thing, since all the semitones are now equal) between the first and the second note. He obtained the lengths of the other strings in a geometrical succession, by dividing 2 by 1.05946 and repeating the same operation serially ten more times. After the 12th operation, the length of 1 *chi* of the initial Huang Zhong is obtained, because $2 : (\sqrt[12]{2})^{12} = 1 : 1$.

Zhu equabily tempered not only strings, but also took up the ancient tradition of varying the diameter of pipes.[34] He must have been aware of the fact that the sound of pipes depends not only on their length but also on their diameter. Thus he also

[34] Part I, Sect. 3.2.

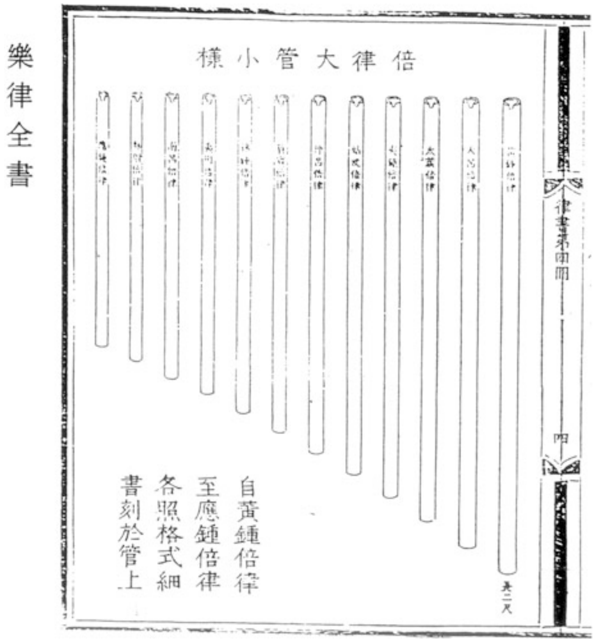


**Fig. 8.2** The pipes tempered equably by Zhu Zaiyu (Zhu Zaiyu, "picture of the 12 pipes", 1584)

tempered the diameters, dividing them progressively, this time by $\sqrt[24]{2}$. Also in this respect, he deviated from Cheng Dawei.[35] (Fig. 8.2)

The Ming noble thus used numbers obtained by the extraction of roots or from proportional means (which would have been called irrational in Europe). But this was not the only novelty with respect to the other Chinese scholars, because it critically depended on considering also the double measure and the half measure among the lengths suitable for music. That is to say, among the others, Zhu also used the notes generated by these latter pipes (in Europe they would have been called the octaves). Few scholars before him had introduced, together with the Huang Zhong of 9 *cun* a Shao Gong [lesser palace] of $4\frac{1}{2}$ *cun*.[36] The tradition of the "decreasing, increasing" system had remained the orthodox one, which did not contemplate it because multiples of $\frac{4}{3}$ or of $\frac{2}{3}$ cannot be included among the multiples of 2. This can be understood from the report of Cheng Dawei, who was unaware of the octave. It is interesting to note that Cheng did not justify the inversion of the rule (two successive upward generations, multiplying twice by $\frac{4}{3}$, where we would expect a decrease of $\frac{2}{3}$ the second time), writing that he did so in order to remain inside the

[35]Robinson 1980, p. 118. But we may note that he should have divided the diameters in the same proportion $\sqrt[12]{2}$ as the length, in order to obtain a good tempering also of the pipes; Tonietti 2003b, p. 240.

[36]Needham, Wang & Robinson 1962, p. 214.

proportion of the octave. On the contrary, he invoked the increase of the *yin* at the summer solstice.[37]

In the Country-at-the-Centre, those who sought mathematical models for music could not have had the idea of the equable temperament if they had not also considered the ratio 1:2. Orthodox tradition did not provide for this, and so it was not possible to divide into equal parts what did not exist. This seems to have been the main reason why here the equable temperament remained unknown up to Zhu Zaiyu. In the Chinese culture, the roots of any number were not subject to those prohibitions seen in Europe. The procedures for their calculation, though approximate, gave results that could be used. This attitude towards numbers was supported by the idea that they, too, like sounds, which were vibrations of the *qi*, and like the whole world, were part of an all-inclusive geometrical *continuum*, where they found their justification. Thus, once the octave was accepted, it would become possible to divide it into equal parts, as Zhu did, without any further problems.

In Europe, the absence, among the orthodox mathematical sciences, of the equable temperament with numbers, stemmed from other reasons. Here, the octave ratio of 1:2 had always been considered among the fundamental ones. But up to this time, we have always been confronted with the rigid separation, made in the Pythagorean-Platonic-Euclidean tradition, between commensurable relationships, which could be represented by numbers, and incommensurable ones, for which this kind of representation was not allowed (nowadays we would speak of rational and irrational numbers). It had thus been prevented to use the instrument by means of which the octave could be divided into equal parts. The division of the Pythagorean tone into equal parts gives a slightly higher value than that of the equable temperament. But, even though it was not strictly speaking equivalent in the precise numbers, the division of the tone into equal parts would have been a good step towards the equable temperament. If the proportional mean could be used, why not calculate it in order to divide the octave? And then to divide again the interval between this note (fa♯ in Europe, Rui Bin in China) and the starting note? This is exactly what Zhu Zaiyu did.[38] The idea of the equable temperament could then have easily occurred to anyone who knew how to extract roots. Anyone who reflected that also the square root of 2 could be considered a number like the others (and therefore usable as the solution to a problem) would have been able to divide the octave into equal parts. And this was what Simon Stevin did.[39]

It was not only the theory that made the two cultural contexts different. Typical instruments of the Chinese musical tradition, like *zhong* [bells] and *qing* [chime-stones] appeared to be more expensive to construct, and more difficult to tune than any string instrument or wind instrument. Thus the desire to play a greater number of melodies on them, with a greater number of modulations between one modality and another, was bound to stimulate the search for an equable tuning. This need found

[37]Tonietti 2003b, pp. 231–232; see above, Part I, Sect. 3.2.

[38]Robinson 1980, p. 115; Chen 1999.

[39]See above in this section. Tonietti 2003b.

partial expression in that unique invention of the Chinese artisans which consisted of bells with an asymmetrical shape. When struck at different fixed points, they are able to produce two different notes.[40]

Even European instruments with a fixed tuning, like organs, not to speak of lutes, seem to be less difficult to tune than sets of bells. Anyway, a comparable stimulus towards the equable temperament was also present in Europe. But here it seemed to show its effects above all among musicians. We have found the greatest openness towards the equable temperament in Aristoxenus and Vincenzio Galilei. And other musicians, too, were only to become convinced when compositions abandoned the static nature of the Gregorian *cantus firmus* and the polyphonic constructions of the Flemish for the variety of melodies and tonal harmonies. When the fixed-tuning harpsichord and the piano progressively took over the stage, becoming the reference instruments for everybody, like today, the equable temperament became an indispensable means to enrich modulations. However, here it was not easy for these requirements to break out of the community of musicians, to make themselves heard by the natural philosophers of the scientific tradition that had long prevailed in Europe. Their ears and their brains had been better prepared to hear the abstract music of the spheres than real compositions.

In China, on the other hand, sciences seem to have evolved in a constant relationship with the numerous practical problems of this culture. Thus, also in our case, it may have been relatively easier for the needs of musicians to influence the considerations of theoreticians. This is exactly what we see Zhu Zaiyu declare, in order to justify his new science of the *lülü*. It would have been practically impossible to find a similar attitude in Europe, because it would have entailed a loss of prestige for the discipline.

It does not seem to be productive for us to level our history to a question of priority. The close proximity in time between Vincenzio Galilei, Simon Stevin and Zhu Zaiyu certainly appears to be striking. The book of the first of the three, *Dialogo della musica antica et moderna* [Dialogue between ancient and modern music], which took up the subject of the equable temperament, came out in 1581. The manuscript of the second of them, which remained unpublished, was probably written a few years after 1585. While Zhu's first text, *Lüxue xinshuo [New summary of the science of lü]*, appeared in 1584.[41] But today, we do not possess any documents even to formulate a hypothesis that one of them had borrowed from the others. It was clearly much less likely that Zhu had been inspired by the Europeans. The Jesuits would have transferred to China the orthodoxy of Maurolico, if anything, rather than the heresy of Stevin, if printing had existed. In any case, both contexts contained the elements necessary to realise the equable temperament with numbers. Consequently, it is much more likely that each of them had invented it by himself.

[40]Chen 1994.

[41]Tonietti 2003b.

## 8.2 Reaping What Has Been Sown. Galileo Galilei, the Jesuits and the Chinese

> Che peccato che l'artiglierie non fussero al tempo
> di Aristotele! Avrebbe ben egli con esse espugnata l'ignoranza,
> e parlato senza punto titubare delle cose del mondo.
> Galileo Galilei, *Dialogo sui massimi sistemi*, "Giornata seconda".
> [What a pity that the artillery did not exist at the time
> of Aristotle! He would have swept away ignorance with it,
> and spoken without any hesitation of the things of the world.
> *Dialogue about overall systems*, "Second day".]

Galileo Galilei (1564–1642), son of Vincenzio, must have learnt from him a great deal of music, and also wrote on the subject, performing practical experiments sometimes. One page dedicated to music is found at the end of the "First day" in the *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* [Discourses and mathematical proofs regarding two new sciences] (1638). Here, Galileo Galilei took up the orthodox tradition of the Greeks, with ratios for musical intervals formed by whole numbers, justifying them by means of the theory of strokes created by sounds in the ears; we have already seen this in Francesco Maurolico.[42]

The famous scholar from Pisa now affirmed, probably taking the idea from his father, that the tension of the musical string did not make the pitch of the note rise linearly (as the ancients believed), just as the depth increases with the length, but in accordance with a lower ratio. Thus, if we want to generate, for example, the higher octave, it is not sufficient to double the tension (with a double weight), length and other conditions remaining equal, but it needs to be multiplied by four. "… vorremmo farla montare all'ottava col tirarla di più, non basta tirarla il doppio più, ma ci bisogna il quadruplo, sì che se prima era tirata dal peso d'una libbra, converrà attaccarvene quattro per inacutirla all'ottava; …". ["… if we desire to raise it to the octave by pulling it more forcefully, it is not enough to pull it twice as hard, but it is takes four times the tension, with the result that if it was previously pulled by a weight of one pound, it will be necessary to attach four of them in order to raise it to the octave; …".]

For his father Vincenzio, this phenomenon had disproved the Pythagorean-Platonic tradition, but not for his son, Galileo. "… non è la ragion prossima ed immediata delle forme de gl'intervalli musici la lunghezza delle corde, non la tensione, non la grossezza, ma sì bene la proporzione de i numeri delle vibrazioni e percosse dell'onde dell'aria che vanno a ferire il timpano del nostro orecchio, il quale esso ancora sotto le medesime misure di tempi viene fatto tremare." ["It is not the direct, immediate reason for the forms of the musical intervals, either the length of the strings, or the tension, or the thickness, but rather the ratio of the numbers of the vibrations and strokes of the air waves that strike the tympanum of our ear, which is made to vibrate at the same rate"]. When the strokes of two notes struck

[42]Part I, Sect. 6.5.

the ear together, creating a coincidence most times, there would be consonances, and in other cases dissonances. "… e crudissime saranno le dissonanze quando i tempi delle vibrazioni fussero incommensurabili; per una delle quali sarà quella quando di due corde unisone se ne suoni una con tal parte dell'altra quale è il lato del quadrato del suo diametro: dissonanza simile al tritono o semidiapente." ["and the dissonances will be truly harsh when the times of the vibrations should be incommensurable; an example of one of these will be when one of two unisonous strings is played with that part of the other one which is to the first as the side of the square to its diagonal: a dissonance similar to the tritone, or semidiapente"]. He preferred the 3:2 ratio for the fifth, "… fa una titillazione ed un solletico tale sopra la cartilagine del timpano, che temperando la dolcezza con uno spruzzo d'acrimonia, par che insieme soavemente baci e morda." ["… it creates a tickle and a titillation over the cartilage of the tympanum, that tempers sweetness with a dash of acrimony, and seems to offer sweet kisses and to bite at the same time."] Incommensurable proportions would violate the harmony of the cosmos, and would lead to chaos. "… ordine disordinato di sregolata intrecciatura, e l'udito con noia riceve gli appulsi intemperati de i tremori dell'aria …" ["a disorderly order of irregular fabrication, and the hearing receives with irritation the untempered impulses of the air trembling"].[43]

Although they are short, these pages clearly reveal Galileo's attitude. He described experiments with water, in order to make the vibrations of glasses and resonating strings visible, arriving at an "… ondeggiamento … per l'aria …" ["… waving … in the air …"]. In giving the above rule governing the tension of strings, he also criticised current explanations as being insufficient, but he did not subvert the general theory. Using geometrical designs as well, he concluded with the orthodox theory of consonances, which prohibited incommensurable ratios in music.

Our natural philosopher, who we may note started to write in Italian, left other pages about music in his *Il saggiatore* [The tester] (1623). He affirmed that the sound of the pipes in an organ depended on their size (and not the material that they were made of), but he did not offer any numbers. However, that sound was made of "… onde, dalla frequenza delle quali nasce l'acutezza del suono e la gravità dalla rarità." ["… waves, the frequency of which produces the acuteness of a sound, while its depth stems from rarity"]. However, this natural philosopher would have like to eliminate the sensations felt in the nose, on the tongue and in the ears. "… restino bene le figure, i numeri e i moti, ma non già gli odori, né i sapori, né i suoni, li quali fuor dell'animal vivente non credo che sieno altro che nomi, …". ["Let the figures, the numbers and the movements remain, but not the smells or the tastes or the sounds, which in my opinion are nothing other than names, outside the living animal."] Thus he judged music by means of numbers, rather than with his ears. As regards sight, he preferred to keep his silence: "… senso sopra tutti gli altri eminentissimo, abbia relazione la luce, ma con quella proporzione d'eccellenza qual è tra 'l finito e l'infinito, tra 'l temporaneo e l'istantaneo, tra 'l quanto e l'indivisibile,

[43] Galileo Galilei 1996 II, pp. 671–680.

tra la luce e le tenebre.” [“... a most excellent sense above all the others, in association with light, but with that proportion of excellence which exists between the finite and the infinite, between the temporary and the instantaneous, between quantity and the indivisible, between light and darkness.”] The degree to which the problem of music was felt in this century, involving Galileo as well, was confirmed in 1619 by Federico Cesi (1585–1630), the founder of the renowned Accademia dei Lincei. “Quella [disputa] delle proportioni musicali et della rispondenza delle corde nelli istrumenti, ...., passammo già copiosamente col *S.^r* Galilei nostro in Tivoli; et veramente sarebbe bene, fusse pienamente distesa in carta ...” [“In detail with our *S.^r* Galilei at Tivoli, we discussed that [dispute] about musical ratios and the correspondence of strings in instruments; and indeed it would be better to put it down in black and white thoroughly ...”].[44] In spite of his father’s defence, there is no trace of Aristoxenus, or of the possibility of dividing the tone into equal parts in Galileo Galilei (judging by the index of name in the national edition of *Le opere*).

Following this line demonstrating the continuing influence of music on the evolution of the sciences in the seventeenth century, at the time of their so-called revolution, the testimony of Vincenzo Viviani (1622–1703), Galileo’s last pupil, is extremely interesting. “Questa del pendolo si è una delle più antiche invenzioni e scoperte in natura del Galileo, e fu circa l’anno 1580, quando era studente a Pisa, nel trovarsi egli un giorno in quel Duomo, dove si abbatté di vedere, lasciata in moto, una lampada pendente da una lunghissima corda. E, come quello che da giovanetto s’era anche esercitato nella Musica, sotto la disciplina di quel gran Vincenzio suo Padre, che si dottamente scrisse poi in *Dialogo della Musica antica e moderna*; perciochè aveva impressa nell’anima l’egualità de’ tempi, co’ quali essa si regola, riflettendo a quel moto, gli fu facile il giudicarlo in mente sua equitemporaneo, sì nelle andate lunghe e larghe al principio del moto, come nelle strette sul fine verso la quiete. In casa poi se ne chiarì in più modi replicate esperienze esattissime, trovando, coll’aiuto de’ suoi compagni, che in un determinato numero di vibrazioni d’un certo pendolo, lasciato andar sempre da una distanza medesima del perpendicolo, quante ne faceva un altro pendolo delle larghe, altrettante in ciascuno ne faceva delle strette e delle strettissime.” [“This finding of the pendulum is one of Galileo’s earliest inventions and discoveries in nature; it was in about 1580, when he was a student at Pisa, that one day, he was in the Cathedral, where he happened to see a lamp swinging freely at the end of a very long rope. As he had also studied Music as a boy, under the discipline of his great father, Vincenzio, who wrote so learnedly in his *Dialogue on ancient and modern Music*; thus he had impressed in his soul the equality of times by which it is regulated, and reflecting on that movement, it was easy for him to judge in his mind that the same time was taken up, both in the long, broad round going at the beginning of its movement, and in the short ones at the end, when it moved towards stillness. Later, at home, he clarified the question in various ways: repeating extremely precise experiments, and finding, with the help of his companions, that in a determined number of vibrations of a certain pendulum,

[44] Galileo Galilei 1968, VI, p. 350 and XII, p. 436.

allowed to start swinging always at the same distance from the perpendicular, the number of broad vibrations of another pendulum corresponded to the number of short and very short movements for each one."][45]

This page allows us to fill in the background, and to understand why, immediately before music in his *Discorsi* …, Galileo spoke of the pendulum, stating: "Io ho ben mille volte posto cura alle vibrazioni, in particolare, delle lampade pendenti in alcune chiese da lunghissime corde, inavvertentemente state mosse da alcuno." ["Many a time have I focused my attention, in particular, on the vibrations of lamps hanging at the end of long ropes in churches, when they were unintentionally moved by someone."] He used the same term "vibrations" both for the oscillations of the pendulum, and for the vibration of musical strings and the air. In the end, in order to explain consonances better by means of the coincidence that he imagined between strokes in the ear, he made the pendulums oscillate together. In this way, their lengths, which simulated the octave and the fifth, would make their movements concordant. "Ma quando le vibrazioni di due o più fili siano o incommensurabili, sì che mai non ritornino a terminar concordemente determinati numeri di vibrazioni, o se pur, non essendo incommensurabili, vi ritornano dopo lungo tempo e dopo gran numero di vibrazioni, allora la vista si confonde ...." ["But when the vibrations of two or more strings are either incommensurable, such that they never return to terminate together during a determined numbers of vibrations, or even if they are not incommensurable, they return there together after a long time, and a great number of vibrations, then sight is confounded"] in the above-mentioned chaotic disorder.[46]

The tempo of music must have seemed to offer a valid alternative to the beats of his own pulse, for our renowned natural philosopher in his aim to measure the movement of bodies. Subsequently, in one of the most famous pages, the one dealing with the isochronism of the pendulum, music played a far from negligible role in the advance towards the new laws of physics. Other historians[47] have attributed the correct importance to music in Galileo. But they generally used it only as the source of his experimental method. Whereas with his refusal of Aristoxenus, who was undoubtedly known to him from his father Vincenzio, and of incommensurable ratios in music, our man from Pisa betraied that he had not listened carefully enough, or experimented sufficiently. He had not believed in, or repeated, what his father had done. Thereby, apart from the new relationship between the pitch and the tension of the string, he remained within the track of traditional Pythagoreans, who had selected those whole numbers for religious, a priori reasons, and not using their ears. Judging by Galileo's letter to Christine of Lorraine of 1615, Boethius was still the

[45]Viviani 1891, 303. Cf. Galileo Galilei 1968, XIX, p. 603. Settle 1996, pp. 30–31. Also in Huygens 1890, III, p. 473.

[46]Galileo Galilei 1996, II, pp. 670 and 680. The sight is confounded, but the hearing is not, Galileo Galilei seemed to say.

[47]Palisca 1961 or Gozza 1989, or Drake 1992, or Settle 1996.

authority in the field of music.[48] If Galileo Galilei had listened, like his father (and like all of us), without any prejudices, to a fifth tuned in the equable temperament of Aristoxenus, wouldn't he have been able to conclude that this, too, offers "kisses and bites"?

Our renowned scholar must undoubtedly have used his ears, too, to listen to the notes of music, and he must have found the exercises that he did in his father's house extremely useful. And yet, if he wanted to attain the ratios of the Pythagorean-Platonic-Euclidean schools that he followed, he too would be led to make himself independent of sensations. Otherwise, he would have heard not the heavenly sounds, but only the earthly ones, like his father's lute, mass in Pisa Cathedral, running water, the blow of wind, the waves of the sea, the rubbing of bodies, the cries of animals, thunder, cannon shots. How could he cancel all these 'noises' that covered up the melodies that floated through his well-ordered cosmos? Thus he abstracted from the environment where these phenomena took place, re-inserting them into a faraway, rarefied, ideal world. He also imagined a higher world of ideas, in which movements were simplified, and emerged from the confused fog produced by our immediate sensibility, to reveal themselves in the eternal laws.

Thinking that the movements of the stars and planets could be observed in this way, not from down here on Earth, the solar system was simplified, as if by magic. Not only did he leave aside the complicated orbits of Ptolemy, but with his simple circular figures, he went beyond Copernicus (1473–1543). Galileo Galilei cancelled Aristotle's natural upward or downward movements. He ignored friction and air resistance. With the eyes of his mind and the ears of his reason, he saw the pendulum oscillate isochronically *ad infinitum*. He abstracted from friction, and the bodies that rolled down an inclined plane (but they could not have rolled without friction!) assumed a law of perfect inertia. Our renowned scholar from Pisa imagined taking away air resistance, and the cannon-balls dropped from the Leaning Tower came down implacably, in a uniformly accelerated movement, regardless of their mass and form.

There was no experiment that could add up directly; he had to seek approximations laboriously, simulating, hypothesising, calculating and measuring. Measuring? How did he measure time? By means of the beats of his pulse! It's just as well that he exploited the help of music. In his *Sidereus nuncius [Messenger of the stars]* (1610), "… novi perspicilli beneficio …" ["with the aid of the new telescope"], he announced to scholars that the moon revealed the shadow of its mountains, and that Jupiter was accompanied by its four satellites, dedicated to the Medici family, lords of Florence.

Instead of just experimenting, Galileo Galilei was capable of representing by means of drawings those things that he considered pertinent to his theory. He

[48]Galileo Galilei 1968, V p. 325. Despite the misleading title, "Galileo and the Demise of Pythagoreanism", and his use of commonplaces taken from others, who distorted and misinterpreted Vincenzio Galilei, even Jordan 1992 ended up by presenting only a Pythagorean Galileo Galilei.

had attended the Florentine Academy of Drawing. He had been influenced by, and also himself inspired, the artists of his period. Among these, he consorted with Ludovico Cardi (1559–1613), known as "il Cigoli". Beneath the feet of the Madonna, this painter placed a moon, exactly as it was observed by our mathematician. Galileo loved to make precise distinctions, separating painting from sculpture, and the classic poems of Ludovico Ariosto (1474–1533) from the baroque cantos of Torquato Tasso (1544–1595). He preferred the former, just as he admired musicians capable of moving us without weeping or singing: "… se tacendo, col solo strumento, con crudezze et accenti patetici musicali, ciò facesse, per essere le inanimate corde meno atte a risvegliare gli affetti occulti dell'anima nostra, che la voce raccontandole. …" ["… if he does this without speaking, only by means of his instrument, with its harshness and accents of musical pathos, as the inanimate strings are less suitable to awaken the hidden feelings of our soul than the voice that explains them."] Giving due consideration to the pictorial and literary tastes of our natural philosopher, we would understand better why he could never have abandoned, in the heavenly orbits, the abstract perfection of circles for those asymmetric ellipses. These were filling churches and palaces, while baroque shapes were being flaunted under his nose by his rival astronomer of Northern Europe. And yet the forms of a lute, like the one that he probably loved playing, were to remain distant from the circle, and closer, rather, to the ellipse.[49]

He even declared at times that he had not conducted the experiment because he knew what the result would be. What did he answer to those who asked if he had dropped a ball from the top of the mast of a ship in movement, to verify the law of inertia? He replied that he did not need to carry out this experiment, because he knew that it would drop exactly at the base of the ship's mast.[50] Galileo was capable of soaring up much higher than that mast, because he dwelt up there, in the heaven of the ideas. Here, he brought support to his imagination, using the instruments that made it possible for him to connect one idea to another. From one of these, he derived some sure consequences through processes of reasoning that followed a course he maintained clear and controlled, and not uncertain or unreliable like the context of life that surrounded him. Therefore he used Euclid's geometry. Up there, he had learnt to read a new book.

In his *Il Saggiatore* our scholar of nature affirmed: "La filosofia è scritta in questo grandissimo libro .... (io dico l'universo), ma non si può intendere se prima non si impara ad intendere la lingua .... Egli è scritto in lingua matematica ed i caratteri sono triangoli, cerchi, ed altre figure geometriche … senza questi è un aggirarsi vanamente per un oscuro laberinto". ["Philosophy is written in this great book … (I mean the universe), but it cannot be understood unless one first learns the language … It is written in the language of mathematics, and its characters

[49]Galileo Galilei 1964, pp. 5, 10, 17ff. At most, he could construct water-clocks. Cf. Settle 1996. Galileo Galilei 1968, XI, p. 342. Holton 1993, pp. 154–155 and 167–174. Lundberg 1992, p. 219.

[50]Galileo Galilei 1996, II, pp. 185–186. Cf. Feyerabend 1973, pp. 63–73; Feyerabend 1979, pp. 58–77.

are triangles, circles and other geometrical figures … without these, it is a vain wandering through a dark labyrinth"].[51] Our renowned scholar from Pisa wanted to distinguish the sciences from the *Iliad* and the *Orlando furioso*, and thus geometry, from the language of the cosmos, became the criterion of demarcation from the rest of culture. Wasn't this a continuation of the influence of Plato? Galileo Galilei often repeated his profession of faith in mathematics, defending it from all the possible objections brought against it.

In his *Dialogo sopra i due massimi sistemi del mondo [Dialogue on the two general systems of the world]*, he made Simplicio say "… con Aristotele che nelle cose naturali non si deve sempre cercare una necessità di dimostrazione matematica. […] … queste sottigliezze matematiche … son vere in astratto, ma applicate alla materia sensibile e fisica non rispondono: perché dimostreranno ben i matematici con i loro principî… che *sphera tangit planum in puncto* ma come si viene alla materia le cose vanno per un altro verso: e così voglio dire di quest'angoli del contatto e di queste proporzioni, che tutte poi vanno a monte quando si viene alle cose materiali e sensibili." ["… with Aristotle that in natural things, it is not always necessary to seek a mathematical proof. […] … these mathematical subtleties … are true in the abstract but, when applied to sensible, physical matter, they do not correspond: because mathematicians will well demonstrate with their principles … that *sphera tangit planum in puncto* [a sphere touches a plane at a point], but when it comes to matter, things take a different course: and thus I affirm that these angles of contact and these ratios all come to nothing when we are dealing with material, sensible things."] In order to convince Simplicio to fly with him up among the angelic beings that populate heaven, Salviati–Galileo tempted him with a mercantile comparison. He answered him decidedly that "… quello che accade in concreto, accade nello stesso modo in astratto. [Come] i numeri astratti [corrispondevano] alle monete d'oro e d'argento ed alle mercanzie in concreto, [ma a patto che quando si andassero a calcolare] gli zuccheri, le sete e le lane …" ["… what happens in the concrete realm happens in the same way in the abstract. [As] abstract numbers [corresponded] to gold and silver coins and merchandise in the concrete realm [provided that when they went to calculate] the sugars, silks and wools…"] they made the necessary tare for the crates and the packing. Thus, when the philosopher-geometrician wanted "… riconoscere in concreto gli effetti dimostrati in astratto, [bisognerebbe che] difalchi gli impedimenti della materia." ["… to recognize in the concrete realm the effects shown in the abstract, [it would be necessary for him] to subtract the impediments of matter"].[52]

In his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze [Discourses and mathematical demonstrations regarding two new sciences]*, Galileo repeated, after including his mathematics even in the title, that "… tutte le ragioni della meccanica hanno i fondamenti loro nella geometria." ["… all the reasonings of mechanics have their bases in geometry"]. He even tried to go beyond his own

[51]Galileo Galilei 1996, I pp. 631–632.

[52]Galileo Galilei 1996, II, pp. 27–28 and 260.

geometrical knowledge, which he had largely derived from the ancients, Euclid and Archimedes. He recalled that Plato wanted his pupils to be "… ben fondati nelle matematiche …". ["… well grounded in mathematics …"]. But unfortunately, he too ended up by getting lost in the labyrinth created by the paradoxes of infinity, which awaited a less Pythagorean, more Aristotelian concept of the *continuum*. Before them, our philosopher was blocked, he felt an aversion, and his literary style furred itself. The atomistic explanation of a problem of cohesion between bodies had led him into the treacherous marshland of an equally atomistic idea for 'real' numbers.[53] From the Greek and Latin tradition, he had inherited at the same time the best results and the prohibitions regarding irrational numbers and infinity. And yet, in spite of these aspects which became for us the defects of his language, our Pisan scientist remained the inventor of mathematical models when he said that it was necessary to "subtract" the impediments of matter. In opening the door to the modern scientific scenario, he also revealed its main limit.

How was it possible to understand what could be safely ignored, without compromising the phenomenon studied? Because it subsequently happened (to him, as well) that some material impediments were deducted, which should have been included, on the contrary, among those things that determined the event to be analysed. In this connection nowadays, we would speak about "stability". In other words, this means that if you intend to ignore something, you must also demonstrate that the effect of the approximation remains, in proportion, minimal: 'minimal' modifications in the causes must correspond to 'minimal' modifications in the results. Thus, passing to the limit, in this case it would be possible to arrive at reliable explanations. Furthermore, stability (in one of its numerous variants) remains a fundamental condition to reproduce the observation. As the parameters of the environment, where the experiment is conducted, are always subject to small variations (the temperature varies, a comet passes, the earth's crust moves, a pleasant lady doctor coughs, …), when stability is not present, the experiment cannot, strictly speaking, be repeated, and in theory, the comparison between events becomes impossible. Without stability, mathematicians and physicists would speak of different, incommensurable events, without any possibility of reaching any agreement. After the studies of Henry Poincaré (1854–1912) or Aleksandr Ljapunov (1857–1918) up to René Thom (1923–2002), the problem of stability (in particular also of the solar system) was taken very seriously (as a rule, with negative results).[54]

We find ourselves faced with two different points of view, which deserve to be studied in their relative worths and weaknesses, that is to say, in the pertinence to their different aims. Now, the objection of Simplicio (Aristotle) is as intelligent and profound as the answer of Salviati (Galileo). The latter could only count on his own intuition and his own fortune to understand what was the wool to be maintained and what the packing to be eliminated in the (mathematical) comparison of money and gain. Sometimes his attempts were successful, other times they were not. One

[53]Galileo Galilei 1996, II, pp. 571, 707, 598–623. Tonietti 2004a, pp. 94–100.

[54]Tonietti 2002a.

brilliant result was achieved by ignoring the friction of air for the fall of solids, and thus he succeeded in arguing by means of geometry that the spaces traversed were proportional to the square of the times of their falls. But the phenomenon of meteorites that burst into flames due to friction with the air escaped his notice, unlike that of Aristotle. Galileo reduced the problem of floating to the specific weight of the body, and to theArchimedes' upthrust.[55] He believed that he could disprove Aristotle's argumentations based on the form of the vessel, with results that boat constructors would do well not to follow. Our great compatriot believed that the tides were a proof of the movement of the earth, which shook the sea like water in a basin, and he made fun of the obscure, hidden influences of the moon, which we shall soon find, on the contrary, in the following sections.[56]

While the dynamics of Aristotle or Leonardo da Vinci had been one of fluids, in mediums with friction like air or water, movement in Galileo Galilei regarded, on the contrary, solid bodies in a void. This new physics should, then, have projected the Earth into the Heaven, as if fired from cannons. However, as their effect did not seem to be sufficient, what was needed was a mathematical abstraction of a different power, so as to make matter levitate among eternal laws. Our man from Tuscany often expressed himself in a fine, clear Italian which is comprehensible still today (except when he attacked the ineffable infinite), but Latin proved to be necessary, above all in the more technical mathematical passages and argumentations: symbolism still had to wait.

In their textbooks, all the students in the world learn that in the following centuries, the point of view proposed by Galileo Galilei was to prevail. Subtracting the impediments of matter was to become the orthodox, dominant scientific approach in Europe. From then on, scholars will believe that scientific truths should be born from procedures that separate. Otherwise, bodies would not move as they are thought to move, but as they are seen. Instead, by suitably eliminating air and friction, feathers would fall like cannon-balls, and would no longer float down capriciously through space. Thus, through this distinction, knowledge led to the separation from the senses and from common sense. By subtracting here and separating there, we end up by gradually cutting the links with the earthly world where we live. In the end, we acquire a more perfect knowledge, constructing another world apart which is in harmony with our ideas: this only happens in laboratories where the experiments are conducted from now on.

Galileo Galilei's first trial was concluded in Rome in 1616. The theory that the Sun was immobile at the centre, and the Earth revolved around it, was condemned as heretical by the Holy Office, and in particular by the Jesuit cardinal, Roberto Bellarmino (1542–1621). Variously known as heliocentricism or Copernicanism, this theory overturned the Ptolemaic system, which taught the opposite and had been taken as 'true' by the Church because it was considered to accord with certain passages of the *Bible*.

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[55] Galileo Galilei 1996, I, p. 451 and II, p. 642.

[56] Galileo Galilei, *Dialogo sui massimi sistemi*, "Ultima giornata"; in Galileo Galilei 1996, II.

Consequently, Galileo was not allowed to teach, defend, or in certain documents, even to discuss this doctrine. It is a particularly famous event, which has been the subject of numerous studies, although these hardly ever arrive at the same conclusions. And yet the story is often vulgarised as the clash between the obscurantism of the Roman Catholic Church and the light of scientific reason, or the attempt of political and religious power to prevent the freedom of research between the truth and error, and so on. Such an important historic episode, however, does not deserve a similar caricature, and in any case, respecting its complexity, my interpretation in part is different.

We are helped to observe some of its original aspects if we take into consideration, on the one hand, the behaviour followed by the Jesuits in China, during those very years of the trial, and on the other, if we consequently observe in greater detail the importance of the religious arguments presented by the parties involved, including those of Galileo Galilei.

Thanks above all to Matteo Ricci (1552–1610), the Society of Jesus had succeeded for the first time in penetrating into that huge country, making itself accepted, and establishing its bases. The empire of the Ming dynasty (1368–1644), which we have already seen with prince Zhu Zaiyu, was about to come to an end in China, and the missionaries would have to face up to the relative court astronomers. But here, the political power was not in the hands of the Jesuits, as it was in the Papal States, with the consequences that we shall see.

High imperial officials were converted to Christianity, and at the same time were instructed in the European sciences. With the help of one of them, Ricci had even succeeded in translating Euclid's *Elements* (the first six books) into Chinese, under the title *Jihe yuanben [The original roots of how much it is].* According to this Jesuit's strategy, Euclidean geometry and Ptolemaic astronomy were to offer a convincing tactical prelude to the higher, celestial truths of the Catholic faith. When translating and publishing in 1574 his first Latin edition of the *Elements*, his master, Christophorus Clavius (1537–1612), following Plato, had sustained that "The mathematical disciplines elevate the soul and stimulate the strength of the mind, to contemplate divine things".[57]

Starting from Macao and the area around Canton, the route followed was initially fairly successful, allowing them to arrive as far as the imperial court in Beijing.[58] Thus Western sciences arrived in the Country-at-the-Centre, taken there by the Jesuits, and there they found other, different sciences. Between our missionaries who offered their Ptolemaic astronomy and the imperial experts at the astronomic office of the calendar, there were meetings and direct clashes. The eclipse of 1610 had been calculated by the Western men of the Church with greater precision than it had been by the imperial astronomers.

Matteo Ricci wrote in his diary of the little appreciation the Chinese, which he had met, felt for the mathematical sciences. He told them that he was bringing not

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[57] Clavius S.I. 1589, p. 15.

[58] Tonietti 2006a, Chap. 4.

only the One True God, but also the only true theorems regarding geometry and the true movements of the stars. "… sì alla matematica come alla medicina non si applicano se non persone che non possono studiar bene le loro lettere per puoco ingegno e habilità; e così stanno queste scientie in bassa stima e fioriscono assai poco." ["… both to mathematics and to medicine, the people who apply themselves are only those who cannot study their literature, due to their limited intelligence and ability; and thus these sciences enjoy little esteem and are limited in their flourishing."] The only Chinese art that was recognized was that of succeeding in casting magnificent astronomic instruments in bronze: "… grandi e belli …" ["… large and fine …"]. To clarify, for the sake of contrast, the reader can make his/her own (anachronistic) choices between the heavens of the Jesuits, made up of solid spheres, and the empty sky of the Chinese, the round Earth of the former and the square one of the latter, the four fixed Greek and Western elements and the five Chinese phases in continuous transformation into one another, between the Euclidean theorem of Pythagoras and the Figure of the string.[59]

Even if they had gained a knowledge of the Chinese cultural differences, these Jesuits of the seventeenth century looked down on them from the heights (literally) of their religious and scientific certainties, and transformed them into inferiority, and errors to be corrected. Criticism was even expressed in the following terms by an imperial official whose name was Xu Guangqi (1562–1633). He had been converted to Christianity and became a pupil of the Jesuits, collaborating in the translation of Euclid's *Elements*. "The *gougu* leaves out explanations, and it can only be seen".[60] That expression of Xu's, "it can only be seen", referred to the *Xiantu* [the Figure of the string], which we know, on the contrary,[61] really succeeds in demonstrating the famous connection between the three sides of a right-angled triangle. Now, for him, too, the glorious Chinese mathematical tradition would have become flawed.

For the Jesuits, the only true theorem of Pythagoras was the one proved inEuclid's *Elements* at the end of Book 1.The sentence quoted above is taken from a book entitled *Gouguyi*, which means *The right gougu*, where right-angled triangles are dealt with in the Western, Euclidean manner. In other words, Euclid was presented to Chinese scholars as the way to make their geometry "correct". It could now be placed on sure bases, provided by the European procedure of proof: that is to say, as a linear discourse which starts from the axioms, postulates and theorems already previously demonstrated, with the aim of arriving at the affirmation of certainties with precision. As Matteo Ricci wrote, this should take all doubt away.

Thus the procedure should be step by step, as if climbing a ladder towards the Heaven of the eternal truths. The image of the ladder was explicitly included byRicci in his "Introduction" to the translation of Euclid's *Elements*. Here he wrote that, however much intelligence they had been endowed with by Heaven, scholars were

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[59] See above, Part I, Chap. 3. Tonietti 2006a.

[60] Tonietti 2006a, Sect. 5.3.

[61] Part I, Chap. 3.

to use "this book [the *Elements*] like a ladder". In the *Bible*, the ladder was the means seen by Jakob in a dream to arrive in Heaven.[62]

It should be only thanks to this ladder that the scholar should succeed in reaching the truth he seeks. But anyone who does not follow this course will not find it, and will remain in error. Thus the theorem of Pythagoras is true, while the right-angled triangle *gougu* and the "Figure of the string" are open to doubt. These did not appear to the Jesuits of yesterday (and to many colleagues of today) to be so precise, well-founded or general. The *gougu* should become *yi*, that is to say, "right and correct," and should acquire significance only if it were adapted to the ladder constructed by Euclid.

After Ricci's death, another of his converted pupils, Li Zhizao (1565–1630), collected the 20 books printed by the missionaries under the title *Tianxue chuhan [The first collection of heavenly knowledge]*, where he included the translation of the *Elements*, together with articles on religious subjects. Thus the "Heaven" of the title, for him, meant the one inhabited by the God of the Christians.[63] The general problem of Christianity in China has been studied by Jacques Gernet.[64] Also Ad Dudink has shown the close relationship that the Jesuits created between the evangelisation of China and their relative scientific novelties. Manuel Dias wrote in 1615, "Not only did the physical eye see only a fraction of the phenomena, but it was also not able to see the Lord of Heaven, who can be seen only through the phenomena he created. Therefore astronomy is an introduction that paves the way to paradise or Heaven".[65]

Chinese scholars remained down here, because their trust in the Earth was greater. European scholars sought perfection in Heaven. By them, the latter was kept separated from the former for centuries. And even when, following the scientific revolution of the seventeenth century, the Europeans tried to unite the two, they studied the Earth by projecting it into the higher celestial world, as Galileo did.

We have already seen[66] that it was quite possible that the *veritas* [truth], both for the Jesuits and for Galileo, would not fit in easily with the Chinese culture, where there was no tendency at all to divide and to discriminate by means of rigid, absolute laws. In Europe, at the time, books were full of truth and falsehood, good and evil, soul and body, friendship and enmity. The Western philosophies of sciences theorised the distinction between primary qualities and secondary qualities, like misleading sensations. In the seventeenth century, European scholars believed that scientific truths derived from procedures that made a separation: "subtracting" the impediments of matter was really what Galileo did.

The Jesuit missionaries took to China precisely that Classic of the West, in which all these differences between the two distant cultures had found their deepest

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[62] *Genesi*, **28**, 12.

[63] Tonietti 2006a, Sects. 4.1 and 4.3.

[64] Gernet 1982 and Gernet 1989.

[65] Dudink 2001, p. 213. Gernet 1982, passim.

[66] Part I, Sect. 3.6.2.

roots: the *Bible*. This was the setting of the famous story about the tree whose fruit Adam and Eve had been told not to eat. Otherwise, "your eyes would be opened and you would become like God, knowing Good and Evil"[67]. However, this page presents something different from the current interpretation, which sustains a generic prohibition of knowledge. The rabbis who can read Hebrew, on the contrary, explain that the original word does not mean "to know", but rather "to mix". Thus the Biblical prohibition regards "mixing Good and Evil". Now the Western Classic has also taught us not to mix the True and the False, and even, as we know, to beware mixing female with male. Over time, this kind of carnal 'knowledge' become proverbial. If they had not been convinced to be converted to Christianity, those imperial officials would not have been happy to abandon the Confucian (or Taoist) art of mixing, amalgamating and connecting things together in order to understand them.

In the following years, however, the initial relative success of the Jesuits was to turn into a heavy defeat. The Chinese officials converted to Christianity, among whom the most influential was Xu Guangqi, together with the Jesuits, tried to carry out the reform of the Chinese calendar. But this should have ended up by involving the organisation of the Rites, which had represented, ever since their 1,000-year-old institution, one of the most important pivots for the orderly functioning of the empire. Confucius had said in the *Dialogues* (Book II): "He who governs virtuously is like the pole star, which remains stable in its place, while all the others revolve around it". Astronomic calculations served for the calendar, which regulated dynastic successions. Imperial power was considered to be a "heavenly mandate". Thus the Jesuits had been too presumptuous and dared too much: they ended up by being arrested in 1616, and were expelled in 1617, with the confiscation of their possessions. Some Chinese scholars openly declared that those mathematical sciences served to introduce Christianity into China.

On the side of the political and religious power, the Jesuits in Rome, with Gregory XIII, had recently reformed the calendar, and as judges were imposing the absolute Ptolemaic truths about the cosmos on Galileo Galilei. At Beijing, on the contrary, they were under accusation in the dock; there they were orthodox, here they had become heretical, and now here suffered what they had practised in Rome. In China, the Jesuits tried to defend themselves from the accusations by sustaining that their astronomic theories were only an instrument to calculate *tuibu* [literally, "a stage in deducing", i.e. mathematics] the different trajectories and movements of the planets, which did not require the real existence of the nine orbits.

Our Europeans believed that they could use the same tactics in China as had proved to be so effective at home. The *De rivolutionibus orbium celestium [On the revolutions of the heavenly orbits]* of Copernicus had been tolerated, with the *imprimatur*, and even the dedication to Paul III, because it was presented as a hypothesis; it was addressed to mathematicians, and the procedure was admitted, seeing that (perhaps?) it was better for calculations. Professional astronomers could

[67] *Genesis*, **3**, 5.

thus (freely?) discuss it. But woe to those, like Galileo Galilei, who claimed to transform the mathematical hypothesis of Copernicus into real *veritas*. The absolute *veritas* must remain an exclusive prerogative of the Holy Scriptures, as interpreted by the Holy Office, and not by heretics, whether they were Lutherans or otherwise. In *Psalm XIX*, David extolled the firmament. God, and therefore the Church, had established what place the Sun occupied in it. Also Galileo Galilei believed in God and in the absolute truths that derived from Him. But our famous natural philosopher claimed to interpret this same passage in a heliocentric, Copernican sense. "*Deus in Sole posuit tabernaculum suum* come in sede nobilissima di tutto il mondo sensibile." ["God placed his tabernacle in the Sun, as in the most noble place of the whole sensible world"].[68] Galileo incurred the wrath of the Holy Office because he claimed to oppose his absolute truth to that of the Jesuits, and furthermore he dared to back it up, not with arguments of pure natural philosophy, but taking his stance on ground that was forbidden to him, because it was the dogmatic field of the Holy Scriptures.

We shall leave aside here the faulty, insufficient experimental arguments like that of the tides, proposed by Galileo to try to demonstrate the movement of the Earth, and also the better ones regarding the choice of the reference system from which the phenomenon was to be observed. Our mathematician would like to condemn appearances. The Earth appears to be still because we are set on it, but if we were on the Sun … And he made use of the example, in Copernicus, of the boat that moves with respect to the river bank. "… che serve non a dimostrare la verità della posizione, ma la non repugnanza tra 'l poterci parere, quanto ad una semplice apparenza del senso, la Terra stabile e mobile il Sole, ben che realmente fusse il contrario." ["… which serves not to demonstrate the truth of the position, but the non-rejection that, to the simple appearance of the senses, the Earth may seem to be still and the Sun to move, whereas in reality the opposite is true"]. For Galileo, this argument would be insufficient to demonstrate the movement of the Earth.[69] Sure enough, after making the movement relative to the observer, it would be necessary to justify why the viewpoint from the Sun is to be preferred. For our renowned natural philosopher, the movement observed from our star should, on the contrary, lead us to an absolute truth. The religious question remained, therefore, of who had placed his tabernacle in the Sun.

Let us return to China, though. Cunningly, having changed their position in the process, the Jesuits now made their Ptolemaic system play the role of a mathematical hypothesis only, losing the prerogatives of the absolute truth. Therefore, they hoped the Chinese imperial officials would treat them in the same way in which Copernicus had been tolerated in Europe. Then the Chinese would thus allow them to continue their work of religious, cultural and commercial penetration. But they were expelled

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[68]Tonietti 2006a, pp. 175–176. Galileo's letter to Mons. Pietro Dini of March 23rd, 1615. Galileo Galilei 1968, V, p. 303. Cf. D'Elia 1947.

[69]Galileo Galilei 1968, V, p. 370.

all the same, because they were felt to be a threat for the empire, just as Galileo was at that time for the Church, which was facing up to heretics and schisms.

The Ptolemaic system was true and real in Rome against Galileo Galilei, but would it become a convenient mathematical hypothesis at Beijing? The *Societas Jesus* presented a double truth. Should we include among its merits also the invention of relativism? In time, the adjective “Jesuitical” came to assume also a derogatory sense of “astute, false, hypocritical”, as we may read in dictionaries. We shall see below what another Christian natural philosopher like Blaise Pascal (1623–1662) thought of the matter.

Hauled before the tribunal again in 1633, and faced with the threat of torture, Galileo Galilei is said to have repudiated his mathematical and naturalistic truths, and submitted to the authority of the Roman Catholic Church. However, he muttered under his breath the legendary “And yet it moves!” and remained convinced of the movement of the Earth, thus suggesting to Bertold Brecht (1898–1956) the excellent aphorism: “Blessed are those peoples who do not need any heroes”. Protected by the Grand Duke of Tuscany, he went on with his research very quietly, though the results were prudently published in Holland.

I have no intention to go over Galileo’s trials again, if for no other reason, to avoid offering some amiable colleague new pretexts to accuse me of wanting to repeat them.[70] But I do not wish to put on trial our poor Jesuits, either, who, in spite of themselves, became those who were persecuted in China. In any case, they suffered a defeat in the field where they felt they were best prepared. The military terminology will be forgiven for a religious order whose project was to declare war on errors and heretics, even using the rank of “general” in their hierarchy. In their excessive preoccupation with their absolutes, which however became relative when it suited them, they did not understand a crucial point: the difference between Chinese culture and their own was much greater than they could imagine.

The European environment was dominated by typical clear-cut dualisms, such as soul and body, good and evil, friends and enemies, man and nature, transcendence and immanence, and so on, arriving at truth and error. Here, downgrading a truth to a hypothesis avoided a head-on confrontation (and the relative serious consequences) with another truth that was incompatible with it. In the Chinese culture, on the contrary, they generally preferred to combine things together, until an overall organism was created, and human beings were not thought to transcend the social and natural context where they live. The dominant Confucian tradition, for better or worse, adapted the rules to the people, and not vice versa, the people to laws. The circumstances thus toned down the contrasts between the truth and error. Thus the Jesuitical stratagem of downgrading the Ptolemaic truths, that were absolute in Rome, to mathematical hypotheses in China proved to be ineffective and useless. They were contrasted, on the opposite side, not by the equivalent truths of Chinese cosmology, but rather by the practices of a 1,000-year-old system of the calendar, sustained by the officials in power.

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[70]Feyerabend 1985. Stengers 1989. Hammond 1992. Redondi 2004.

In spite of this episode, it should be maintained that, in the end, the Western sciences imposed themselves in China, too. Of course. Therefore, we now face the question of understanding its reasons and the historical circumstances in which the truths and the errors were decided by arguments as convincing as those heard in the ecclesiastical tribunals.

Starting from 1618 in the north-east, the Manchu were invading China, and winning battles, conquering some important Chinese cities. As a result, the Chinese now needed that military competence which the prevailing Confucian tradition had neglected, or relegated to the lower ranks of the imperial hierarchy. Matteo Ricci had realised that warlike activities. considered so noble in the Christian West, did not enjoy an equal consideration there in China. And the Jesuit had included this among the differences to be taken into account. When introducing the translation into Chinese of the *Elements*, however, he had explicitly written that it was essential for generals of the army to be precise, and thus it was important for them to study mathematics. Anyone who knew history was bound to know that those who disposed of the newest, most ingenious weapons possessed the means to win battles, or better defend themselves.

Ricci's expressions might seem to us pure advertising blurb, and for him they must, in any case, have remained theoretical, albeit useful with interlocutors who appreciated above all the practical side. But this was not the case with the other translator of the *Elements*, the convert Xu Guangqi, who followed them literally. Starting from 1619, this imperial official was responsible for the army, because he was considered to be an expert on arms, with the task of defending the capital. He insisted that the armament should be renewed by importing arms from the West, and that the military experience of faraway lands should be used in the war.

It was above all Sun Yuanhua (1581–1638), a pupil of Xu, who was to prove to be the most active, productive element in the noble Christian art of killing enemies. He pressed for the use of telescopes, built fortifications, and even went so far as to cast cannons, and master the art of shooting with them. Here, Euclid's theory of ratios, and the art of performing calculations with a pen on paper, instead of sticks, were exploited to obtain the quantity of gunpowder needed. Sun wrote books on military subjects, such as *Jingwu quanbian [Complete collection of military matters]* and *Xifa shenji [Divine machines with the Western rules]*. Sun was successful in his enterprises: his cannons were used victoriously in battle against the Manchu. More cannons were ordered from him, and he became responsible for their casting. Naturally, Xu and the others spoke in favour of the purchase of the cannons, and of the recruitment of Western experts from Macao. Sun ended up by commanding an army of 8,000 soldiers.

In this way, the Jesuits sneaked back into China, at first only unofficially, and a host of famous characters arrived. In 1621, Johannes Schreck (alias Terrentius, 1576–1630), a member of the Academy of the Lincei and a colleague of Galileo, arrived in China. Then in 1622, Adam Schall von Bell (1592–1666) Arrived. These two obtained permission to stay in the Country-at-the-Centre as military experts. With the help of a Chinese collaborator, in 1643 Schall produced a compendium on firearms, and how to use them in warfare. Others, like Guido Aleni (1582–1649), again started to convert to Christianity those Chinese who showed an interest in geometry. One of these converts, Han Lin (1600–1644), began to study military sciences. and was instructed in the use of cannons. There was a growth in literature about military topics, written by the missionaries who were Ricci's successors and by those who they directly converted to Christianity.

The needs of warfare had forced the Ming empire to open up to the West in an unprecedented manner. Thanks to Schreck, this new opening now led to the entry of trigonometry. He translated Galileo's *Compasso geometrico militare* [Military geometrical compass], and the *Misura del cerchio* [Measure of the circle] by Archimedes. A committee was set up to translate Western scientific books: this was composed of three or four Westerners and some Chinese who were almost all converted. Xu and Sun assumed increasingly important imperial responsibilities. The former again put his hand to the reform of the calendar. As its director, he excluded an elderly Chinese scholar who would have liked to take part without conforming to Western methods. Ricci's pupil set out the rules for the applications of mathematical sciences. (1) forecasts of floods and droughts, (2) dykes and canals, irrigation, (3) improvement of music, (4) military camps, arms, fortifications, (5) financial administration and taxes, (6) building of houses and bridges, (7) mechanical instruments, (8) geographical maps, (9) medical astrology, (10) the measurement of time. But in spite of the help of the Jesuits and the European scientific experts, in 1644 the Ming lost control of the empire to the Manchu, subsequently called Qing in Chinese (1644–1911). Apart from Xu Guangqi, who died of old age in 1633, the other Chinese converts met with a particularly violent end, as a result of the war.

However, the presence of the Jesuits continued also under the Qing, the last empire, destined to endure until the formation of the Republic in 1912.[71] The pictorial style followed in the *Zhoubi* for proofs was taken up by Mei Wending (1633–1721). Yet he now gave this a linear written representation, even copying some characters from the *Jihe yuanben* by Ricci and Xu. He could no longer ignore the Western mathematical sciences, but he was to find another way of bringing them back to China, and positioning his Country-at-the-Centre at the centre of the Earth. He believed that the Chinese must, in all cases, have arrived first; then their inventions would have spread everywhere, arriving in Greece. Also Euclid, therefore, might have re-elaborated in his own way ideas which must have come from the Country-at-the-Centre. Lastly, the Jesuits would have brought that Chinese geometry, clothed in Greek apparel, back to its real country of origin. In the meantime, unfortunately – Mei sustained – the Chinese scholars would have forgot their ancient books, and were therefore unable to recognize them as those brought from the West.

We may easily harbour the suspicion that this giddy historical hypothesis had simply been invented for reasons of national prestige. Not only are documents lacking, as usual, to prove the single origin of all universal geometry in China.

[71] Tonietti 2006a, Chap. 5.

On the contrary, we have extant printed texts, where any interested literate could have found again, more or less easily, those ancient mathematical treatises of the Han period, as besides the interlocutors of Matteo Ricci did.[72] Anyway, Mei Wending would have tried to reconcile Chinese tradition with Western mathematical novelties. He brought again Euclid's geometry toward the tradition of the *gougu* in order to make it comprehensible: "while they [the Westerners] make a secret out of it teaching about God, and we [Chinese] reject it as a *yixue* [foreign doctrine]".[73]

The translation of Euclid's *Elements* was finished half-way through the nineteenth century. In fact the ultimate and final exportation of Western mathematical sciences to the Country-at-the-Centre, which alas! in that period had discovered itself off-centre, took place as a result of the stimulus of, and following, the infamous opium wars: the most shameful and darkest page of European economic globalisation, the one that in other periods, with less hypocrisy, were called colonial imperialism. The preface to the complete translation of Euclid was written by no less than the famous warlord Zeng Guofan (1811–1872). Zeng and the movement *Yangwu* [foreign affairs] sustained that, in order to restore the power of the Qing, it was necessary to make use of Western sciences and techniques.

Unfortunately, the events that took place in China in the seventeenth century have generally been ignored or underestimated by science historians (except Sinologists), above all by scholars of Galileo and the so-called scientific revolution. But on the contrary, they make it possible for us to reformulate, with greater understanding, the crucial questions that lie at the basis of the relative historiographic discussion.

What circumstances influenced the evolution of the sciences in Europe? What role did a transcendent religion like Christianity play? Why does war break out so often? What is hidden behind the continual claim to possess the truth, whether scientific or religious, when those who believe in it do not hesitate to contradict it or adapt it to their own advantage, depending on the circumstances? Can we avoid answering these questions, if we wish to understand how, from time to time, groups of orthodoxy are established, and heretics are put aside, when attempts are not made to eliminate them completely? If we could know, and take into due consideration the historical circumstances, why can we not arrive at the point of thinking that the results of scientific research are not so objective and absolute as they are commonly believed to be? Even in the truths of science, do we not end up by believing above all by faith?

The hard nucleus of the dispute between Galileo Galilei and the Church was: who possessed the monopoly of absolute truth, controlling the means to arrive at it, and discriminating between truth and error? That was clearly written in our mathematician's correspondence with Benedetto Castelli, Piero Dini and Christine of Lorraine. The defenders of the Ptolemaic system wielded blows with the Holy Scriptures, like the expression "Sun, stand thou still" used by Joshua in the *Bible*. They "… si tengon sicuri d'avere in mano l'assoluta verità della quistione che

[72]Tonietti 2006a, pp. 170–171.

[73]Martzloff 1981a, p. 34. Tonietti 2006a, pp. 182–197.

intendono disputare, ...” [“... are sure about possessing the absolute truth of the question they intend to dispute, ...”]. Let them say “... se loro stimano gran vantaggio aver colui che in una disputa naturale s’incontra a sostenere il vero, ..., sopra l’altro a chi tocca sostenere il falso? So che mi risponderanno di sì, e che quello che sostiene la parte vera potrà aver mille esperienze e mille dimostrazioni necessarie per la parte sua, e che l’altro non può aver se non sofismi, paralogismi e fallacie.” [“... whether they consider it a great advantage to have the one who, in a dispute about natural matters, argues sustaining the truth, rather than the other one, who is left to sustain what is false. I know that they will answer affirmatively, and that he who sustains the true part may have thousands of experiments and thousands of necessary proofs on his side, and that the other cannot have anything better than sophisms, paralogisms, and fallacies”]. Then, Galileo Galilei asked Castelli, “... perché, nel venir poi al congresso, por subito mano a un’arme inevitabile e tremenda, che con la sola vista atterrisce ogni più destro ed esperto campione?” [“... why, in coming to the discussion, do they immediately take out an inevitable, dreadful weapon, which, just at its sight, appals all the most accomplished and expert champions?”] In all these general considerations, the religious and naturalistic questions were inextricably mixed, becoming intolerable to suffer for the political and religious power of the time.[74]

For us, it is also interesting that Galileo Galilei’s way towards the “absolute truth” always brought him close to music. “Chi è quello che non sappia, concordantissima essere l’armonia di tutti i veri in natura ed asprissimamente dissonare le false posizioni da gli effetti veri? Concorderà, dunque, in ogni spezie di consonanza la mobilità della Terra e stabilità del Sole con la disposizione di tutti gli altri corpi mondani e con tutte le apparenze, che sono mille, che noi ed i nostri antecessori hanno minutissimamente osservate, e sarà tal posizione falsa e la stabilità della Terra e la mobilità del Sole, stimata vera, in modo alcuno non potrà con le altre verità concordarsi?” [“Is there anybody who does not know that the harmony of all true things in nature is perfectly concordant, and that false positions are harshly dissonant from the true effects? Will the mobility of the Earth and the stability of the Sun will be in agreement, therefore, in every kind of consonance, with the disposition of all the other worldly bodies and with all the appearances, which are innumerable, that we and our predecessors have observed in great detail? And will this false position, of the stability of the Earth and the mobility of the Sun, if considered to be true, in any way agree with the other truths?”][75] How this sentence could not bound to bring to our heads again the music of the spheres, so dear to Plato and the Pythagorean sects?

We are venturing along untrodden paths that lead us to listen to music, and also to observe other, faraway, exotic scientific cultures, also in order to help us answer

[74]Galileo Galilei 1968, V, pp. 285 and 281–370. Feyerabend 1985, pp. 250–256. Cf. Redondi 2004.
[75]Galileo Galilei 1968, V, p. 356.

these problems and succeed in breaking away from the well-known commonplaces found in so many books.

> … you will know the truth and the truth will set you free.
>
> John **8**,32

## 8.3 Johannes Kepler: The Importance of Harmony

In 1619, Johannes Kepler (1571–1630) published his *Harmonices Mundi Libri Quinque [Five books on the harmony of the world]*. With the surprising (for us of the twenty-first century) idea of continuing the *APMONIKA* of Claudius Ptolemy, which had remained incomplete, he wrote a complex treatise of geometry, music, astrology and astronomy. His theory for the division of the octave, and the justification of consonances was highly original, because it was based on the "demonstratio" ['constructibility'] of regular polygons with a certain number of sides. In "Book V", lastly, he included his third law of planetary movement. "Sed res est certissima exactissimaque, quod *proportio qua est inter binorum quorumcunque Planetarum tempora periodica, sit praecise sesquialtera proportionis* mediarum distantiarum, id est *Orbium* ipsorum; …" ["But it is absolutely certain and exact that the *proportion between the periodic times of any two planets is precisely the sesquialterate proportion of their mean distances, that is, of the actual spheres*, …"].[76] In post Cartesian symbols, we might write today that for the periods of revolution $T$ and the mean distances from the Sun $R$:

$$T_1 : T_2 = R_1^{3/2} : R_2^{3/2}$$

Thus this famous result was to be found in a book that was also about the theory of music.

Western modern sciences were now transformed into the art of perceiving intuitively how much, and what, could be omitted in a phenomenon, in order to make it absolute, and transport it into the realm of pure ideas. Attempts were made to achieve results that were certain and universal. And yet, depending on the individual, the tendency was to "subtract" different impediments, and consequently not to construct the same model for the same phenomena. The history, culture and interests of various subjects made the music that spread from the heavenly spheres sound different. Kepler did not live in the same manner, did not reason in the same way, and did not share the same values, as Galileo Galilei. Apart from Copernicanism that they had in common, they often arrived at contrasting conclusions. The German believed, unlike the Tuscan, that it was the mysterious influence of the Moon that moved the tides, not the movements of the Earth, just as the 'chaste goddess' regulated the female cycle and the growth of the plants. For him, all the stars, and

[76] Kepler 1619 [1969], V cap. III, 8, pp. 189–190. 1997, p. 411. The Italics are in the text.

the planets in general, influenced the Earth, and even our minds, which they affected by means of their harmonious movements. Could a better proof have been offered than the one provided by music?

In his *Mysterium cosmographicum [Mystery of the cosmos]* (1596) Kepler wrote: "In this booklet, my reader, I propose to demonstrate that the most excellent and great Creator, in creating these movements of the world, and setting out the heavens, had reflected on those five regular bodies, famous from Pythagoras and Plato to our days, and had proportioned to their nature the number of the heavens, their ratios and the reason of their movements." [...] "... I admire more than anything else analogies, my most faithful masters. They know all the secrets of nature, and must not be at all neglected in geometry". "The main aim of all investigations into the external world will have to be to discover the rational order and the harmony that God has set on it, which He reveals to us in the language of mathematics."[77]

Also for him, the harmony of the cosmos was expressed in mathematics, and again, it was above all Euclid's geometry. However, Kepler read it in his own way, arriving at surprising results: he succeeded in driving the five Platonic solids into the orbits of the six then known planets, as follows: by analogy, he placed the circular orbit of Mercury around the Sun on the sphere inscribed within the regular octahedron, and the orbit of Venus, again circular, on the sphere circumscribed around the octahedron. This last sphere was also inscribed within the icosahedron, and so on, placing the circular orbit of the Earth between the icosahedron and the dodecahedron, that of Mars between the dodecahedron and the tetrahedron, that of Jupiter between the tetrahedron and the cube, and lastly that of Saturn on the last sphere possible, circumscribed around the cube (Fig. 8.3).

To Kepler, this mathematical model of the solar system must have seemed particularly harmonious, but only in a qualitative way. That is to say, in trying to make it quantitative, in relationship to the astronomic measurements, he must have realised that those geometrical forms were not the only ones possible, and therefore our original German astronomer proposed others. In his *Astronomia nova [New astronomy]* (1609), he distributed a good number of planetary observations along elliptical orbits, and no longer along the traditional circles, which had remained such also for Copernicus and his followers. He had come into possession of those numbers and angles at Prague, by fraud,[78] as he himself related, while he was an assistant to Tycho Brahe (1546–1601), subsequently taking his place as director of the observatory. The orbit of Mars, above all, must have seemed to him too eccentric to be placed on a circle. Ellipses did away with need for the epicycles, eccentrics and equants to which Copernicus and Galileo still had recourse. Thus he enunciated his first two famous laws: the orbits covered by planets are ellipses around the Sun, which takes one of the foci; the radius vector between the Sun and the planet sweeps equal areas in equal times. The planet thus moves faster at the

[77] Kepler 1938, v. I, pp. 9, ... and passim.

[78] With the permission of the emperor, but without that of the heirs.

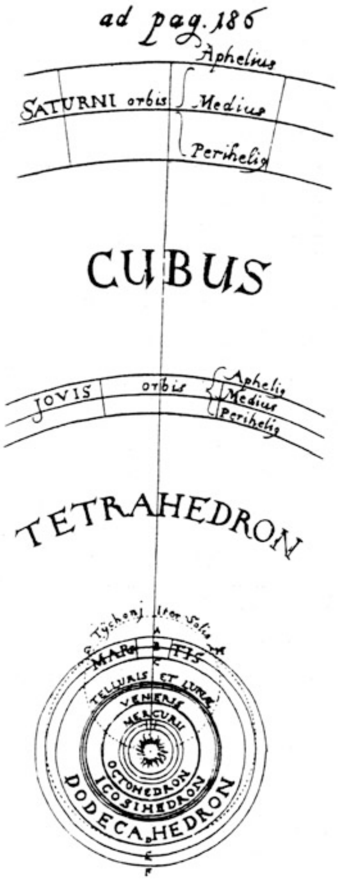


**Fig. 8.3** How Kepler distributed the regular polyhedra among the orbits of the planets (Kepler 1619, V p. 186)

perihelion, when it is close to the Sun, and more slowly at the aphelion, when it is furthest away.

But in so doing, hadn't he destroyed that perfect Greek beauty of Platonic solids, to use figures that were not equally harmonious? Undoubtedly, it cannot have been easy for him to give up the sphere and the circle, seeing that these geometrical figures maintained the divine quality of Heaven. Actually, Kepler immediately dealt with the problem of how to reconcile the elliptical orbits again with the harmony of the cosmos. And this became the even more surprising story of the third law. Totally unsatisfied with the solution found, and not at all convinced that the adventure could finish there, with a better agreement of astronomic data, Kepler then wrote the most interesting and most important book for our history: the *Harmonices Mundi Libri Quinque*.

The not enough historians who have drawn close to our German scientist would quickly underline the great attention that this work should deserve, and what a serious defect it is to ignore it. But then few have studied the text with the necessary care. There are histories of scientific thought that speak about everything (including Vladimir Ulianov, known as Lenin, 1870–1924) for thousands of pages, but then dedicate exactly nine pages to Kepler![79] The *Harmonices ...* is treated even worse, because almost all confess candidly that they have not read it. It was more often to be found in the libraries of musicians like Alban Berg (1885–1935).[80] In Italy, the anastatic reprint was brought out in a series of books about music. Until quite recently, its translations existed only in German, and (a mediocre one) in French; the English only published the fifth chapter. In Italian a paltry compendium came out, which was too partial, and questionably edited. We follow the original Latin, and the complete English translation which appeared a few years ago.[81]

The book deserves to receive more attention, just because in it Kepler expressed the idea of a strong, crucial link between geometry, astronomy and music. In the first two books, he expounded a theory about the 'constructibility' of polygons, and regular solids, also demonstrating their properties of filling the plane and space. Having shown that the heptagon does not possess the necessary quality of 'constructibility', Kepler presented, in Book III, his geometrical division of the monochord, and therefore he proved himself capable of excluding the dissonances generated by the division into seven parts. In Book IV, he explained how the planets and the stars influenced the souls of men living on the Earth, by means of the harmony generated by their movements, similarly to what music is actually able to accomplish. In Book V, every planet was assigned its own particular melody: fewer notes for Venus, because its orbit is almost circular, a longer sequence for Mars, because its orbit is fairly elliptical. The Earth sang (and continues to sing, unfortunately) one strident soft semitone like $mi - fa$, $mi - fa$ "MIsery and FAmine, MIsery and FAmine" (Fig. 8.4). The third law, seen above, was almost hidden without any emphasis towards the end.

Kepler had thus succeeded in establishing his famous law, while again trying to model the heavenly spheres according to the proportions of music. He had found the sesquialtera 3:2 of the fifth again, in the ratios between times and distances.

While he reflected on the question, he also read Galilei, however not his colleague Galileo Galilei, rather his musician father, Vincenzio Galilei, who, as we have seen, had assumed a stance in the musical controversies of the period, with his work *Dialogo della musica antica et moderna [Dialogue between ancient and modern music]*. In the context of the Camerata dei Bardi, our Florentine composer and theoretician sustained the emerging accompanied monody of "recitar cantando" [reciting in song], which preluded lyric opera: in other words, the music practised by the ancients. However, Kepler undoubtedly preferred the modern music of the period instead: that is to say, polyphony. And, as he had transformed the orbits from circular into elliptical, he had to give up the unison of the classical conception, according to which movement would produce a single note. Now, he had finally demonstrated

[79] Geymonat 1970, II, pp. 105–107 and 508–514.

[80] Tonietti 2004, pp. 665–669.

[81] Kepler 1619 [1969]; 1939; 1952; 1979; 1994; 1997. Holton 1993, pp. 3–23.


**Fig. 8.4** How Kepler imagined the melodies of the solar planets (V, p. 207; 1997, p. 440; Kepler, facsimile 1969, lib. V, pp. 186/187)

that the different melodies, sung by the various planets, harmonised in a general composite polyphony in following their orbits. Was it not true that the heavenly ellipses behaved up there in an analogous manner to the music written down here by the most renowned composers to elevate souls? Macrocosm and microcosm were thus in perfect harmony in singing God's praises.

The harmony of the cosmos had now finally been regained and maintained. Above all, it appeared to be very interesting that Kepler divided the monochord in an original manner by means of geometry, moving away from the usual well-known considerations about the numbers 1, 2, 3, 4 of the classical Pythagorean tradition. He still used the general Euclidean approach of ratios, like Galileo, but he obtained a music that resounded to be in part changed. He had few doubts that the Third law indicated the conductor of the orchestra had to be the same for all the planets, and should thus be situated in the Sun. Without any fear of betraying a trust in some occult agent, our German scientist believed that it was a kind of influence analogous to magnetism, or light. Thanks to his faith in harmony, an important step forward had been taken towards the modern construction of the cosmos. The next step would be taken by a scholar using other mathematical instruments, in order to make it quantitative. But for this aim, it was necessary to learn to use other symbols in geometry, other ideas that came in part from far away, from environments which would not necessarily all continue to express themselves in Latin, which remained Kepler's main written language.

For our German astronomer, the harmony of the cosmos was to include all kinds of phenomena, nothing excluded. "First, then, you will remember that the hard third arose from the pentagon, and the pentagon uses the division into extreme and mean ratios, which form the divine proportion.[82] However, the splendid idea of generation is in this proportion. For just as a father begets a son, and his son another, each like himself, so also in that division, when the larger part is added to the whole,

[82]The golden section $1:x = x:(1-x)$, indicating the whole as 1 and the larger part as x.

the proportion is continued: the combined sum takes the place of the whole, and what was previously the whole takes the place of the larger part.[83] Although this ratio cannot be expressed in numbers,[84] yet some series of numbers may be found which continually approaches nearer to the truth [the real value]; and in that series, the differences of the numbers from the genuine terms (which are not countable, but inexpressible) by a wonderful alternation, breeds males and females, distinguishable by the members which indicate sex. Thus if in the first place the larger part is 2, and the smaller 1, the whole is 3.[85] Here, plainly, 1 does not stand to 2 as 2 stands to 3[86]; for the difference is unity, with the result that the rectangle of the extremes 1 and 3 is less than the square of the mean, 2.[87] Then, by adding 2 to 3, the new total becomes 5; and by adding 3 to 5, the total becomes 8, etc. The rectangle of 1 and 3 creates a female, for it falls short of the square of 2 by unity (Fig. 8.5);

The rectangle of 2 and 5 creates a male, for it exceeds the square of 3 by unity; the rectangle of 3 and 8 a female, for it falls short of the square of 5 by unity. Again from 5 and 13 arises a male, in respect of the square of 8; from 8 and 21 a female, in respect of the square of 13; and so on infinitely.[88]

Such is the nature of this division, which relates to the construction of the pentagon; and God the Creator has shaped the laws of generation in accordance with it; in fact, in accordance with the ratio of inexpressible terms, which is genuine and perfect in itself [divine proportion], he has shaped the ratios of the seeding of plants which have been commanded individually to have their own seed within themselves. In the accordance with the combined ratio of pairs of numbers (of which the falling short of one by unity is compensated by the excess of the other), he has shaped the coming together of male and female: what is surprising, then, if the progeny of the pentagon, the *dura* [hard] third 4:5 and the *mollis* [soft] 5:6 moves minds, which are the images of God, to emotions which are suitable for the business of generation? Here it must be repeated, from Chap. III, that, although 1:6 comes from the hexagon,

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83 $(1 + x):1 = 1:x$; that is to say, $x = \frac{1}{1+x}$.

84 In other words, it cannot be represented in a 'rational' manner, because $x^2 = 1-x; x^2+x-1 = 0; x = (-1 + \sqrt{5})/2; x = 0,6180339\ldots$.

85 $1 + 2 = 3$.

86 $1:2 \neq 2:3$

87 $3 \times 1 = 2 \times 2 - 1$.

88

$$5 : 3 \neq 3 : 2; 2 \times 5 = 3 \times 3 + 1$$
$$8 : 5 \neq 5 : 3; 3 \times 8 = 5 \times 5 - 1$$
$$13 : 8 \neq 8 : 5; 13 \times 5 = 8 \times 8 + 1$$
$$21 : 13 \neq 13 : 8; 21 \times 8 = 13 \times 13 - 1$$
$$\ldots$$

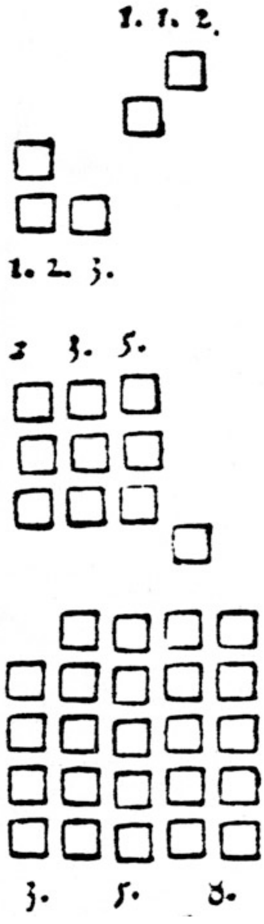


**Fig. 8.5** How, in his arrangement of the *squares*, Kepler represented the numbers of sexual generation by means of Fibonacci's series (Kepler, ibid. lib. III p. 77)

yet its remainder 5:6 is consonant, not on account of the hexagon, but on account of its derivation from three tenths of the circle, by the doubling and halving of terms. Thus, this remainder also, and its progeny, the *minor* third, comes from the pentagonal class of figures. On establishing, then, that the association of two thirds represents the association of male and female, it is now no trouble to assign to each sex its own third. For the *major* third will prove manly, the *minor* feminine, since the proportion of their actual bodies is also the same as that of the material and spiritual powers. And as the *major* comes from a figure with an uneven number of sides, that is to say, the pentagon, but the *minor* originates from the decagon which has an even number of sides, this is also in agreement with the views of Pythagoras, who said that uneven numbers were male, and even ones female[89] (which is confirmed by this study of excesses and shortfalls, since the odd is also in excess) with the result that the former is considered to be of the masculine sex, the latter of the feminine."[90]

In other words, using post-Cartesian and post-Arabic algebraic modern notations, Kepler 'showed' that the fractions $\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{8}, \frac{8}{13}, \frac{13}{21}, \ldots$ became increasingly closer to the number $x = \frac{1}{1+x}$, that is to say, to the continuous fraction capable of defining an irrational:

$$\frac{1}{2} = \frac{1}{1+1}; \frac{2}{3} = \frac{1}{1+\frac{1}{1+1}}; \frac{3}{5} = \frac{1}{1+\frac{1}{1+\frac{1}{1+1}}}; \ldots$$

Going on to infinity, we would obtain

$$x = \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ldots}}}}}}$$

thus $x = \frac{1}{1+x}$; that is to say, $x^2 = 1 - x$ from which $\frac{x}{1-x} = \frac{1}{x}$, and in other words, $1 : x = x : 1 - x$. Therefore, $x$ represented the value of the divine proportion. Not only so, the fractions that approximated to its value as a continuous fraction were composed of ratio between two adjacent terms in Fibonacci's succession, just the one inspired by the love-making rabbits. Furthermore, the divine proportion entered into the construction of the pentagon. Here, Kepler seems to want to render heavenly also those activities which others in his period would have considered all too animal and earthly.

Establishing unexpected relationships between the pentagon, the divine proportion and Fibonacci's succession, Kepler seized "the law of generation", valid for the seeds of plants, and for the union between male and female. This law did not appear to be the prisoner of whole, rational numbers, but rather tended to the ineffable, the divine, the infinite. The pentagon also achieved that division of the monochord which justified the new consonant harmonic ratios of the third, borrowed from Ptolemy, and made current in that period by Zarlino. Thus, even his

[89]Five was the Pythagorean number for marriage.

[90]Kepler 1619 [1969], III cap. XV par. VIII, pp. 76–77; 1997, pp. 241–242; translation slightly changed.

mathematical model for generation derived from the search for a harmony in every aspect of the cosmos, even biology, that was inspired by music. And with this model, he explained why the activity tending towards generation was stimulated by the intervals of the major third -*do/mi*- and the minor third -*mi/sol*. Kepler interpreted the world through mathematical relationships, which illustrated its harmony. It regarded all phenomena, and not just astronomic and physical ones. Even between human beings, animals and plants, our German astronomer assumed geometrical relationships, which were to be in agreement with those of the musical notes and the planets, in a global conception.

For music, Kepler made reference to the Pythagoreans, to Ptolemy, to Boethius, and so on, all the way down to Zarlino and Vincenzio Galilei. He explained how and why he could not wholly accept their proposals about the origin of harmonious proportions and consonances. In contrast with the Pythagoreans, who reduced harmony to whole numbers, he invoked the ear. They did not justify why certain numbers were acceptable and others were not. On the contrary, he offered the causes. “Cum enim intervallorum Consonorum termini sint quantitates continuae, causas quoque, quae illa segregant a Dissonis, oportet ex familia peti continuarum quantitatum, non ex Numeris abstractis, ut quantitate discreta; …” [“For since the terms of the consonant intervals are continuous quantities, the causes which set them apart from the discords must also be sought among the family of continuous quantities, not among abstract numbers, that is in discrete quantity; …”]. He thus finally broke out of narrow pattern of the *quadrivium*. He could then explain, by means of continuous geometry, that consonant intervals were “scibiles” [“knowable”] and dissonant ones “aut improprie scibiles, aut inscibiles” [“either improperly knowable, or unknowable”].[91]

Having curved and closed in a circle the vibrating string that generated sounds, our German astronomer divided it in accordance with the ratios previously fixed by regular polygons. For him, the “constructible” polygons became the cause of “knowable” intervals, and thus of consonances. He knew that the musicians of his period had by now gone beyond the limits of the Pythagoreans, making a large use also of the intervals of the third and the sixth, only to remain silent at the moment of the new heavenly harmonies. The garments worn by Kepler were still those of Euclid, woven with definitions, axioms and propositions, but he used these to obtain different results, seeing that he explicitly wrote that he repudiated the numbers of the Pythagoreans. The regular polygons with 5 or 6 sides thus justified the new consonant relationships of the third and the sixth, whereas those with 7, 9, 11, … sides gave dissonances.

But it was not only as a Platonic that he presented his “sublime” contemplation of the new axioms of musical theory; for him, it was also “… Fideique Christianae analoga, …” [“… analogous to the Christian faith …”]. This geometry, therefore, eternal like the God of Kepler, provided the models from which the laws could be derived. Spirits, souls and minds, as images of God the Creator, rejoiced in the

[91] Kepler 1619 [1969], III, pp. 7–9; 1997, pp. 137–139.

same proportions that could be found in the senses, bodies and movements. "… per materiae quantitatem, inter infinitas proportiones non harmonicas, occurrerint etiam harmonicae istae suis temporibus, & sic non in ESSE sed in FIERI consistant." ["… due to the quantity of material, among an infinity of proportions which are not harmonic, those harmonic proportions also occur at their own time, and thus subsist not in BEING but in BECOMING."] For these proportions guaranteed by God, Kepler provided models, both with the heavenly movements [in Book V], and with the "Radiationibus Astrorum" ["radiations of the stars", in Book IV] and also with the human soul, which takes pleasure from musical notes.

Considering the resonance between two vibrating strings, our natural philosopher from Northern Europe explained that the vibrations were propagated from the first one, when it was plucked, also to the second one, left untouched, but in unison; its cause was: "… species immateriata corporis chordae, …" ["… an immaterial image of the body of the string …"]. He wrote of "species", and not "ictus" [strokes] as Maurolico, Benedetti or Galileo Galilei did, as we have seen above. Kepler believed that the relative theory was insufficient to make the strokes coincide in order to explain the consonances. Otherwise, the best consonance would have been playing in unison. For him, instead, it was not necessary to count the (discrete) number of strokes that arrived at the ears with the sound, but rather to follow that "species" of the vibrating string as far as the mind, where it was judged by the tribunal of the spirit.[92]

Following his own ears, Kepler had discovered the errors of the ancients, and insisted above all that he had discovered the causes of harmonious relationships. "… ut certus sit, ea, de quorum causis laboramus, sensuum experientia certissima niti, nec sponte mea (cujus criminis Pythagorei in parte rei sunt) esse conficta, proquam veris obtrusa." ["… so that he can be sure that those things, whose causes we are struggling over, rest on the surest test of the senses, and are not improvised inventions of my own (a crime of which the Pythagoreans are partly guilty) which have intruded into the place of truth."][93] He had undoubtedly found the same musical intervals as Ptolemy and Zarlino, but now he felt sure about them, as he had justified them in a balanced way, between the senses and geometry.

Thus our natural philosopher cannot have concerned himself only with an abstract, inaudible music of the spheres, because very often he indicated on Guido D'Arezzo's stave the notes corresponding to the numbers of the intervals. He even arrived at the triads of the major and minor chords, refusing, however, any Pythagorean interpretation of them based on the number three.[94]

Kepler remembered that incommensurable magnitudes existed in geometry, and therefore it was a vain hope to expect to find a "… minimum Elementum …" ["… smallest element …"] with which the other consonant or melodious intervals

[92]Kepler 1619 [1969], III, pp. 13–15; 1997, pp. 147–150. Here *species* is misunderstood as "emanation".

[93]Kepler 1619 [1969], III, p. 28; 1997, p. 165. Translation slightly changed.

[94]Kepler 1619 [1969], III, pp. 30–31; 1997, pp. 168–170. Dickreiter 1973, pp. 153–154.

could be formed. Therefore it was necessary to accept that "… Intervalla consona (praeterquam quorum unum est alteri Multiplex) esse, ut proportiones illas ipsas, incommensurabilia; …" ["… Consonant intervals (except in cases where one is a multiple of the other) are incommensurable, like the actual proportions; …"]. In the example that 1:4 was not commensurable with 2:3, we can see the arrival of the notorious *comma*. For Kepler, music was made up of consonant intervals, which preceded any division into smaller intervals. Music was not destroyed by breaking it down into quantities; on the contrary, the relationships between notes possessed a qualitative nature. Breaking down a consonance would have meant ruining it. Thus tones, semitones and diesis arose from the consonances, and not vice versa, as the ancients believed.

With this (Aristotelian?) idea in his head, that the quality of the whole could not derive from the quantitative union of the single parts, our revolutionary astronomer of the seventeenth century was able to go back over the more traditional melodious intervals, including the various *comma*, *limma*, and diesis of Ptolemy's treatise. But he continued to repeat his accusations against the Pythagorean "… superstitione …" ["superstition"] of numbers, which were to be substituted with his geometry. Albeit with the due criticism, he followed Ptolemy, and ignored the Platonic-Pythagorean scale.[95]

Of the three Greek genres, Kepler cancelled the enharmonic one, and reinterpreted the diatonic and the chromatic genres as "durum" [hard] and "molle" [soft] through the decomposition of the fifth into a "hard" and "soft" third. Consequently, in a similar way to Zarlino and other musicians of the period, also our astronomer contributed to the transformation of the Greek-ecclesiastical modes into the subsequent 'major' and 'minor' keys.

In Chap. VIII of "Book III", Kepler states his position regarding the controversy that we have seen running through European culture for centuries, which had recently been inflamed again by Vincenzio Galilei. Here, then, he wrote that in practice, musicians were to temper the notes of the scale on their instruments, and to make all the tones equal, dividing them into two semitones that were likewise equal. And now the astronomer took as his explicit target Vincenzio Galilei, who he knew to have been inspired by Aristoxenus, and tried to demonstrate that the hearing should reject this solution which was most convenient for players of music. He calculated the numbers of the lengths for the strings of the lute according to Vincenzio, who had taken the constant ratio 17:18 as the semitone. This was clearly an approximation to the correct number for the tempered semitone. For a string 100,000 long, decreasing by the geometrical progression of $\frac{17}{18}$, he obtained the length of 50,363 for the higher octave, and not 50,000, in accordance with the due ratio of 1:2, as in the classical Greek scales taken by Kepler.

[95]Kepler 1619 [1969], III, pp. 33–37; 1997, pp. 173–179. Here *superstitione* is translated as "fascination".

He also noticed the imperfection of the tempered fifth 67,025, instead of his Greek-Pythagorean 66,667.[96] "… & haec differentia sonorum facile deprehenditur ab auribus, …" ["… and this difference between the notes is easily discerned by the ears, …"]. "… sciunt boni Arithmetici falsum esse, cum 1:2 & 17:18 sint incommensurabiles." ["… good arithmeticians know this to be false, since 1:2 and 17:18 are incommensurable"], that by repeating 17:18 12 times, we obtain 1:2. "… auditus, si ratione acuatur … probans partes, quas mei numeri signant, repudians Mechanicas primi ordinis … Ego quidem usum ejus Mechanicum agnosco, ut organis eadem pene libertate in tensionis, quae est in humana voce, possimus uti; ad speculationem vero, imo ad Naturam cantus dignoscendam perniciosam exsistimo: quoque hoc efficiat, ut Organum ingenuitatem humani cantus vere nunquam assequatur." ["… the hearing …, if it is sharpened by reason, …"] approving in the table ["… those parts which my figures represent, and repudiating the mechanical figures [of Vincenzio Galilei] in the first column. … I indeed recognize its mechanical function, so that in instruments we can enjoy almost the same freedom of tuning as can the human voice. However, for theorizing, and even more for investigating the nature of melody, I consider it ruinous; and the effect of it is that the instrument never truly attains the candour of the human voice."] At the beginning of Chap. IX, however, our 'good arithmetician' admitted: "Non sine lite res est; sed sequor ego rationes necessarias." ["The matter is not uncontroversial; but I am following necessary calculations."][97]

After all the observations written in favour of geometry and continuous magnitudes, like those suitable also for the treatment of music (and not only astronomy), nobody should imagine that Kepler was not capable of extracting the roots of numbers, and did not know how to divide the octave into 12 exactly equal parts, either. After all, he also had a direct knowledge of Aristoxenus. He simply did not want to do so, therefore, and preferred to remain close to Zarlino in his variant of the classical Greek *diapason* [octave], although he had found and justified it otherwise. Why?

From time to time, he repeated, even in this musical "Book III" as well, that harmony had been fixed by God, also for the movement of the planets. "Rursum autem hoc discrimen utriusque generis harmoniae, Deus ipse in motibus planetarum expressit, ut lib. V audiemus." ["Now again, the distinction between the two kinds of harmony [hard and soft] has been expressed by God himself in the motions of the planets, as we shall hear in Book V".] Thus the basic reason that guided our versatile scholar of Northern Europe was to become clear only in astronomy. All that he thought and did in the theory of music was to prove to be consistent with the novelties imagined for the movement of the planets and vice versa. "… Proportionibus, totoque apparatu Harmonico ex Musica, opus habeo ad explicandas causas proportionis Orbium coelestium, Eccentricitatumque & motuum

[96]For Vincenzio's tempered fourth, the table shows a printing error: 75,242 instead of 75,142, compared with the number 75,000 of the ratio 3:4.

[97]Kepler 1619 [1969], III, pp. 46–50; 1997, pp. 195–200 . Kepler 2009, pp. 20–31.

in Apsidibus." ["... I also labour over the proportions and the whole harmonic panoply in music, in order to explain the reasons for the proportions of the celestial circles, and their eccentricities and their motions at the apsides."][98]

Historians of music may find it interesting to study the variants introduced by Kepler with respect to the Greek modes, the ecclesiastic modes, Ptolemy and the other theoreticians of his time.[99] We may limit ourselves here to the observation that our mathematician continued to maintain the tortuous theoretical complications of the orthodox Greek tradition, refusing the route chosen by contemporary musicians and composers. If he, too, like them, had accepted the equable temperament re-proposed by Vincenzio Galilei, as if by magic, the system would have been simplified. What Kepler succeeded in achieving with the orbits of the planets contained in only three laws, he did not succeed in doing with earthly musical harmonies.

It was a curious paradox that, thanks partly to music, our astronomer had made the image of the solar system more simple. And yet, at the same time, as a result of the link imagined with astronomy, he had to maintain much of the ballast of the Pythagorean-Ptolemaic theories of music, in a period when musicians instead, freeing from it, would have given rise to a brilliant season of creativeness. They sought variety in the modulations between one tonality and another, which were considerably facilitated, above all by the possibility of transposing intervals on the different notes of the scale. But our natural philosopher from Northern Europe would have preferred to stop all those transpositions which were incompatible with his system. "Quaeras ultimo, quid impediat illas transpositiones, toties jam a me rejectas? Videntur enim aures illas non impedire ..." ["Finally, you may ask what prevents these transpositions which I have already rejected so often? For the ears seem not to prevent them, ..."]. And in the end, Kepler had to invoke "naturae leges" ["laws of nature"] as a support for his system. But which laws? Those of astronomy, of sounds or of music?[100]

In the wake of a tradition that went back to Plato's *Republic*, our astronomer of the seventeenth century was not averse to expressing his opinion about the effects provoked by the various tones, intervals or modes on people. Having reduced the genres to "hard" and "soft", he identified the former with the 'male' and the latter with the 'female'. In support of this, it was clearly not sufficient to quote the classic episode in which the singer Timotheus was driven out of Sparta because his chromatic tempered melodies made the young men effeminate and unsuited for warfare. "Nam ut foemina ad patiendum potissimum facta est, mas ad agendum, praesertim in generationis negocio, sic Molle genus passionibus animi foemineis, Durum actionibus virilibus accommodatur; ...". ["For as woman is made chiefly to be passive, and man to be active, especially in the act of generation, so the

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[98]Kepler 1619 [1969], III, pp. 44, 58, 60, 85, V, pp. 233–234; 1997, pp. 189, 212, 215, 254, 476–478; passim.

[99]Kepler 1619 [1969], III, Chap. IX–XIV; 1997, pp. 200–237. Dickreiter 1973.

[100]Kepler 1619 [1969], III, p. 74; 1997, p. 236.

'soft' kind is fitted for the feminine emotions of the mind, and the 'hard' kind for masculine activities; ..."] To prove this, then, he elaborated the explanation quoted at the beginning, which culminated in the expressive Fig. 8.5, where the 'soft' minor third stood for womanliness, and the 'hard' major third for virility.[101]

At the end of his *Les six livres de la république [The six books of the republic]* (1576), Jean Bodin (1530–1596) expounded a theory which used the three means, the arithmetic, geometric and harmonic mean, to explain and justify democracy, aristocracy and monarchy, respectively. In the "Political digression on the three means" at the end of "Book III", Kepler resumed, and criticised the Frenchman for various reasons, partly mathematical, but often ended up by finding that the harmonic means, like those found for the monarchies, gave the best results, also in political questions. Some historians or political scientists may still find some of its details interesting. Among these, it is worth underlining that in an authentic democracy, the fundamental criterion for choosing should be by drawing lots: a simple procedure, which would immediately improve the political class of the planet, and would solve countless problems. Beyond this, in our particular history of sciences, the digression would throw further light to clarify certain characteristics of our astronomer. Plato's *Republic* continued to make its presence felt, with all its weight, as did also, partly, Aristotle's *Politics*. Ratios still remained the fundamental criterion to understand the phenomena, in this case those of politics.

Music facilitated the ability to extend ratios and analogies to peace and war. "Hic cum Ptolemaeo (...) ex generibus harmoniarum & cantus, Molle quidem Paci daretur, Durum bello; dissidiaque generum eadem quae in civitate utrorumque temporum." ["Here in agreement with Ptolemy (...), of the kinds of harmonies and melody, the soft would in fact be given to peace, and the hard to war; and the dissensions between the genres are the same as those in the state of both times."] Thus, the dissonances played an important function, not only to give a colour to the melodies, but also in relation to the events of the world, in accordance with the plan of divine providence.[102]

And it was even more significant that, thanks to the ratios between numbers, the laws of the State and of jurisprudence merged in the pages with those of the whole cosmos. These were all guaranteed by God, and this thought gave rise to the habit of indicating as laws (of Kepler, of Newton, of inertia, of movement, of gravitation, ...) those rules, models, or relatively constant forms of behaviour in the movements of the planets, in vibrating strings and in other cases, too. We shall soon find the two last aspects again, in the other German natural philosopher and jurist of our history: Leibniz.

In "Book IV", our North European theoretician of music broke away from earthly sounds, to consider the stars, which influenced earthly phenomena by emitting rays. The harmony always remained the same, immutable and valid for everything, but now it changed its name: for sounds, the word 'octave' was used, for the rays

[101] Kepler 1619 [1969], III, p. 77; 1997, p. 240.

[102] Kepler 1619 [1969], III, p. 101; 1997, p. 276.

emanated by heavenly bodies in the same ratio, the term used was the 'aspect in opposition'. Before giving us, with relative exactness, particulars about the 'aspects' of the planets among themselves, in other words the angles formed between them, Kepler was anxious to point out that the relationships between sounds needed the mind and the soul in order to be achieved, otherwise they would remain separate. "… in genere relatio omnis, sine Mente nihil est, …" ["… in general, every relationship is nothing without the mind …"].[103]

Thus our natural philosopher appears to recognize the true harmonies, which the sense of hearing brought him, because their ratios, if correct, corresponded to the eternal, universal archetypes deposited by God in the soul and in the mind. These archetypes, or paradigms, revealed the pure, secret harmonies of things, and in order to attain them, of course, it was necessary to make use of the mathematics expressed in proportions. Kepler's archetypes did not exist outside the soul. One way or another, harmonies could be adequately represented only by regular figures inscribed within the circle, and present in the mind. For this reason, among Plato, Aristotle and Ptolemy, our mathematician preferred to cite the neo-Platonic Proclus at length. In him, he even found Christian cues, and he interpreted Plato's *Timaeus* as a comment on Moses' Biblical *Genesis*, transcribed, however, in accordance with Pythagorean philosophy. He even succeeded in remembering the *Acts of the apostles*, where Paul spoke to the Athenians about his God. The archetypes of all the numbers (1, 2, 3, 4, 8, 9, 27) and of the circle, as a model of all movements, would lie, according to him, in the soul.[104]

Of Aristotle, he rejected the image of the soul as a white sheet of paper waiting to be written on, but he shared his criticisms of the Pythagoreans. As regards the continuous magnitudes of geometry, for Kepler numbers remained secondary, and he refused their symbolic interpretations (present also in Plato and Bodin), like that of the climacteric number 7. The "Archetypalis Harmonia" he included in his figures, and these were to be "Neque tantum scibiles sed etiam scitas …" ["Not only knowable, but also known, …"] by the mind, because they dwelt in it. The senses were not sufficient, then, to establish their criteria. "Geometria ante rerum ortum Menti divinae coaeterna, Deus ipse (quid enim in Deo, quod non sit Ipse Deus) exempla Deo creandi mundi suppeditavit, & cum imagine Dei transivit in hominem: non demum per oculos introrsum est recepta." ["Geometry, which before the origin of things was co-eternal with the divine mind and is God himself (for what could there be in God which would not be God himself?), supplied God with patterns for the creation of the world, and passed over to Man, along with the image of God; and was not, in fact, taken in through the eyes."][105] From his treatise on optics, *Ad Vitellionem Paralipomena [Integrations to Witelo]* (1604), our Christian natural philosopher again assumed the sphere as the image of the Holy Trinity, Father, Son and Holy Spirit.

[103] Kepler 1619 [1969], IV, p. 109; 1997, p. 291.

[104] Kepler 1619 [1969], IV, pp. 114–117; 1997, pp. 298–302.

[105] Kepler 1619 [1969], IV, pp. 118–119; 1997, pp. 303–304. Tangherlini 1974.

By horoscopes, we mean the ascendants, or rather the aspects, that is to say the angles at which the planets and the constellations of the zodiac can be observed from the Earth at the moment of birth. About 800 of these, cast by our renowned astronomer during his whole lifetime, are extant. He proved to be particularly good at this, and met with success; consequently, he was sought, both for the compilation of almanacs with weather forecasts, and to foretell political and personal events. For 1618, he had foretold turbulences in May, and on May 23rd, the famous Defenestration of Prague marked the beginning of the Thirty Years War. In 1608, he was working for the famous general Albrecht von Wallenstein (1583–1634), horoscope included. Interpreting again the same horoscope at his request in 1625, Kepler predicted that the terrible confusion in the country of March 1634 (between Catholics and Protestants, with various armed forces, Swedish, Spanish, French, Italian, imperial, and others, running amok all over Central Europe) would have serious consequences for our imperial general: Wallenstein was assassinated on February 25th, 1634.

The event that this astronomer was not only capable of foreseeing the observable position of the planets with great precision, but even claimed to derive predictions about the cold weather in winter, and the destiny of men and empires, has always created embarrassment for historians of scientific progress, who are used to distinguishing astronomy from astrology. With Kepler, these experts are forced to leap acrobatically from one document to another, and from one written work to another, usually ending up by concluding that he was forced to do so, to earn a living at the court of the powerful. And yet, nevertheless, emperors and nobles often did not pay him the sums that they owed. But when he was about to choose a second wife for himself, he had no problem in admitting the influence of the stars on him.[106] Thus it is anachronistic to distinguish between Kepler, the astronomer, and Kepler, the astrologer.

Actually, he did not declare himself either in favour or contrary to astrology: the title *Tertius interveniens [A third possibility exists]* is already fairly indicative. He refused the determinism of judicial astrology, for which one's whole destiny is decided by the horoscope, but he admitted that the stars influenced earthly phenomena. On the other hand, Isaac Newton, if not Galileo Galilei, was to follow him in explaining the tides as due to the influence of the Moon on the sea. It was possible to believe in the influence of stars without falling into superstition; it was sufficient to understand the reasons of it.

His philosophy consisted of assigning the form of the circle also to the soul, in order to find in it those musical ratios justified by geometry, which would be the same also for the heavenly movements. "Valde enim confirmant hanc Philosophiam Geneses, cum videamus characterem confluentiae radiorum coelestium ad idem punctum, velut ex circulo communi imprimi in nascentis animum; ...". ["For this philosophy is strongly confirmed by horoscopes, as we see that the character of the concurrence of celestial rays at the same point, as if from a common circle,

[106]Kepler 1992. Kepler 1997, pp. 308–309.

is imprinted on the mind of a newborn; ...”].[107] The proof of this was that the harmonies touched the minds of everyone, including those who were ignorant of philosophy and geometry. Amorous attractions were thought to be of the same nature.

Depending on the angles at which the stars were observed from the Earth, the archetypes of the human soul would come into resonance with the heavenly ones, and were influenced by them. Kepler attributed a soul also to the Earth; consequently, as a result of the same reasons and the same mechanism, also meteorological phenomena on the Earth, such as clouds, wind, rain, heat and cold, were believed to be influenced by the astrological aspects. “In Terrae enim corpore ponenda est haec anima: quia nec anguli harmonici radiorum in ulla alia parte mundi quam in Terra existunt & opera Naturae, quae ad configurationes radiorum sequuntur, ex Terrae visceribus, montiumque cavernis ortum trahunt.” [“For this soul must be placed in the body of the Earth, as the harmonic angles of the rays exist in no other part of the world but on the Earth, and the works of Nature which follow upon the configurations of the rays take their origin from the bowels of the Earth and the caverns of the mountains.”][108]

However, our astronomer made a distinction between music, as played by musicians, and the alternation of the various angles in the movement of the planets. In the *continuum* of sounds, there were jumps from one note to another, and the infinite possible intervals between the two were passed over in silence. In the world of the planets, on the other hand, the discordant moments mixed with the harmonious ones, which happened only at particular instants, although they generated more lasting effects. “Breviter, configurationes praecinunt, Natura sublunaris saltat ad leges hujus cantilenae.” [“In brief, the configurations [of the planets] sing the leading part, sublunary Nature dances to the laws of this song.”][109]

Even if in a slightly different way than the one thought for music, our German-speaking astronomer-astrologer again used his geometrical ratios of the first two books, to select the aspects, that is to say, those configurations which influenced the earth: 180° (opposition), 90° (quadrature, from the square), 120° (trine, from the triangle), 60° (sextile, from the hexagon) and so on. He even took pains to remind us that the term “aspectus” derived from an Arabic translation of the Greek σχηματισμός [appearance], used by Ptolemy both “... in *quadripartito [Tetrabiblos, Four books]* ...” and “... in *Opere magno [Almagest, Great work]* ...” and “... inque *Armonicis [Armonikon, Harmony]* ...”. The abstraction of the Greek Schema became, for the Arabs, a far more concrete “face”: in German *das Angesicht*, which Italians would call – Kepler wrote – *Mascaras* [masks].[110]

Kepler specified here that no perfect correspondence existed between the musical consonances and the astrological aspects. They would have different origins, seeing

[107] Kepler 1619 [1969], IV, pp. 121–122; 1997, p. 308.

[108] Kepler 1619 [1969], IV, p. 131; 1997, p. 323.

[109] Kepler 1619 [1969], IV, p. 132; 1997, p. 325.

[110] Kepler 1619 [1969], IV, p. 133; 1997, p. 326.

that the circle that generated the former eventually had to be opened to become a vibrating string. Instead, for the latter, a more important role was played by the congruences of the second book between the figures, and in any case, an aspect was an angle. Thus, the only consonance that corresponded perfectly to an aspect remained the octave, generated by the division of the circle into two parts, which would also produce the opposition of 180°. On the other hand, the trine would not correspond to the fifth, but only to the interval of the octave plus a fifth; the quadrature would not correspond to the fourth, but to the double octave, and so on. Although the same numbers were not counted for the aspects as for the musical harmonious divisions, all the same a great affinity remained between the two, because they both were born from the "noble" figures of the circle.

For music, marriage would have been admitted – Kapler continued – between a noble patrician, like the fourth part of a circle cut away from the square, and a plebeian woman like the remaining three quarters. But with the aspects, there would be no female remainders, and thus no marriages could have been combined. "Contra in Meteorologia mos est alius. Nam ut quisque ipse nobilis seu ortu seu meritis (Scibilitate scilicet aut Congruentia) ita plurimum valet authoritate, reliqui volitant velut umbrae: foeminarum ratio habetur nulla." ["On the other hand, the fashion in the study of the Meteorology is different. For as each nobleman is worth himself by his origin or by his merits (that is, by the knowability or congruence), so [the aspect] has great power by its authority, and the rest flit like shadows. No account is taken of females."][111]

He previously wrote, for the *Ephemerides [novae motum coelestium] [New almanac of heavenly movements]*, that "... Creatorem Deum, aut ex Harmoniis Cantus infra Octavam, libro III descriptis, desumpsisse leges ordinandorum Aspectuum; aut ad coelestes Aspectus attemperasse aures hominis, Concordantiarum illarum judices." ["...: God the Creator either took the laws for ordaining the aspects from the harmonies of the music within an octave, which are described in Book III, or attuned the ears of man, which are the judges of those consonances, to the heavenly aspects."][112] But the author of these almanacs, however successful they were, must have had doubts about too close a correspondence between two partly different phenomena. Thus he searched more carefully for the causes. One reason was that he wanted to make a better defence of his third position, in other words intermediate between the out and out denigrators of astrology and the followers of judicial astrology. Among the former, he quoted also Giovanni Pico della Mirandola (1463–1494). Thus he had partly modified what he had written, starting from 1606 in *De stella nova [On a new star]*.

To combat the adversaries of both sides, Kepler endeavoured to demonstrate that the Earth possessed a soul, like living creatures. because certain characteristics of this soul could be observed externally. In this. he was similar to Leonardo da Vinci, who, however, he could not gain a good knowledge of, because his codices were not

[111] Kepler 1619 [1969], IV, p. 155; 1997, p. 355; translation different from ours.

[112] Kepler 1619 [1969], IV, p. 151; 1997, p. 350.

yet in circulation in his times.[113] Among the arguments proposed by Kepler, which are most convincing for us today, because they have been taken up by the subsequent orthodox evolution of sciences, we may read: "… ille mirabilis fluxus & refluxus Oceani semidiurnus, qui etsi ad Lunae motus se accomodat, sic ut probabile mihi sit visum, in Praefatione commentariorum Martis, undas a Luna trahi, ut ferrum a Magnete, virtute corporea unitionis corporum; …" ["… the wonderful twice daily ebb and flow of the ocean; yet it fits in with the movements of the Moon in such a way that it seemed probable to me, in the preface to my *Commentaries on Mars*, that the waves are drawn, as iron is by a magnet, by a corporeal virtue of the combination of bodies; …"].[114] Among the most vertiginous and bizarre analogies, here we find rocks and minerals that spout from the bowels of the earth, as if from a pregnant woman, with figures and images impressed on them like those present on newborns.

Of course, also the soul of the Earth has a circular form, like the zodiac, containing its regular archetypes. He preferred to call the essence of the soul $'\epsilon\nu\epsilon'\rho\gamma\epsilon\iota\alpha\nu$ [energy], which he compared to the flame in movement, and to God. "Deus quippe est substantialis Energia, …" ["God, of course, is substantial Energy, …"].[115] This is how the circle of the zodiac could succeed in influencing not only the circular soul of man, but also the meteorological phenomena of the earth.

As an example, our astrologer-astronomer offered his readers his own horoscope, comparing it with his capacities. Briefly, he was a Capricorn with a strong presence of Jupiter, so much so that "… magis delector Geometria in Physicis rebus expressa, quam abstracta illa, & Saturni siccitate prae se ferente, magis inquam Physica quam geometria; et quod Luna gibba in clara frontis Taurinae constellatione impleat Animae Phantasticam facultatem imaginibus, quarum tamen multas Naturae rerum consentaneas re ipsa expertus sum, velut ex Procli Paradigmatibus delapsas." ["… I delight more in geometry expressed in physical things than in the abstract, and showing in its appearance the dryness of Saturn, more, I say, in Physics [natural philosophy] than in geometry; and because the gibbous Moon in the famous constellation of the brow of Taurus would fill the fantasizing faculty of the soul with images, though I have experienced that many of them agree with the nature of things, as they have been taken from the patterns of Proclus."]

But he also added immediately that astral influences by themselves would not be sufficient to explain many other important events that had happened to him, which depended, rather, on circumstances like the studies carried out, the places where he had lived, the emperors he had met, and other things, too. The only effect of the horoscope "… instigavit animum ad laborem indefessum, auxitque desiderium sciendi; breviter, non inspiravit animum non ullam dictarum hic facultatum, sed excivit." ["… urged my mind on to untiring toil, as well as increasing my desire for knowledge. In short, it did not inspire my mind, or any of the faculties stated

[113]In out great epoch, the Earth, considered as a single, complex living organism, has been resumed, and dubbed the 'Gaia hypothesis'.

[114]Kepler 1619 [1969], IV, p. 162; 1997, pp. 365–366.

[115]Kepler 1619 [1969], IV, p. 163; 1997, p. 367; here *Energeia* has been translated as "activity".

here, but roused it."] As a direct attack on deterministic astrology, Kepler described a person, born under his same constellation, who first of all was a woman "… inquietissimo sane ingenio, sed quo non tantum nihil proficit in literis (non mirum hoc est in foemina), sed etiam totum turbat municipium suum, sibique author est miseriae deplorandae." ["… a temperament which was certainly very restless, but from which she not only draws no advantage in book learning (that is not surprising in a woman), but she also disturbs the whole of her town, and is the author of her own lamentable misfortune."][116]

This is enough as regards astrology, but Kepler's character will continue to accompany us, because he took pleasure in revealing it also through more scientific arguments. We can easily imagine that all this brought into discussion the complex religious problems of the age, including predestination, free will, original sin, grace, etc.: all elements disputed by Catholics, Lutherans and Calvinists. Which God did Kepler believe in? He quoted Luther's horoscope, in the version of Girolamo Cardano, deriving from the conjunction of many planets (Venus, Mars and Jupiter) his authority over the people and his favour with princes. He ended "Book IV" with a prayer to the Triune God for the salvation of souls. At the beginning of "Book V", he confessed "… me vasa aurea Aegyptiorum furari, ut Deo meo Tabernaculum ex ijs construam, longissime ab Aegypti finibus." ["… I am stealing the golden vessels of the Egyptians to build a tabernacle to my God from them, far, far away from the territory of Egypt."] The sentence was a clear reference to the *Bible*. But how to restrain ourselves from interpreting it as indicating that he, like a new Moses, had stolen the astronomical observations of Tycho Brahe, to construct a different planetary system?

Now, more than 20 years later, Kepler thought back over all his astronomical studies. He made modifications where necessary: "… pluribus sit opus principijs, praeter quinque regularia corpora." ["… more basic principles are needed in addition to the five regular solids."] Then he again took up his initial work, *Mysterium cosmographicum*, to correct the proportions between the distances of planets, and to justify them in another way. Among other things, he seized the opportunity to create marriages between the Platonic solids. By transforming (in duality) the faces of one into the vertices of the other, and attributing a sex to them, the vertices of the male solids, like the cube and the dodecahedron, corresponded to, and could enter into, the faces of the female octahedron and icosahedron. Instead the tetrahedron was transformed into itself, and was thus a hermaphrodite (Fig. 8.6).[117]

As in the Greek and Latin tradition still present in Maurolico, our Central European astronomer combined the numbers of the vertices, the edges and the faces in the ratios of music. But as he had been forced, by the precise observations of Tycho Brahe, to abandon circular orbits in favour of the eccentric ones of the ellipse, he was in search of other elements related to the movements of the

[116]Kepler 1619 [1969], IV, pp. 170–171; 1997, pp. 376–378 and pp. xxi–xxiii. Cf. Kepler 1984, pp. 87–90. Cf. Caspar 1993, pp. 76, 181–185 and passim.

[117]Kepler 1619 [1969], V, pp. 179, 181, 187; 1997, pp. 391, 396–397, 407.

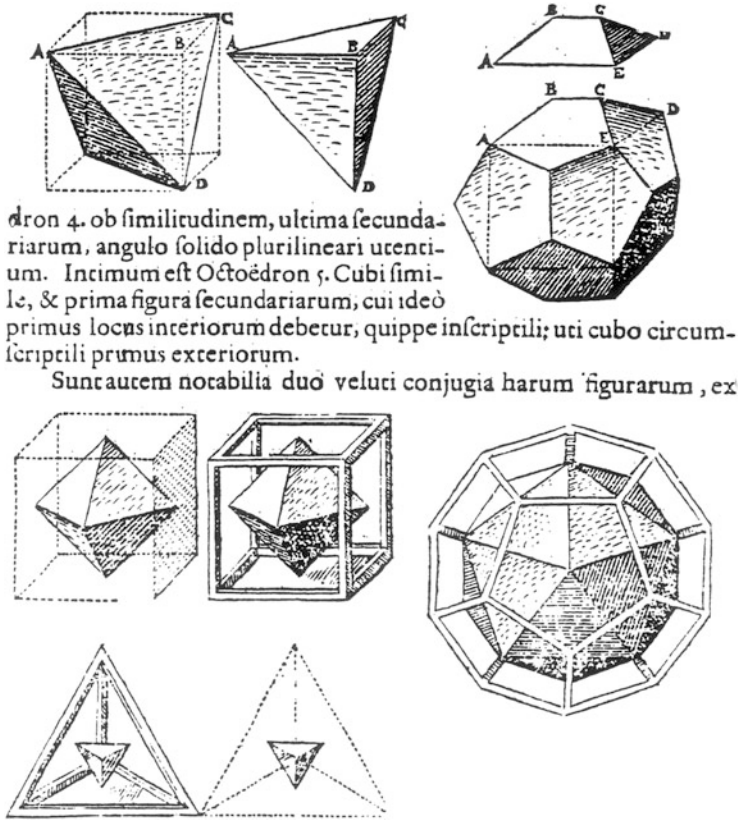


**Fig. 8.6** How Kepler coupled the regular polyhedra, attributing male, female and hermaphrodite properties to them (Kepler, ibid. lib. V p. 181)

planets, that could reconstruct the desired harmony. In solving this problem, he also nonchalantly corrected the ratio that he had written in *Astronomia nova*, where the times taken by planets to complete one full orbit had been made to increase with the square of the mean distance from the sun. They would have increased too much as they moved away from our star, and as a result, here in the *Harmonices* … the proportion became the sesquialtera (intermediate between the linear proportion and the quadratic one) of the third law, as we have already seen at the beginning of this section.[118]

Anyway, faith in the music of the spheres continued to burn brightly in the heart of our German mathematician. He was now only deciding what notes the planets should emit if they moved in accordance with the new laws invented by him. The heavenly harmonies would not be lost, because they would be found here

[118]Kepler 1619 [1969], V, p. 189; 1997, p. 411.

among the ratios of angular velocity. He excluded the periodic times, the mass and the planetary distances, because among these physical elements he could not find any harmonious ratios, though he continued tenaciously to search for them. “Cum autem Deus nihil sine Geometrica pulchritudine constituerit, quod non ab alio priore quadam necessitatis lege sit nexum; … […] … oportet igitur vel in his Moris, vel si quid est ijs prius in Mente Opificis, inveniri concinnitates Geometricas.” [“Since, however, God has established nothing without geometrical beauty unless it is bound up with some other, prior law of necessity, … The geometrical harmonization must therefore be found either in these times, or in something prior to them in the mind of the Maker.”][119]

Between the maximum distance of the Earth from the Sun (aphelion) and the minimum distance (perihelion) of Venus, there would exist even the ratio 1,018:719, which, equalling 1.4158, would be a good approximation of $\sqrt{2}$:1, that is to say the *semidiapason*. However, Kepler ignored it, like other approximations to the octave and to the fifth, because he was searching, rather, for harmony between the movements, as music is born from these. In the end, being he guided by instinct and by reason, he had the fate to find the notes among the angles followed every day by the planets, as seen from the Sun: the source of all movement. “… ut ita haec apparentia, beneficio luminis ad corpus Solis perlata, cum ipso lumine recta in creaturas, instinctus hujus participes, sic influere possit: sicut libro quarto, schema coeli in foetum, beneficio radiorum diximus influere.” [“… so that this appearance, brought by the agency of light to the body of the Sun, can along with light itself influence directly the living creatures, who share in this instinct, just as in the fourth book we have stated that the pattern of the heavens influences a foetus by the agency of the rays.”][120]

Effectively, among the seconds of the arc covered every day by the planets, our obstinate and skilful mathematician succeeded in identifying major and minor thirds, fifths, octaves and other intervals. For example, between Saturn at its perihelion and Jupiter at its aphelion, the ratio $2'15''$:$4'30''$ of the octave; between the aphelion and the perihelion of Mercury $38'1''$:$25'21''$ ($2{,}281''$:$1{,}521''$) the fifth. The fourth can be found in the movement of the Moon. Not all the intervals prove to be perfect, but – Kepler wrote – it would have been difficult to perceive the margin, although in the case of Mars and Jupiter, it arrived at a diesis. Jupiter and Mars also would produce the dissonant interval 1:7. Between aphelion and perihelion, each single planet was said to have produced one note after another in monody, like the ancient Greeks. Modern (in that period) polyphony, instead, comes from combining the various planets together.

Our Central European astronomer translated the ratios between angular velocities of the planets into the notes of the scale, which he then transferred to the stave, distinguishing the hard genre from the soft one. There would no longer be any reason to marvel at the beautiful melodies composed by men, because they would

[119] Kepler 1619 [1969], V, p. 194; 1997, p. 419.

[120] Kepler 1619 [1969], V, p. 198; 1997, p. 424; translation a little different.

do "... nihil aliud quam Dei Creatoris simias agere & ludere veluti drama quoddam ordinationis motuum coelestium." ["... nothing but aping God the Creator, and, as it were, acting out a particular scenario for the ordering of the heavenly motions."] In this way, Kepler assigned to the planets the melodies of Fig. 8.4 and to the Earth, a strident 'soft' semitone, better interpreted as *mi fa mi*.[121]

Having discovered the "... genuinus Archetypus fabricae Mundanae ..." ["... true archetype of the construction of the world ..."], our German-speaking mathematician then addressed the "modern" musicians of his time, to propose to them "... prima universitatis exempla genuina ..." ["... the first true offprints of the universal whole ..."] offered by nature. As he declared, having read Vincenzio Galilei's *Dialogo della musica antica et moderna* [Dialogue between ancient and modern music], modern composers were, for him, the musicians of polyphony with several voices, those composers that the Tuscan was pushing to one side in that period, to make room for the monodies inspired by the ancient Greeks. Kepler, in fact, had referred several times to the Flemish polyphonist, Roland de Lassus, whose compositions, "In me transierunt", "Ubi est Abel" and "Tristis est anima mea", he quoted in his text, together with notes and titles".[122]

Roland de Lassus (1530/32–1594) had flooded Europe, from Italy to the Flanders, with his 2,000 sacred and profane compositions, madrigals, motets and masses. Having eventually settled down at Munich, he became the precentor of the Catholic Counter-Reformation. He concentrated, in time, on writing spiritual madrigals, and became so famous in his epoch that he was called 'the prince of music'. In the years of his education, and often subsequently, he visited Italian musical environments, which were already enriched by the presence of other famous Flemish composers. He had met not only Giovanni Pierluigi da Palestrina, but also Adrien Willaert, Cyprien de Rore or Jacques Archadelt. Even the Protestant Churches were glad to have contact with him. If necessary, they changed the texts to be set to music. "La lettre, accommodée à la Musique d'Orlande ... estoit sotte, lascive et profane, ... j'ai rendu ces chansons honnestes, et chrestiennes pour la pluspart ..." ["The text set to music by Roland ... was frivolous, lascivious and profane, ... I have made these songs honest and Christian for the most part ..."]. For this characteristic of his, he would undoubtedly have been appreciated also by Kepler.

And yet that fine tenor voice from Mons represented the peak of polyphonic culture of the sixteenth century; at the same moment, the radical innovation of the accompanied monody was appearing on the European scene, as elaborated by Vincenzio Galilei, by the Camerata of the Bardi, and lastly by Claudio Monteverdi (1567–1643). Two aspects present in the work of Lassus had interested Kepler. The first was the profound Christian faith, transcending a fleshly reality, though there was no desire to renounce its joyful presence. Together with about 50 Masses, a 100 or so Magnificats and innumerable penitential motets, our musician had added profane motets, Italian madrigals, popular Villanelle and French songs about

[121] Kepler 1619 [1969], V, pp. 204, 207; 1997, pp. 435–436.

[122] Kepler 1619 [1969], V, p. 208, III pp. 64, 72, 75 and 84; 1997, pp. 441, 221, 234, 239 and 253.

licentious topics. Preferring to set verses of Petrarca to music, we may identify the second aspect as a common classical-style platonism.[123]

Right from the moment of creation, God was thought to have made the world sing in polyphony with several voices, the same that whispered to the human mind through the ears. Thus, astronomic tables in hand, our religious philosopher prepared to demonstrate what music the Maker had composed for the different planets. He even invited composers to make use of it. "Anne flagitium fecero, si a singulis hujus aetatis Componistis artificiosam aliquam Motetam pro hoc Elogio exigam; Textum huc aptum Psaltes Regius, caeterique Sacri libri suppeditare poterunt. Verum heus vos in coelo plures quam sex non concordant. … […] … Qui propius Musicam coelestem exprimet hoc opere descriptam, huic Clio sertum, Uranie Venerem sponsam spondent." ["Shall I perhaps be committing an abuse if I demand some ingenious motet from individual composers of this age for this declaration? The royal psalter and the other sacred books will be able to supply a suitable text for it. Yet take note that no more than six parts are in harmony in the heavens. … If anyone expresses more closely the heavenly music described in this work, to him Clio pledges a wreath, Urania pledges Venus as his bride."][124]

Between Mars, the Earth and Mercury, consonant harmonies occurred quite often, but to obtain them between four or five planets, it would be necessary to wait for centuries, or tens of thousands of years. As a result, the consonant harmony between all six would occur only once, at the beginning of time, and would characterise creation. Among the various angles covered every day at the various points of their orbit, Kepler found those that generated the chords $mi - sol - si$ and $do - mi - sol$ in the hard genre, and in the soft genre, $mi\flat - sol - si\flat$ and $do - mi\flat - sol$. The harmonies were classified as hard and soft, because they referred to the male Earth or to the female Venus. If it were possible to silence this talkative goddess who is an eternal source of disagreement, with only five planets, and thus only five voices, the chords obtained would be $sol - si - re$ and $sol - si\flat - re$.[125] Our very human imperial mathematician must have really fallen in love with his sexual reading of the two genres, already seen above and re-proposed here. Between Earth and Venus, there would be discord, when the husband accomplished his virile duties, while Venus remained distant in her gynaeceum. Yet sometimes the planet Earth descended in its perihelion towards Venus "… voluptatis causa, positis clypeo paulisper & armis operibusque viro convenientibus: tunc enim Harmonia mollis est."

[123]Borren 1944, p. 65 and passim.

[124]Kepler 1619 [1969], V, p. 208; 1997, p. 441. As I have done so far, at times I will modify the 1997 translation to make it closer to the Latin text, without continuing to point this fact out.

[125]The chords lend themselves to a modern harmonic interpretation as 'hard' major tonalities or 'soft' minor ones. However, this would be anachronistic, because Kepler referred to the Franco-Flemish polyphony that preceded the harmonic revolution of Bach and Rameau: from the Greek-ecclesiastical modes to major and minor tonalities.

["… so as to make love, laying aside for a little while his shield and arms, and those tasks which are proper for man: for then the harmony is soft."][126]

In our mediocre, wretched times, however, dissonances could not have been avoided. "Nihil igitur aliud sunt motus coelorum quam perennis quidam concentus (rationalis non vocalis) per dissonantes tensiones, veluti quasdam Syncopationes vel Cadentias (quibus homines imitantur istas dissonantias naturales) tendens in certas & praescriptas clausulas, singulas sex terminorum (veluti Vocum) ijsque Notis immensitatem Temporis insigniens & distinguens; …". ["Therefore, the motions of the heavens are nothing but a kind of perennial harmony (in thought, not in sound) through dissonant tunings, like certain syncopations or cadences (by which men imitate those natural dissonances), and tending towards definite and prescribed resolutions, and towards each of those six terms (as with vocal parts) marking and distinguishing by those notes the immensity of time; …"]. This is the reason why man, "… Creatoris sui Simia …" ["… aping his Creator …"], invented in the end that polyphony which was unknown to the ancients. "… ut scilicet totius Temporis mundani perpetuitatem in brevi aliqua Horae parte, per artificiosam pluriumque vocum symphoniam luderet, Deique Opificis complacentiam in operibus suis, suavissimo sensu voluptatis, ex hac Dei imitatrice Musica perceptae, quadam tenus degustaret." ["… so that he might play, that is to say, the perpetuity of the whole of cosmic time in some brief fraction of an hour, by the artificial concert of several voices, and taste, up to a point, the satisfaction of God, his Maker, in His work, by a most delightful sense of pleasure felt in this music, imitator of God."][127]

For Kepler, Saturn and Jupiter would sing bass parts, Mars tenor, Earth and Venus contralto, and Mercury soprano. Having excluded from his theory both the distance from the sun and the mass of the planets, he tried to justify it through the number of the notes attributed to them. As everything was established by God by fixing the archetypes, in the end our German philosopher abandoned astronomical measurements to show that the planets must follow those exact ratios, and not other possible ones. The style remained that of Euclid, with axioms and propositions, but often the arguments recalled the Pythagorean sects, Platonic apriorisms and Aristotelian logic. *Breviter*, Earth and Venus would show small-scale eccentricities in their orbits, for the precise purpose of being able to sing major and minor sixths, of the hard and soft genres. Vice versa, Mars and Mercury should have to possess greater eccentricities, because of their intervals. Saturn and Earth would espouse the hard genre, Jupiter and Venus the soft one. In short, our Lutheran astronomer's idea was that the eccentricities observed in the orbits of the planets derived from the musical harmony fixed by God at creation.

Music gave the harmonic ratios, and from these, Kepler now derived also the distances of the planets from the Sun, which he compared with the measurements, "… qui omnes valde prope accedunt ad illa intervalla, quae inveni ex observationibus Braheanis." ["… all of them approach very closely the distances which I found

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[126]Kepler 1619 [1969], V, p. 211; 1997, pp. 446.

[127]Kepler 1619 [1969], V, p. 212; 1997, pp. 446–448.

from the observations of Brahe."] There was a "… parvula differentia …" ["… tiny difference …"] only with Mercury, which the imperial mathematician justified by the scarcity of the relative astronomical observations.[128]

He explained here his preference in the end for the musical ratios of harmony to geometrical proportions, not on the basis of astronomical data, but by means of general philosophical arguments. Geometrical proportions are infinite, whereas harmonies are few. The former include also the incommensurable ratios. "At Harmonicae proportiones omnes sunt effabiles, omnium termini commensurabiles; ijque desumpti ex figurarum planarum certa & finita specie. Infinitas vero sectionis materiam, commensuratio vero seu Effabilitas terminorum formam repraesentat. Ut igitur materia Formam, ut rude saxum justae quidem quantitatis Ideam humani corporis, sic Geometricae Figurales proportiones Harmonias appetunt; …". ["But the harmonic proportions are all expressible [constructible], and the terms of all of them are commensurable. Also, they have been taken from a definite and limited class of plane figures. Now infinite divisibility signifies matter, but commensurability or expressibility [constructibility] of terms surely signifies form. Therefore, as matter strives for form, as a rough stone of the correct size indeed strives for the idea of the human form, so the geometrical proportions in the figures strive for harmonies."][129]

Consequently, Kepler did not take into consideration the rival musical theory of Aristoxenus, but rejecting it as we have seen above, because it was based on the incommensurable division of the tone and the octave. He had direct knowledge of this ancient Greek, because his manuscript had recently been published, together with those of Ptolemy, in the edition of the *ΑΡΜΟΝΙΚΑ* that he used.[130] Here in *Harmonices mundi* …, Aristoxenus was mentioned only twice: as a source used by Vincenzio Galilei to tune the lute in the equable manner, or so that Kepler would not be confused with the Greek musician when he, too, considered 12 intervals, which actually he did not make all the same.[131]

The whole argument, about how heavenly musical harmonies guided the movements of the planets, in the end led to the Sun. "Nam uti Sol in seipsum revolutus, per emissam ex se speciem, movet Planetas omnes, sic etiam mens […] omnes facit intelligi." ["For as the Sun, in its revolution about its own axis, moves all the planets by emitting a glance from itself, so also the mind … causes all things to be understood."] "… quietem Solis in centro mundi, pulchro responsu consequitur intellectionis simplicitas, …" ["… from the immobility of the Sun at the centre of the world follows the simplicity of the understanding by a beautiful relation …"]. "… in Sole vero Intellectum simplicem *πῦρ νοερόν* seu Νοῦν habitare, omnis Harmoniae fontem, quicunque ille sit." ["… surely on the Sun dwells simple

[128] Kepler 1619 [1969], V, p. 240; 1997, p. 486.

[129] Kepler 1619 [1969], V, p. 242; 1997, p. 489.

[130] Kepler 1997, p. 131.

[131] Kepler 1619 [1969], III, pp. 48 and 67; 1997, pp. 197 and 226.

Understanding, the 'intellectual fire' or 'mind', the source of all harmonies, of whatever kind they may be."][132]

"The epilogue in the Sun" was called "conjectural" by Kepler. He was trying to explain his philosophy of considering the Sun as immobile. And yet to us this would seem to be, rather, a chapter about God and the Christian religion. "IN SOLE POSUIT TABERNACULUM" ["In the Sun He has placed His tabernacle"] was the same Psalm of the *Bible* used by Galileo Galilei to defend heliocentricism.[133] Would the Central European Lutheran fare better or worse than the Tuscan Catholic?

We could easily expect an astronomer of the seventeenth century, who was a follower of Copernicus, to move away from the planetary model of Ptolemy. But the event that he continued to take his inspiration from him to continue his programme of research on the music of the spheres should come as a surprise to us. "… hic Ptolemaeus ante 1,500 annos materiam mei lib. V propriam tractare instituerit, si per suam astronomiam potuisset." ["… here [in chapter XV of *APMONIKA*] Ptolemy, 1,500 years ago, would have set about handling the subject matter belonging to my Book V, if he had been able to, through his own astronomy."][134]

That is to say, Ptolemy would already have started to explain the movements of the planets through music, although he would not have succeeded in this. Kepler, on the contrary, could now do so, because he had discovered the cause of movements in the Sun, and the reason for consonances between notes, no longer in the numbers of Pythagoreans, but in regular polygons.

The falsity of Ptolemaic astronomy was revealed in the "… imperfectio Symbolismorum illorum …" ["… imperfection of his symbolisms …"]. "… contra Ptolemaeum disputo, symbolismos scilicet illos potissima parte non esse necessarios, nec causales, nec naturales, sed poëticos potius & oratorios." ["… I disagree with Ptolemy, arguing of course that those symbolic associations are for the most part not necessary, because neither causal, nor natural, but rather poetic and rhetorical …"]. "… ostendi Ptolemaeum luxuriare exercitio comparandi poëtico vel oratorio." ["… I have shown Ptolemy exaggerates in using poetic or rhetorical comparisons."][135]

Kepler sustained that he had demonstrated the "poetic" conjectures about the music of the spheres handed down by the Pythagoreans and Plato, and taken up in the end by Ptolemy. How seriously our imperial mathematician took the astronomer-astrologer-musicologist from Alexandria can be even more clearly understood from his project of translating his works, commenting on them, and completing them

132 Kepler 1619 [1969], V, p. 247; 1997, p. 496.

133 Kepler 1619 [1969], V, p. 245; emphasis in the text. Kepler 1955, v. XVII, p. 326. Tangherlini 1974, p. 166. See above 8.2.

134 Kepler 1619 [1969], V, p. 251; 1997, p. 503.

135 Kepler 1619 [1969], V, pp. 249, 250, 251; 1997, pp. 500–502.

analytically, although this was only started, and not concluded. Unsatisfied with the current translation, he borrowed the Greek original.[136]

Among the natural philosophers of the seventeenth century, Kepler was not the only one who showed faith and interest in the music of the heavenly spheres; for example, the Jesuit Athanasius Kircher (1602–1680) has left us his *Musurgia Universalis*.[137] But he, like Ptolemy, did not meet with the unconditional approval of Kepler. His explicit criticism was reserved for the doctor from Oxford, Robert Fludd (1574–1637). This English scholar dealt also with practical music, whereas the Central European scientist studied above all the theoretical aspects. The former dressed himself in a certain obscure symbolism, typical of chemists and alchemists, and of the hermetic tradition of Paracelsus, whereas on the contrary, the latter – Kepler repeated here – sought the causes. Our mathematician, in the pay (so to speak) of the Holy Roman Empire (the Habsburgs), investigated, through the soul, the "noble theme" that the whole cosmos could be found in man, and that the Earth was a living organism. The English doctor dressed up the correspondences between Microcosm and Macrocosm with a formal rhetoric. Like the ancients, he continued to construct his cosmic music on the abstract numbers of the Pythagorean sects, and subsequent variants, down to the Rosicrucian sects. Their music of the spheres sounded different, both in style and in their results, and for a few years even a written dispute developed between the two.[138]

Kepler also mentioned Cicero's *Somnium Scipionis* with reference to the music of the heavenly spheres.[139] But now, he had substituted his dreams on this subject with his own mathematical proofs. He dreamt, instead, about people living on the Moon, as he wrote in his *Somnium seu opus posthumum de astronomia lunari [Dream, or posthumous work about lunar astronomy]*. He imagined that he was observing from there the movements of the Earth and the Sun, in support of Copernicanism. "This is the gist of all the *Somnium*: proving the movement of the Earth, or confuting the contrary arguments, based on subjective perception."[140] Experience by itself, and crude observation by itself were, for Kepler, "ignorant experience". Science would be born when this ignorance is allowed to die, and is, rather, the fruit of theoretical imagination.[141] His *Somnium* is thus to be compared with the *Discours de la méthode* by Descartes, and with the numerous subsequent scientific methods emphatically flaunted by a host of philosophers and science historians. All of his precious astronomical observations were not sufficient for Tycho Brahe, who,

[136]Kepler 1619 [1969], V, pp. 249–251; 1997, pp. 130–131, 190, 390–391 and 499–503. Stephenson 1994, pp. 37–38 and 98–117.

[137]Kircher 1650.

[138]Kepler 1619 [1969], V, pp. 251–254; 1997, pp. 503–508. Baumgardt 1951, pp. 181–182. Caspar 1993, pp. 290–293. Pauli 1955. Ammann 1967.

[139]Kepler 1619 [1969], IV, p. 106; 1997, p. 284.

[140]Kepler 1984, p. 117; cf. 137.

[141]Kepler 1984, pp. 67 and 75.

according to the imperial mathematician, did not possess sufficient imagination to detach himself from the ancient planetary systems.

The *Somnium* was, at the same time, Kepler's first and his last work. His first, because he penned the most ancient version of it in 1593, when he enrolled in the University of Tübingen, his last, because he continued to comment on it for the rest of his life, in 1609, 1611, and above all after 1620, 1621, 1622–1624, 1627, until his death in 1630, while he was still trying to have it printed. But it was only published posthumously by his brother-in-law and his son.[142] Yesterday (like today), when it was a risk to express one's opinions freely, it was better to take refuge in satire.[143] This does not make this booklet insignificant or misleading, but on the contrary, it makes it a priceless mine of information to arrive at the deepest convictions of the imperial mathematician. And actually, the clear defence of the Copernican system was immediately censured by all the Protestant professors of Tübingen (except one), who were as liberal and tolerant as their Catholic colleagues (and as those of today).

Thus, the Moon greatly attracted Kepler, who felt that its influence was exerted over the sea, over every body and every other thing, just as he believed that the Sun did, as well. "I define gravity as a capacity for mutual attraction, like the attraction of magnets. But the force of this attraction is greater between bodies that are close together than between those that are far away".[144] He imagined, by analogy with magnetic attraction, a universal gravitation, to which Isaac Newton was to give a quantitative formulation. The greatest force was exerted when the heavenly bodies were lined up in conjunction, 'with syzygies' as astronomers say. Our German natural philosopher, who wrote in Latin, used the term "copulis" ["with copulations"] for this phenomenon.[145] As he had elsewhere assigned sexual attributes to the planets, we may suspect that he was alluding here to more common relationships and unions of a completely different kind, those that the word has maintained in English.

Anyway, he dreamt. He dreamt that the inhabitants of the Moon called their country Levania, as in the Hebrew language. Considering the Earth from there, a planet four times bigger, which revolved rapidly on itself, he gave it the name Volva, from the Latin *volvo*, [to rotate]. But once again, our imperial mathematician chose, for his satire, a word that presented a salacious 'double entendre', also in Latin, which it is not necessary for me to spell out. We all know that in our dreams, we can take the liberty of behaving in ways that would be repressed if we were awake, or permitted only in analysis.

Our Lutheran natural philosopher considered the "… vow of virginity to be contrary to natural inclinations …" and if one would be forced to observe it, he should be cured with a "… well-tried medicament …: continuous, intense, ardent

[142] Kepler 1984, pp. 15–22. Caspar 1993, pp. 351–353.

[143] Kepler 1984, pp. 22, 70, 97, 99, 107, 109.

[144] Kepler 1984, pp. 57, 102–105, 161.

[145] Kepler 1984, p. 161.

speculation."[146] In his allegory, what was Kepler alluding to? To his own impossible dreams? To the rules imposed by the Catholic Church on its religious orders? To that joyful coupling of planets in their dance around the Sun, sustained by the Copernican system, but included among the mortal sins by the geocentric system?

In general by him, the Muses and the sciences were considered "... gentle and innocuous ...", above all astronomy. But then Kepler specified, in order to avoid any accusation of lying, their negative aspects, too. "... medicine also teaches poisons, ..., metaphysics, ..., disturbs the dogmas of Catholicism with too many inappropriate subtleties, ..., astrology sponsors superstition; optics deceives; music serves love; ... arithmetic serves avarice."[147] He did not approve, therefore of all music; rules governing it should exclude the kind capable of influencing illicit passions. And here we are bound to hear echoes of Plato, Guido D'Arezzo, or Cardano, criticising music that was capable of corrupting young people.[148]

With the assistance of his witch-mother and the Muse Urania anyway, our astronomer-astrologer travelled to the Moon, and was able to observe his beautiful Volva (*alias* the Earth) which moved joyfully in various ways, while Levania (*alias* the Moon) would appear to be immobile. And then all immobilities would be apparent and deceptive. Backed up by an endless classical culture, the ability shown by Kepler in speculating on the phenomena of the Moon, and how they might be seen from there, can but leave us wondering in admiration. However, it is impossible not to notice that his argument cut both ways, and could be overturned to argue against heliocentricity. Of course, it would have been possible to dream of travelling to the Sun, and observing from there that the Earth was moving. However, why should the inhabitants of the Sun enjoy a special status called reality and truth, (the Sun stands still while, instead, the Earth moves), while the inhabitants of the Earth should suffer from the deception of appearances (the Sun moves and the Earth remains motionless)? Kepler did not go so far in his hot speculations as to distribute a certain value equally to the various points of view. For him, the viewpoint from the Sun could not be appearance, like the others, whether they were based on the Earth or on the Moon, because there, God had placed His tabernacle, as we have seen above. It was undoubtedly a question of flattery, in order to obtain help in collecting the eternal credits from the Court of Prague, that in 1618, Kepler even placed the councillor Matthias Wackher on the Sun. "If I were a pagan, ignorant of Christian teachings, I would say that on the Sun – the excellent globe of the universe – there would be a refuge open to all Wackher [valorous] souls".[149]

Albert Einstein (1879–1955), born three centuries after our imperial mathematician in the same region of southern Germany, he too dreaming of moving amid improbable reference systems, was to speculate about similar problems, but he was to invent the absolutes of Relativity.

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[146] Kepler 1984, p. 109.

[147] Kepler 1984, p. 83.

[148] See above, Volume I, Sect. 2.3, 6.2, 6.5.

[149] Baumgardt 1951, pp. 130–131.

Our Lutheran natural philosopher was not successful with his Levanias and his Volvas, which continued to mean something else, at least in neo-Latin countries. On the contrary, the term "satellite", introduced by him, was adopted with its current meaning, like the word "focus" for the point of the ellipse occupied by the Sun.[150]

*Strena seu de Nive Sexangula [Gift, or on hexagonal snowflakes]* was a booklet in which Kepler explained why snowflakes exhibited forms with a hexagonal symmetry. Here, he was looking for geometrical archetypes in the hexagonal cells of bees, in pentagonal flowers, and in the rhombohedral berries of the pomegranate. "Geometria est archetypus pulchritudinis mundi" ["Geometry is the archetype of beauty for the world"].[151] Our imperial mathematician distinguished the numerical symbols of the kabbala from the geometrical archetypes, in a letter of 1608 written to Joachim Tanckius. With the former he might play, in the latter he firmly believed, searching for their effects on phenomena. "But the geometry of [astrological] aspects is an objective cause, which impresses a certain impetus on the underground archaeum, from which all the phenomena mentioned in general derive; at times one, and at times another, depending on the circumstances."[152]

"… ne in Nive quidem hanc ordinatam figuram temere existere credo." ["… I do not believe that this orderly figure may exist by chance, in snow too"]. He connected the order impressed, even on the snow, with the soul, the image of the Creator. But in living creatures, order would derive from a purpose, which eliminates chance, remaining the reason. What could the purpose of snow be? "Respondeo, Rationem formatricem non tantum agere propter finem sed etiam propter ornatum, nec solum tendere ad corpora naturalia efficienda, sed etiam solere ludere in fluxis, quod multis fossilium exemplis patet. Quorum ego universorum rationem à ludicro (dum dicimus Naturam ludere) ad hanc seriam intentionem transfero: quod puto, Calorem qui hactenus tutabatur Materiam; ubi à circumstanti frigore vincitur: …" ["I answer that the creative reason does not act only for a purpose, but also to embellish; it does not tend only to produce natural bodies, but it also enjoys itself, flickering lightly, as we can see in many examples of minerals. The reason for which I transfer in general from a joke (we say then, that nature plays its jokes) to this serious intention: as I believe that the heat which so far protected matter is overcome by the surrounding ice: …"].

For snowflakes, our German natural philosopher ended up by discarding the Platonic octahedron, since it was a solid in space, and fell back, in the end,

[150]Kepler 1984, pp. 160 and 201.

[151]A scattered sentence between the *Mysterium Cosmographicum* and *De Stella Nova*, quoted by Robert Halleaux in Kepler 1975, p. 27.

[152]Kepler 1954, v. XVI, n. 493, pp. 154–165. Here he already wrote down some of those elements referring to music and sexual generation, in accordance with Fibonacci's numbers, seen above in the *Harmonices mundi* … . As regards the geometrical archetypes in God, in creation and in nature, see also Kepler's letter to Christoph Heydon of 1605 . Kepler 1951, v. XV, n. 357, p. 235. Quotations also from Halleaux in the introduction to Kepler 1975, pp. 26–29.

on hexagonal plane figures. Thanks to the "… Anima formatrix …" ["forming soul"], "… pugna calidi vaporis et frigidi aeris in planitie existit …" ["… a struggle exists between hot steam and icy air in the plane …"]. But why would forms with a hexagonal symmetry derive from such conflicts, which were more Heraclitean than platonic? After gradually eliminating other hypotheses, seeing that the struggle between opposites was to take place on a plane, Kepler reflected on the geometrical plane figures, concluding in favour of the hexagon. "… formatrix facultas sexangulum eligat, nulla materiae spaciorumque necessitate coacta, sed solum decentia hac invitata, quod alias sexangulum struat planitiem excluso vacuo, sitque (ex iis figuris, quae idem possunt) circuli simillima." ["… let the forming faculty choose the hexagon, not constrained by any necessity of matter or of space, but only attracted by this convenience: that furthermore, the hexagon builds the plane, excluding empty space, and that (among those figures that can do this) it is the most similar to the circle".][153]

The booklet clearly confirms our imperial mathematician's faith in geometry, as the model of the soul created by God for all things, living and not living. About 10 years before the *Harmonices* …, he already presented here some of the themes that were to be developed there. Among these, we may recall the search for harmony, in this case spatial symmetry, the properties of figures in constructing the plane and space, the relationship between the divine proportion (the golden section), Fibonacci's succession, and the pentagon, as an expression of the biological generation among plants and animals.

This brief essay allows us to glimpse Kepler's inventive process, as marked by deeply rooted themes, which returned in continuous variations and confirmations, until they received complete, satisfactory expression. He thus appears to be a tenacious scholar, who did not lose sight of the purpose he had set himself until he had reached it. He also liked joking and enjoying himself. That Christmas present for a close, and important, friend of his in Prague, was presented as a "nihil" [nothing], not only as insignificant as a snowflake, but also as an allusion to another short poem, appreciated by the addressee, the title of which was, appropriately, *Nihil*. The jokes that recurred in the text, between one who did "nothing at all" and such an illustrious character of the imperial court, found learned variants, and puns: between the Latin *nix* [snow] and the German *nichts* [nothing]. Perhaps this also concealed a satire against the void of nothing and the atoms, seeing that he also quoted the famous saying "nature abhors a void" of Aristotle. Even this "nothing" thus appeared to be full of valuable things, all the same. And there was bound to be, on our Volva, the story about a woman who was made pregnant by a snowflake, sufficiently lively, obviously, to perform the task. And after the initial "… lascivi Passeris lusum argutissimum simul et venustissimum ..", ["… penetrating, and at the same time, delightful amusement of the lascivious cock …"], the reader may imagine by himself whatever he pleases.

[153]Kepler 1941, IV, pp. 275, 278, 279; Kepler 1975, pp. 73–74, 79–80.

While he was a fluent writer on many subjects – somebody counted a total of 86 works written by him – the style used by Kepler is characteristic, and different from that used by other natural philosophers, and above all by future modern scientists. In connection with his own inventions, he also told us about the historical circumstances considered pertinent to the subjects presented. Thus, in his books about science, he revealed here and there some episodes of his life, which had been a particularly dramatic, adventurous one, not at all simple or comfortable.

Kepler began his *Dream* with the conflict between the emperor Rudolph II of Habsburg (1552–1612) and his brother, Matthias. "In the theses [his first dissertation on the Moon, of 1593], I referred this to politics, seeing that public affairs are exposed to frequent large-scale changes, and private affairs often flourish." Through horoscopes, he tried to justify the reign of Rudolph II. But when the latter lost power, and had to abdicate in favour of Matthias, Kepler adapted his writings, practising self-censure.[154]

He informed us about the disputes sustained at Tübingen regarding Copernicanism and the *Bible*. Addressing the Jesuit Paul Guldin (1577–1643) in the geographical appendix to the *Dream*, Kepler wrote of a "… turbulent epoch, in which the whole Court is troubled by thoughts of war." With the death of Matthias in 1619, now Ferdinand II had become Emperor. In 1623, he lived in Vienna, where Guldin taught mathematics.[155] The war had started in 1618, with the repression of Bohemian Protestants, and the victories of the Catholic Counter-Reformation; after various phases, it was to come to an end only in 1648, thus deserving to be called the Thirty Years War.

In Prague, he had listened to the singing of the Muslim muezzin in the retinue of the Turkish ambassador, and compared this with the Easter hymns of the Christian liturgy. He was forced to exclude the possibility of finding harmony in similar intervals "… insolitis, concisis, abhorrentibus …" ["… unusual, truncated, abhorrent …"]. "Nihil dicemus de stridulo illo more canendi quo solent uti Turcae & Ungari pro classico suo: brutorum potius animantum voces inconditas, quam humanam Naturam imitari." ["We shall say nothing of that grating style of song which the Turks and Hungarians customarily use as their signal for battle: imitating the uncouth voices of brute beasts rather than human nature."][156] The stave of a warlike Turkish melody was contrasted with that of a devout "Victima Paschali laudes …". In that period, the vast Ottoman Turkish empire also included Budapest and all Hungary, and was drawing dangerously close to Vienna. Our German scholar undoubtedly hoped that, with goodwill, some consonant agreement might in the end be found between Catholics and Protestants, but he believed that this was not possible with Islam.

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[154] Kepler 1984, pp. 167–168. Caspar 1993, p. 204. As regards how Kepler wrote his books, see Koestler 1991, p. 309.

[155] Kepler 1984, pp. 180–181 and 187.

[156] Kepler 1619 [1969], III, p. 61; 1997, p. 217.

Also in his treatises packed with axioms, propositions, theorems and corollaries, like the *Harmonices mundi* . . . , this famous astronomer and mathematician relates to us the circumstances of his inventions. "22 years . . . " passed between the *Mysterium cosmographicum [Mystery of the cosmos]* of 1596 and the *Harmonices* . . . , which was finished in 1618 and published in 1619. He gave the name of the person who had passed him the relative book by Ptolemy, in which, however, he did not find "... the true causes of the harmonies."[157] The stages towards the final goal were described in detail. "... 16 years ago . . . "; "... my appetite was particularly intensified and my purpose stimulated by the reading of the *Harmonicorum* ... [*Harmony*] of Ptolemy . . . ". "Now, eighteen months after the first light, 3 months after the true day, but very few days after the pure Sun of that most wonderful study began to shine, nothing restrains me." "... and if you want the exact moment in time, it was conceived mentally on the 8th March in this year, 1618, but submitted to calculation in an unlucky way, and therefore rejected as false, and finally returning on the 15th of May and adopting a new line of attack, stormed the darkness of my mind. So strong was the support from the combination of my labour of seventeen years on the observations of Brahe and the present study, which conspired together, that at first, I believed I was dreaming, . . . ".[158]

He had received Ptolemy's book in 1607, and thus Kepler was assigning the dates of late 1616 and March 8th, 1618 to his other attempts to compose the harmony of the planets. At last, on May 15th, he "dreamt up" his third planetary law. He related the story of his discoveries in order to explain better why in the end he had abandoned Platonic solids for the harmony of music. Starting from Copernicus in 1594, he had searched for the suitable proportions, to establish that the distances between the planets were not equal, nor were they "... unequally unequal . . . ". Subsequently, he had first thought of plane figures, and then Platonic solids.[159] His interest in Ptolemy's theory of music was by no means ephemeral, and he translated a part of it from Greek into Latin in 1608. However, "... my change of abode, combined with a great many troubles . . . " prevented him from finishing this work. Nevertheless, his intention was to include at least something about Ptolemy as an appendix to the *Harmonices mundi* . . . : to demonstrate "... the imperfection of his symbolism ... [derived from the] ... falsity of the basic principles of Ptolemaic astronomy . . . ". Again, however, other problems had cropped up, and "... meanwhile the neighbouring Bohemian war broke out, . . . ". Roads were closed, printers were recruited as soldiers, *und so weiter*. Anyway, even without Ptolemy, the book was terminated on May 17th/27th, 1618 and published on February 9th/19th, 1619. Why the double dates, 17th–27th and 9th–19th? Because in that period, Europe was divided by wars of religion, and now had two calendars. Catholics, who followed the new Gregorian calendar, counted 10 days more, while

[157]Kepler 1997, pp. 163–164 and xxiii.

[158]Kepler 1619 [1969], V, p. 189 . Kepler 1997, pp. 389–391, 411. Caspar 1940, pp. 462–484. Cf. Caspar 1993, pp. 91–96 and 264–290.

[159]Kepler 1997, pp. 489 and xvi–xxiv.

the others, who had remained faithful to the Julian calendar, 10 days less. In contrast with theologians and his colleagues at Tübingen, our Lutheran astronomer preferred the new papal calendar.[160]

Thus, here and there, wars broke out. Not only with the Turks, but also other melodies were defined as warlike.[161] Signs of war were visible, above all, in the dedication to "… Iacobum Magnae Britanniae" [James I Stuart (1566–1625)], king of Great Britain, who was the son of Mary Stuart, a Catholic, but had been brought up as a Calvinist Protestant. His daughter had married the Prince of the Palatinate (Germany), Frederick, the champion of the Protestants in the fight against the Catholic Habsburg emperors. But rather than imprudently taking sides with the defenders of his own Protestant faith, our imperial mathematician always "dreamt" of harmony, both in the Heavens and on Earth. In spite of the conflicts of which he had direct personal experience among the Habsburgs, Kepler foresaw that there would soon be peace. "… ut intelligeretur ex cursu non inperturbato Pacis ornamentorum per has provincias, famam intestini belli sinistram, cum ipsa re procul dubio brevi extinctum iri: Dissonantiamque hanc paulo duriorem, ut in Pathetica Melodia, jam jamque in suavem Clausulam desituram." ["… he should understand, from the non-imperturbed progress of the preparations of peace throughout these provinces, that the sinister rumour of civil war would undoubtedly soon be extinguished together with its reality, and that this slightly too harsh discord, as in an emotional melody, is on the very point of resolution into a pleasing cadence."][162]

While he had hoped, for 20 years, to discover harmony in the movement of the planets, until he finally succeeded, he was not allowed by God, or by a cruel destiny, or by history, to live serenely through a period of peace. And yet, as dissonances make consonances more pleasant in music, in the same way, divine providence was to make use of dissonances in human affairs. The Christian king, James I, was admired by Kepler, because he succeeded in reconciling two hostile kingdoms like England and Scotland. On the contrary, the life of our natural philosopher appeared to be increasingly dogged by dissonances. "Vellem hic, Dissonantiam publicam, Vocum trifariam obstrepentium, paulo mihi mitiorem esse, ut audiri publice ex animi mei sententia possem: …" ["I should wish, here, the three-part public dissonance of clamouring voices to show me a little more moderation, so that I could make the results of my own thinking publicly heard, …"]. If he did not succeed in surpassing that public screaming, then his voice, too, would simply increase "… molestias absurdi concentus …" ["… the annoyance of a false chorus …"] "… proh dolor,

[160]Kepler 1997, pp. 498–500. Caspar 1993, pp. 228–232. Baumgardt 1951, pp. 27–28 and 118. In a letter dated August 4th, 1619, however, Kepler wrote that the *Harmonices mundi* … had not been published yet. Baumgardt 1951, p. 141. As regards the extent of Kepler's studies on Ptolemy, see Stephenson 1994, pp. 98–117.

[161]Kepler 1997, pp. 219 and 239.

[162]Kepler 1619 [1969], "Dedicatio" [a–b]. Cf. Kepler 1997, p. 2. Here the translation is different: "non-inperturbato" was changed into "uninterrupted". Caspar 1993, p. 288.

tumere adhuc vulnus decussatum, an malumus sacratiori foeliciorique vocabulo, cruciforme tumere inquam, multiplici labro; & nullo illorum connivente, medicinam hactenus irritam, omnibusque partibus irrisam; ... " ["... ah! what sorrow – that criss-cross wound is still swollen, or if you prefer a more sacred and more felicitous word, the cross-shaped wound, is swollen, I say, with its multiple lips; and though none of them winks at it, the medicine has so far been useless, and mocked on all sides; ..."]. And yet Kepler still hoped that "... Dissonantia haec diuturna (...) in meram & durabilem Harmoniam terminetur." ["... this enduring dissonance (...) will end in pure and abiding harmony."][163]

Again in 1621, when introducing the second edition of the *Mysterium Cosmographicum*, in the words "... Now that the weapons have been laid down [sic!] ...", he still hoped for peace. He believed in it so strongly, that he misunderstood Plato's *Republic*, as if the Greek philosopher had indicated geometry as the way of peace, instead of training young men for war. "(A benevolent fate has made me do this work during the sharp dissonances of the last 2 years) ...".[164] If he was led to invoke the mediation of the British king, James I, to confirm "... studium in se Concordiae & Pacis Ecclesiasticae & Politicae ..." ["... the zeal for concord and for peace in church and state ..."],[165] what dissonances may our devout German imperial mathematician have experienced at his own expense?

At the root lay the religious contrasts that he himself mentions in the three clamouring, rather than singing, voices of the Protestants, the Catholics and the Calvinists. These were to lead to the Thirty Years War. In this context, not at all serene, during the last few years of work on the *Harmonices Mundi* ..., as if all this were not enough, a trial also started against his mother, who was accused of witchcraft. Kepler felt Lutheran, and unlike other friends and acquaintances, he would never be converted to Catholicism, in spite of the obvious advantages that he would receive from this. In imperial Catholic Austria, at Graz as well as at Linz, he was persecuted, losing his possessions and his positions, and was forced to emigrate from one place to another.[166]

The Lutheran theologians of Tübingen did not treat him any better either. He had never accepted their Formula of agreement about Christ and the sacraments, and had therefore been excluded from them. "The Jesuits clamour and the Calvinists clamour they are discredited by the Formula of agreement, as regards the article of faith about the person of Christ." Here, our conciliatory Lutheran invoked his own conscience, going so far as to defend, in following the Holy Scriptures and the Church Fathers, those other Christians of a different Church, in opposition to his own pastors. Banished from imperial Catholic Graz, he was never to obtain the desired, and requested, place at the Lutheran University of Tübingen, but was forced, against his will, to become the assistant of Tycho Brahe at Prague. Subsequently, at

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[163]Kepler 1997, pp. 3–5. The idea is also found in Leibniz; See Part II, Sect. 10.1.

[164]Baumgardt 1951, pp. 128–129. Caspar 1993, p. 306.

[165]Kepler 1619 [1969] "Dedicatio" [c–e]. Cf. Kepler 1997, pp. 3–5.

[166]Caspar 1993, passim. Baumgardt 1951, pp. 53–57 and 172–174.

the death of the aristocratic Dane in 1601, he became the Mathematician of His Holy, Christian, Imperial Majesty, thanks to the recommendation of his Catholic friend, Georg Herwart von Hohenburg (1553–1622).[167] But this was to prove to be a position that gave more prestige than a good, regular salary. Kepler was forced to attend to the credits that he had accumulated over the years, for the rest of his life. They were not even paid to his heirs after his death.[168]

In brief, Kepler was a heretic for everybody, and for this reason, was subject to continual persecution, and without many friends. Certain Lutherans went so far as to sustain that the main reasons for the mental problems from which his first wife suffered were the religious ideas of our astronomer.[169] However, he also accepted "… the doctrine of Catholicism, not only with my heart, but also with my head …". "… I was introduced to the Catholic Church by my parents with the baptismal water … I remain in the Catholic Church, but by rejecting all that I do not acknowledge as Apostolic … I cling to the Catholic Church, even if she rages and beats …".[170] In 1619, prudently, in order to avoid the probable Catholic censure against Copernicanism, he suggested that Italian booksellers should sell his *Harmonices Mundi* … only to high-ranking prelates, to mathematicians and to metaphysicians. At Graz, in 1623, copies of one of his almanacs had even been burnt in public. At Linz, in 1625, his library was sealed off.[171]

He still wrote his books even more frequently in Latin, though he felt that he was German. Every now and then, he let slip a few words in his native language: "… *das Klavier* [in Gothic], & instrumenti genus Clavichordium …" ["… *das Klavier*, and the clavichord kind of instrument …"]. Elsewhere, when using the three means, arithmetic, geometric and harmonic, to discuss the best form of government, and to give a correct retribution, he displayed German habits, writing of "… nos Germanos …" ["… we Germans …"].[172] In instructions specially for Italian booksellers, he said that he had written "… as a German, after the German fashion, and with German frankness." He preferred to live in German-speaking countries, and refused a teaching post in Italy at Bologna, and a probable position in England. To be more precise, he felt at home only in Southern Germany, and postponed for some time a position as mathematician-astrologer at the University of Rostock, situated on the Baltic.[173]

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[167]Letter to Mästlin dated December 12th/22nd, 1616; Kepler 1955, v. XVII, pp. 203–204. Caspar 1993, pp. 49–52, 77–118, 122, 188–189, 258–264, 359–360, … and passim. Baumgardt 1951, pp. 58–65, 101, 106–107.

[168]Baumgardt 1951, pp. 87, 90, 96, 129–131, 138, 163, 171, 174, 179, 189, 192–195. Caspar 1993, pp. 104, 109, 122–123, 141–142, 156–159, 188, 225, 356–357, 362–363, … and passim.

[169]Baumgardt 1951, pp. 112–113.

[170]Baumgardt 1951, pp. 133, 172–173. Cf. Caspar 1993, pp. 112–113, 148, 337, … *passim*.

[171]Baumgardt 1951, pp. 133–136 and 150. Caspar 1993, pp. 299, 307, 317, 335–337.

[172]Kepler 1619 [1969], III, pp. 51 and 98; 1997, 202, 272.

[173]Baumgardt 1951, pp. 133, 144–148, 189. Caspar 1993, pp. 220, 236 and 347.

From his "Provocatio" [challenge] launched to all rival scholars, we may imagine Kepler's character, too. He invited mathematicians and philosophers to come forward, and try to modify his harmonies, finding others that were better, and closer to astronomy, if they could. For his part, he put it in writing that he would change "… illa, quae prioribus diebus oscitanti cura, vel properanti ardore praepostere concepta deprehendere potui." ["… anything which I could discover which was incorrectly conceived in the preceding days, if my attention yawned or my enthusiasm was hasty."][174]

Having arrived at the last section of the chapter, $49 = 7 \times 7$, "Epiphonema" [Envoi], he took his leave as if it were the Sabbath, the seventh day when God had terminated his creation, quoting the relative passage of the *Bible*. We cannot underestimate the religious faith of the imperial mathematician and astronomer, which not only marked his life, but which he readily exhibited in his works of astronomy. Now he presented himself as an imitator of God, in search of perfection in his works, just as He had filled the world with harmony. He addressed all those who, in the spirit of Christ, rejected every kind of sectarianism, rivalry, envy, and anger, in order to restore harmony among Christians. He prayed that God would keep the dissonances of discord away.[175] That part of the infinite, which his limited mind had succeeded in grasping, he had conveyed to those who had read his demonstrations. But he himself had behaved like a "… vermiculo, in volutabro peccatorum nato & innutrito …" ["… miserable worm, born and nourished in a slough of sins, …"]. He only hoped that all his demonstrations would serve to extol His glory, and to save souls, instead of misleading them.[176]

So much did it mean for him to be considered a devout religious scholar, that in the end, he even strained the dating of Proclus, so often quoted by him (fifth century, backdated to the fourth century). He thus succeeded in interpreting him in a Christian sense, in spite of the event that this Greek neo-Platonic philosopher had written against the new form of worship of his period, which was by now consolidated in Europe. But the forced interpretations and the historical errors only underline Kepler's profound intention to make his Christian faith compatible with his mathematical and natural philosophy: the truth of both would be to be sought by transcending the sensible world. Thus his *Harmonices mundi libri quinque* terminated with a hymn to religious and scientific transcendence, which was a paraphrase of a psalm of David, suitably modified. "… in saecula saecolorum. AMEN." ["… for ever and ever. AMEN"].[177]

Kepler knew very well that if he tried to reconcile the *Bible* and Copernicanism, he would run serious risks, and he would be criticised by theologians, and by both Protestant and Catholic colleagues. Thus he had already written to Michael Mästlin in 1598: "… we shall imitate the Pythagoreans also in their customs. If someone

[174] Kepler 1619 [1969], V, p. 241; 1997, p. 488. Caspar 1993, pp. 180–181 and 323.

[175] Kepler 1997, p. 452.

[176] Kepler 1619 [1969], V, p. 243; 1997, p. 491.

[177] Kepler 1619 [1969], V, pp. 245–248; 1997, pp. 493–498.

asks us for our opinion in private, then we wish to analyse our theory clearly for him. In public, though, we wish to be silent. … Let us behave, therefore, in questions of astronomy, in such a way as to maintain support for it, and avoid dying of hunger."[178]

From a character like this, historians, philosophers, mathematicians, physicists, astronomers and popular writers of our great epoch have considered themselves authorised to take whatever they needed, and to ignore the rest. Among the very few who have tried to depict a complete Kepler in the round, considering for him as significant harmony, religion, astrology, wars and the other dramatic circumstances of his life, the palm must be awarded, above all, to Max Caspar.[179] We refer to him, for brevity, so that our *Musical history* … (already too extensive) will not become an interminable history which is the prelude to a volume about the imperial astronomer. To Caspar, we refer readers who are interested, not in an apology, but in understanding without excluding at all the possibility of laughing or weeping. Here, we underline only those interpretations which are too distant from, or too close to, my own. I shall behave in the same way with the secondary literature, which fluctuates between metaphysical rational reconstructions and the novel.[180] And there is even an opera dedicated to him.[181] But, surprisingly, we are unable to quote a single film of which he is the protagonist, in spite of all those events that might lend themselves to a successful script.

Galileo Galilei's trials are well-known, but also Johannes Kepler may be said to have suffered the consequences of a trial, which indirectly involved his own good name, to which he was forced to dedicate all his energies. The person on trial was his mother, Katharina Guldenmann (1547–1622), who was accused of witchcraft. "The prisoner appears, alas [sic!], with the support of her gentleman son, Johannes Kepler, the mathematician." Our mathematician had to defend her, never doubting that the accusation of witchcraft might not be legitimate. Besides, he had also made her a character of his *Somnium* … on the Moon. We can well imagine why, despite the acquittal finally obtained for her with great effort by her son, she died only a few months after the conclusion of the trial.[182]

Many letters written by Kepler, totalling about 400, are extant, as well as 700 received by him. Through these, the various events have been reconstructed.[183] They reinforce the impression already received from the reading of his most famous works. Curiously, there is a singular coincidence between the fortunes that befell our imperial mathematician, who was able to use Tycho's observations to calculate the orbit (the first elliptic one) of Mars, the god of war, and the dramatic events

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[178] Caspar 1993, pp. 68–69. Kepler 1984, pp. 68–69.

[179] Caspar 1993. Cf. Gingerich 1970.

[180] For example: Banville 1993; Koestler 1991; Lombardi 2000.

[181] Hindemith 1957/1961.

[182] In that area, and in that period, 38 poor women were sentenced to death for similar accusations. Caspar 1993, pp. 240–258. Baumgardt 1951, pp. 156–162. Banville 1993, pp. 154–156. Lombardi 2000, pp. 59–66.

[183] Baumgardt 1951.

that afflicted him, as a result of the numerous wars in the Europe of that period. His father was a mercenary soldier, and for this reason was absent from home. In the end, our mathematician was given hospitality at Sagan by the Duke of Friedland [sic! Land of peace], the famous General Wallenstein (1583–1634), who was fighting the Thirty Years War, with success, on behalf of the Habsburgs. This did not make it any easier for him to receive the payment of his salary, or to carry out his studies in peace.[184] But perhaps it fortified his obstinate efforts to find a harmony that was impossible on Earth, which could only be imagined in the Heavens. Kepler passed away at Regensburg, during his umpteenth vain attempt to find some money and a safe haven. He was buried outside the city walls, because he was a Protestant, and the war, which merrily continued, soon destroyed his tomb. As an epitaph, he had it written that only his body was there in the dark earth, whereas "Mens coelestis erat" ["My soul was from heaven"]. The follies of men could no longer affect him. However, he was persecuted until the very end. Two Lutheran journalists succeeded, all the same, in taunting him, both in an article about his "vain" spirit of peace, and by expressing doubts about his religious faith.[185] Here, too, and not only with his new astronomy, Kepler was opening up the way to our great modern ages.

From certain indications here and there, we may begin to suspect a little-disguised inclination of the imperial mathematician towards that sex which is, in general, the more beautiful, the stronger, and the more important for historical evolution. His two wives both complicated and gladdened his existence. More than the first one, made happy his life the second one, much younger than him, who he had married on the day of an eclipse of the Moon. Numerous daughters and sons were the objects of his affectionate thoughts. The count arrived at 13, but at least half of these were destined for a short life. As a student, he had played female parts on the stage at University. These troubled him, without doubt, but for ever he was to love the more "… polished, noble, dear, timid and so easily frightened, tender sex." Through horoscopes, he gives us information about his first experiences of love, which we cannot define as outstanding.[186] In this connection, the natural philosophers of his period displayed different attitudes and customs. In general, they came from the monastery tradition, and had maintained these habits.[187] Also in this respect, Kepler appears to be a heretic.

[184]Caspar 1993, pp. 34–36 and 338–350. Baumgardt 1951, pp. 174 and 179.

[185]Caspar 1993, pp. 355–361. Baumgardt 1951, pp. 192–197.

[186]Baumgardt 1951, pp. 20, 24, 94, 112–117. Caspar 1993, p. 43. Walker 1967, pp. 243–245. Koestler 1991, pp. 231 and 236.

[187]Noble 1994. In this book, which deals with the absence of female figures in the development of Western modern sciences, Kepler was not even mentioned, perhaps because he appeared to be such an exception for the author's thesis.

It has been observed that none of the scholars who were his contemporaries gave much credit to Kepler's inventions.[188] As we shall see, neither did Newton, although he was a debtor to him for some essential elements of his gravitation.[189]

In the *Somnium*, our inventor of the three planetary laws referred to a booklet entitled *Ignatius His Conclave*, a satire against Ignatius of Loyola (1491–1556), the founder of the Jesuits.[190] By force of circumstances, it was published anonymously in 1611, though its author was, in fact, John Donne (1572–1631).[191] This famous English poet and mystic had passed from a family with Catholic traditions, actually related to Thomas More (1478–1535), the adversary that Henry VIII caused to be executed, to the Anglican Church. He became a *protégé* of King James I of England, to whom we know that Kepler dedicated the *Harmonices mundi* .... Donne was afterwards appointed chaplain in the entourage of the legation sent to Germany in 1619, with the – unfortunately vain – aim of making peace. It might be possible that on this occasion, this English poet and preacher may have met our German astronomer: the latter, in these years, was wandering from one town to another, worried about his mother's trial, in search of peace, of money, and of printing works where he could publish his books. But it appears to be unlikely, seeing that the only place where they could by chance have met was Augsburg. Here, Kepler might have arrived in the summer of 1619, to let his book about comets appear, whereas Donne was there between the end of June and July.[192] Or at least, perhaps Donne had his *pamphlet* with him, in which he spoke ironically about the Jesuits of the period. Remembering Giordano Bruno burnt at the stake, even if they were relatively well protected, it would be more prudent to put certain things in writing anonymously, in the form of satire. Thus the humorous, ironic side of Kepler in the *Somnium* may have encountered the same element in Donne. In the *Ignatius* ..., he joked about how the Jesuits were treating Copernicus. This must have naturally attracted the attention of the astronomer, who was always in search of support, against such powerful adversaries.

The episode should be seen, not only as a curious detail, which is only of interest for learned professors of this subject. Rather, it reveals to what extent Copernicanism had by now arrived beyond the bounds of mathematicians' university lecture-halls, and was discussed practically everywhere by the educated classes: we even find it among the writings of an English churchman. It is clear, then, that the Catholic Church, and others, as well, were worried by this. Again in 1611, the renowned English poet described its decadence, in "An Anatomie of the World".

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[188] Baumgardt 1951, pp. 51 and 124. Caspar 1993, pp. 135–138.

[189] Barone 1989.

[190] Kepler 1984, p. 70.

[191] Donne 1969.

[192] Kepler 1963, v. VIII, pp. 135 and 461. Caspar 1993, p. 301. Bald 1970, pp. 338–365.

And new Philosophy cals all in doubt,
The Element of fire is quite put out;
The Sunne is lost, and th'earth, and no mans wit
Can well direct him where to looke for it.

Then, the beautiful spherical proportions of the world had been lost, "… to finde out so many Eccentrique parts […] For his course is not round … […] All their proportion's lame, it sinckes, it swels."

The best parts of beauty, proportions, are dead, like the moral parts with recompenses and penalties. Even pains were out of proportion.

Shee by whose lines proportion should bee
Examin'd, measure of all Symmetree,
Whom had the Ancient seene, who thought soules made
Of Harmony, he would at next have said
That Harmony was shee, and thence infer,
That soules were but Resultances from her,
And did from her into our bodies goe,
As to our eyes, the formes from obiects flow:[193]

Written in the same period as *Ignatius* …, this poem reveals the same themes dear to Kepler: harmony as the soul of the world, a harmony that was being lost, at least on the Earth. The poem "Loves Alchymie" made reference to the music of the spheres, and as a result, the Ancient mentioned would be Pythagoras. In Donne's "Hymne to God my God, in my sicknesse", the poet takes part directly in the music of spheres at his death, rising into Heaven.[194]

While different in their native tongues and in their professions, were these two characters linked together only by a vague neo-Platonism? Was it only a singular coincidence? Perhaps there we are facing something more to it. The indications that Kepler knew Donne's *Ignatius* …, and that the latter travelled to Germany at the beginning of the famous War, would deserve greater attention than has been reserved to them in relative literature. The English Ambassador on the continent (above all for Venice), Sir Henry Wotton, who had tempted Kepler to leave his homeland for England, had been an intimate friend of Donne's for many years. As a result, he knew some of the people who were in contact with Kepler directly. Even if they probably never actually met, they were only one step away, to use a modern way off putting it.[195]

In his anti-Jesuit *pamphlet*, anyway, Copernicus was banished to hell by the English churchman, like Lucifer by John Milton, and made to dialogue with the chief of the devils. The Catholic Church had rewarded Ignatius in 1609, placing him among the blessed, that is to say, in paradise, but here Donne turned him into

[193]Donne 2002, pp. 44, 48, 52.

[194]Donne 2002, pp. 124 and 144.

[195]Donne 1969, pp. 101, 129, 139. Bald 1970, passim. Only Nicolson 1940 and Edward Rosen in Kepler 1984, p. 70 have dwelt on Donne, the former in view of literary interests. Cf. Koestler 1991, p. 361. Cf. Gingerich 1970, p. 306.

Satan's gatekeeper. In the style of Dante, famous characters would have wished to sit close to the leader of the devils: Paracelsus, Machiavelli, Pietro Aretino, Columbus, Filippo Neri. Of course, it was a pretext to magnify the diabolical virtues of Ignatius, the Jesuits and the Roman Pope. Donne was doing his job as an Anglican preacher at the time of the Counter-Reformation, writing satires in profusion about the copious partisan theological literature written in that period. He immediately translated the Latin original himself into English; he made fun of, and popularised, religious themes or commonplaces, which were the object of ferocious attacks, and controversies that were not at all peaceful. Every now and then, a king, a prince, a churchman, witches or natural philosophers were assassinated, or put to death, if they said the wrong thing, or were in the wrong place at the wrong time. Controversies of the kind thus appeared to be far from theoretical opinions, but assumed aspects that were decidedly political and social. Two characters who have already come into our story crossed swords over these questions: the Jesuit, Cardinal Roberto Bellarmino (1542–1621), the protagonist of the trials against Galileo Galilei, and the King of England and Scotland, James I, Kepler's dedicatee. Donne's *pamphlet* corresponded to their official written disputes.[196]

In the English poet's satire, Ignatius proved to be such an expert in selecting those that did (or did not) deserve to go to hell, that he was about to steal the position from Lucifer himself. The Jesuits, the worthy heirs of Machiavelli, would have surpassed their master in their ability and dialectic tricks. Consequently, to avoid contrasts, they were relegated to the Moon, where they would organise another Church of their own, and their personal Hell.

The first one who knocked at that fearsome door was Copernicus, but then, in the end, the Jesuit excluded him, on the grounds that he was not sufficiently meritorious. It was true that he had derided Ptolemy, and closed him in a corner, he had set the Earth in movement, thus bringing Lucifer from the bottom of the universe up to Heaven. Would the door remain closed for the one "… totam mundi machinam versanti, & pene novo Creatori, …?" "'… who had turned the whole frame of the world, and … thereby almost a new Creator?'" Had he not displayed enough pride, like Lucifer? But Ignatius of Loyola intervened, in spite of his ignorance on the subject, seeing that he had learnt something from his Jesuits who, like him, had ended up down there. Maybe, as the Earth moved, sins and Hell would disappear? "Omnia ut ante credunt, peragunt. … etiam vera esse possunt, quae asseris.." "Do not men believe? do they not live justly, as they did before? … opinions of yours may very well be true." As is known, neither the devil nor the Jesuits loved the truth too much … In his opposition to the truth and to Copernicus, their Clavius would have deserved the honour, rather, of sitting next to Satan. Furthermore, this Jesuit with the new Gregorian calendar, by which "… Ecclesiae pacem, & civilia negotia egregie perturbavit." "… both the peace of the Church, & Civill businesses have beene egregiously troubled: …" "*Horrende Imperator, Mathematiculus iste*". "… this little *Mathematician* (dread Emperor) …". He perhaps might be allowed in,

[196]Donne 1969, pp. xix–xxix.

only when the Jesuits would succeed in obtaining an appropriate anathema against Copernicanism from the Pope.[197]

Thanks to their good offices, Rome did actually include the famous treatise by Copernicus in the Expurgatory Index in 1616, but all the same, Donne did not modify this page in the subsequent editions. In a letter, he expressed another judgement on Copernicus. "Copernicus in the Mathematiques hath carried earth farther up from the stupid centre; and yet hath not honoured it, because for the necessity of appearances, it hath carried heaven so much higher from it."[198]

"[Ne] fiat iniuria Galilaeo" "In order not to do any wrong to Galileo", nothing was said at the beginning about the planets or the stars. Only at the end did the Tuscan natural philosopher return, so that Lucifer could suggest him to the Pope as a valid helper in transferring the Jesuits up there, as he had observed the Moon from close up. Kepler deserved something different. "... cui (ut ipse de se testatur) a *Tychone Brahe* mortuo, *cura incessit, ne quid novi in coelis se inscio existeret*." "... who (as himselfe testifies of himselfe) *ever since* Tycho Braches *death, hath received it into his care, that no new thing should be done in heaven without his knowledge*." Here Donne was quoting almost word for word what Kepler wrote in 1606, when he referred to a star in the constellation of the Cygnus. In astronomy, he appears to have accepted more readily the compromise that Tycho Brahe did between Ptolemy and Copernicus.[199]

About 10 years later,[200] Sir Henry Wotton was to invite the German mathematician to go to England. But would the ambiguous irony of the poet and militant Anglican preacher, as regards Copernicus, Galileo and Kepler, be enough to convince our Lutheran German that he would find a friendly environment in London? That city would give him a warmer welcome, different from those of Tübingen and Prague, Graz and Linz, or Rome and Bologna? So it was that Kepler stayed at home.[201]

In his *Astronomia Nova*, our imperial mathematician wrote, "After the divine goodness had given us in Tycho Brahe so careful an observer, that from his observations, an error of calculation amounting to eight minutes [of an arc] betrayed itself, it is seemly that we recognize and utilize in a thankful manner this good deed

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[197] Donne 1969, pp. 12–19. Emphasis in the text.

[198] Donne 1969, p. 109.

[199] Donne 1969, pp. 6–7, 80–81, 102, xxx. Emphasis in the text

[200] Caspar 1993, pp. 252 and 256.

[201] The hypothesis that Donne may have drawn his inspiration directly from Kepler's *Somnium* for his *Ignatius*, who likewise ended up on the Moon, seems unlikely to me. Note 8, added by Kepler to the *Somnium* ... much later, says, "Fallor an author [sic!, auctor] Satyrae procacis, cui nomen Conclave Ignatianum, exemplar nactus erat [sic!, esset] huius opusculi; pungit enim me nominatim etiam in ipso principio." ["I am not sure whether the author of an impudent satire, the title of which was *Conclave Ignatianum*, may by chance have come across a copy of this leaflet; he teases me by name, right from the beginning of the work."] I prefer to maintain Kepler's doubts. The relative dates and the limited circulation of a manuscript, which was only printed posthumously, make this possibility too complicated and too remote. Cf. Nicolson 1940 and Donne 1969, pp. 159–160.

of God's, that is to say, we should take pains to search out at last the true form of the heavenly motions."[202] These were the values of the intellectual context in which Kepler put together the first laws of his astronomic system. He had always sought the truth, without being satisfied with appearances, to be calculated by means of mathematics, as the prudent Copernicus had done. For him, the truth could not be guaranteed by anyone less than God in person. If His divine providence, through the influence of the stars, had subjected him to such a hard test, giving him such a dramatic life, it was also true that His infinite goodness had led him, by tortuous pathways, to meet Tycho and his treasure of precious measurements.

As we are not at the same level as Kepler before the universe, and we are incapable of revealing the secrets of destiny and chance, we historians must, with due modesty, limit ourselves to relating the attendant circumstances. These were subject to continual unexpected *coups de théâtre*, almost always completely independent of the will of our natural philosopher, who could have equally well been a Protestant theologian at Tübingen, instead of finishing up as an astronomer-astrologer at the imperial Court in Prague. Between one persecution and another, between one city and another, between one journey and another, between one printer and another, between one war and another, between one request for payment and another, between one period of mourning and another, between one marriage and another, between one dwelling-place and another, the known events of his life were so unstable and adventurous, that they would have severely tested the most judicially-minded, that is to say, the most deterministic, astrologer.

In 1605, he confided to his friend and protector, Herwart. "My goal is to show that the heavenly machine is not a kind of divine living being, but similar to a clockwork, insofar as almost all the manifold motions are taken care of by one single absolutely simple magnetic bodily force, ... I also show how this physical representation can be presented by calculation and geometrically."[203] In presenting the world as a clockwork machine, Kepler was starting out in a direction which, in time, even if not immediately, was to meet with success among other natural philosophers, perhaps beyond his own intentions. For would it not be easier, as he thought, to attribute a soul to an animated cosmos, like a living organism, than to an inert mechanical device? Among all the other aspects present in our famous German astronomer's work, this one was to be preferentially amplified by many of the subsequent scholars. In this way, European mathematical sciences were to move farther and farther away from that image of the world as a complex organic being, completely inter-connected, which we found predominant in the approaches of the Chinese.

In 1608, Kepler wrote to the astronomer-astrologer David Fabricius (1564–1617), "Natam ais tibi filiam ex Geometria matre? Vidi: pulchra est. At meretrix pessima futura est, abductura quam plurimis meis filiabus ex matre Physica susceptis, maritos suos. [...] Me igitur nuda duxit Natura, nullis instructa

[202] Caspar 1993, p. 128. Gingerich 1970, pp. 294–297.
[203] Caspar 1993, p. 136.

vestibus hypothesium; [. . . ] . . . denique ut novo ornatu placeret delicatis, nuditatem fastidientibus: imo vero ut pro Natura, generosissima puella, substitueretur spuria tua, meretricio ornatu et moribus, ad voluptatem comparatis, non ad ingenuitatem: hoc est Fabricio interprete, ut arti consuleretur, non alienatis philosophis." ["You say that a daughter has been born to you by geometry. I have seen her; she is lovely, but she will become a very bad wench, who will carry off the men of the many daughters that the mother, physics, has given me. . . . Therefore, bare nature has guided me, free of all the attire of a hypothesis. . . . and then so that [your theory], thanks to the new embellishment, could appeal to lascivious men, who disdain simplicity: or rather, in the place of this Nature, a highly respectable girl, your whore with her wench's apparel and her conduct leading to lust, not to virtue, is supposed to appear: that is, in the speech of Fabricius, one should address science, not crazy philosophers."].[204]

He too, of course, was subject to the weaknesses of the flesh, and even our devout natural philosopher was to allow himself some little casual (paid?) *affaires* with young and beautiful, more frivolous daughters of Geometry. Geometrical theory, curiously, was contrasted here, in the debate with his colleague, with a presumedly virtuous (repressed?) Lady Nature. Between what he was later to write in the *Harmonices mundi* . . . , above all, as we have seen above, in "Book IV", and the world as clockwork, we do not find much consistency. We are forced, then, to imagine an evolution in time. We find even less agreement when he defines as a "whore" that daughter of Geometry who was to introduce him to pleasures that were not only theoretical, but also sensible, because they are found in Music, and included in the Third Planetary Law.

"Do with me what you want. Even if you were to pull one vein after another out of my body, I would have nothing to admit."[205] These words pronounced by a poor 70-year-old woman, Kepler's mother, registered in the proceedings of a trial for witchcraft, should be compared with the famous, mythical "And yet it moves" muttered by Galileo Galilei after his well-known recantation. When they were led in front of the torture instruments, the Italian natural philosopher and the German sorceress reacted differently. Of course, there were differences of character, sex, language, faith, culture, values, contexts, different worlds. But in the tribunals of the period, the battle for religious, scientific, or judicial truth, or simply for the plain truth, continued with every means available, including extreme violence.

"I ask God to make my spirit strong, so that I direct my glance at the pure truth, from whichever side it should be presented, . . . ". "I wanted to become a theologian; for a long time, I was restless. But now, see how by my pains, God is being celebrated in astronomy also." Kepler thought of himself as a priest who was reading the book of nature, in order to sing God's praises. We need to take into consideration his neo-Platonic philosophical positions, together with his theological convictions about the divine presence of Christ in the world, which were more Lutheran than

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[204] Kepler 1954, v. XVI, pp. 205–206. Cf. Caspar 1993, p. 171. Translation slightly different.

[205] Caspar 1993, p. 255.

Catholic, and more Calvinistic than Lutheran. How did the incarnation of God the Son take place? In the bread and the wine of the Eucharist, was Christ present as 'transubstantial', 'consubstantial', or only as a symbol? In any case, Kepler refused the worship of the images of saints, the worship of the Eucharist, and the idea of God's presence everywhere. At this point, there can be little doubt that the truth imagined for all his life byKepler was absolute and transcendent. For him, the world was a corporeal "imago" [image] of God, and the soul was the incorporeal "imago". In this term "imago", he included not only the image, the representation the design, but probably also the phantom, the dream and the appearance of the Latins.[206]

Plato's transcendence of geometry should afford the best "imago" of the Christian Trinity, which became a sphere with its centre in God the Father, the visible surface in the Son, Christ, and the Holy Spirit in its rays.[207] Having created us in His image and after his likeness, "... God wanted to have us recognize these laws ...", he wrote to Herwart.[208]

In our imperial mathematician, the evident recurring dualism between truth and falsity in sciences was inextricably linked with the dualism between good and evil, or soul and body, of his religious faith. With him, it is impossible to throw out all theology, astrology, the weather forecasts of his almanacs and music, in order to keep only his studies on astronomy. We would be making the same mistake that he imputes to others in his longer title of the *Tertius Interveniens*: "Das Kind mit dem Badt ausschütten" ["To pour out the child with the bath"].[209] Consequently, as is more common among science historians, we should not distinguish, either, between the Kepler of progress, who anticipated Newton, and the Greek-Aristotelian one, between the tireless calculator who compared his own numbers with Tycho's observations, and the animistic metaphysician, convinced that the whole world was harmoniously connected together by the beautiful proportions of geometry and Greek music, between the mathematician of precision and the Lutheran theologian, between the tenacious scholar poring over his books and the ironic, polemical vagabond of the courts and the parishes, between the intelligent astronomer and the affectionate husband, ...

Even without Kepler's permission, if we wanted (diabolically) to mix good with evil, we might tell how he took possession of the precious papers left by Tycho Brahe. "Ego sic censeo de Tychone, Divitiis abundare, quibus non recte utatur, more plerorumque divitum. Danda est igitur opera (quod et ego pro mea parte, sed cum debita modestia feci) ut divitias illas illi extorqueamus, ut emendicemus, scilicet, ut vulget suas observationes sincere, et omnes." ["My opinion about Tycho is this: he has abundant wealth. Only, like most rich men, he does not know how to make proper use of his riches. Therefore, one must take pains (what I did

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[206]Caspar 1993, pp. 336, 373–378. Banville 1993, pp. 176–180 and 150.

[207]Kepler 1619 [1969], IV, pp. 119–120. Caspar 1993, pp. 272 and 379. Pauli 1955. Tangherlini 1974.

[208]Caspar 1993, p. 380.

[209]Caspar 1993, p. 182.

myself, with proper sagacity) to wring out his treasures from him, to get from him, by begging, the decision to publish all his observations without reservation."] "Non diffiteor, me Tychone mortuo, haeredibus vel absentibus, vel parum peritis, Observationum relictarum tutelam mihi confidenter, et forsam arroganter usurpasse; adversis haeredum voluntatibus; ...". ["I do not deny that when Tycho died, shamelessly, and perhaps arrogantly, I took charge of directing the remaining observations, seeing that the heirs were either absent or too unskilled; against the will of the heirs; ..."].[210]

Whether they served as ephemerides for navigation, or as almanacs for weather forecasts, or were used to establish the festivities in delicate, controversial religious calendars, or for astrological predictions, astronomical observations had a great strategic importance. On the basis of them, General Wallenstein decided his military campaigns. They were like the data used for constructing supercomputers or nuclear weapons today. Apart from the disputes between natural philosophers and theologians, they all guarded them jealously. Galileo Galilei hid his observations on Saturn and Venus behind anagrams, which Kepler in vain sought to decipher. "Haec immatura a me jam frustra leguntur o y." ["These immature things have now been explained by me not at all or y."] By this, he meant: "Cynthiae figuras aemulatur mater amorum." ["The mother of loves imitates the figures of Cynthia [Diana, the Moon]."] He had thus observed the phases of Venus, which, revolving around the Sun, was illuminated by it, like the Moon.[211] Followers of the Christian religions should be well acquainted with the long story of sins and violence related in the *Bible* and in the *Gospels*; this is no reason for their faith to be shaken. But how many followers of the religion of science would be ready to include this theft in their history of progress?

Though I am aware of all the limitations, the choice to leave Kepler in his historical context allows me to avoid the apologetic and promotional caricatures, which bring with them inevitable hints of nationalism: like the German Kepler versus the Italian Galileo, the Frenchman Descartes, or the Englishman Newton. However, it would be useless to ignore that the budding scientific community was to filter only that part that we all know from school textbooks. To understand better how this happened, and the differences with respect to the characters of the following chapter, we shall take up again the comparison with the cultural and scientific values of the Chinese.

In 1623, the Jesuit Father Johannes Schreck (alias Terrentius), whom we have already come across, wrote a letter from faraway China, a copy of which was destined to arrive also in Kepler's hands. ["Let us consider correcting the Chinese calendar, in which the main point is the forecasting of eclipses and the precession of the equinoxes. In this, your Reverences could be of no little help in this task, if you sent at least a short letter on the subject, if it is burdensome to write more,

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[210] Kepler 1945, XIII, n. 113, p. 292. Kepler 1951, XV, n. 357, p. 232. Caspar 1993, p. 87. Koestler 1991, pp. 339–340. Banville 1993, pp. 127–129.

[211] Koestler 1991, pp. 369–370.

and if you had anything new besides what is found in the supplement to Magini [Giovanni Antonio, 1555–1617]: especially about the diameters of the Sun, when they are at the distances from the Earth in the apogee and the perigee, and about the other things that are necessary to calculate eclipses. Perhaps some new hypothesis about the Moon will be presented. I have no doubt that during our absence, others will have been published: for example, Kepler's *Hypparchus*, or other works by [Galileo] Galilei. It would take too long to wait for these books; they are held up, or destroyed, much more frequently than they are delivered, by the majority of means of consignment; it is easier to send a letter, and it arrives here more quickly, without doubt within 3 years. After 2 years, after completing my course of Chinese grammar, there will be the advantage, both of understanding and of writing many things. Now, however, I will indicate just a few.

The Chinese start their histories with Yao, the first famous king; they sustain that these are certain: although they count six kings from Fochi [Fu Xi] to Yao; their histories are full of fables, and no indication of dates is certain. Furthermore, from Yao to the present day, they count about 4,000 years, advancing by cycles of 60, as we do by 15 and 28. In that period, they observed the solstice close to the cusp of Sagittarius, I don't remember the degree: as a result, up to this point, it would have already advanced by 52°. This is in good agreement with the Tycho's results. Shortly after Yao, an extraordinary eclipse was recorded which, they say, touched the second degree of Scorpio. They indicate the year; I don't know whether the hour and the magnitude are known. This matter must be deferred until I talk to the royal mathematicians in Beijing. They have a similar arithmetic to ours, and in the quantities of the year they used fractions with precision up to the times of Yao. They also have certain geometrical problems; recently, I have seen 15 of them which are more than 3,000 years old. Among these, the first [problem] is the penultimate [proposition] of the First [Book of the *Elements*], that is to say, the theorem of Pythagoras.]"

The event that the problems seen in the Chinese book by Terrentius totalled "15" makes it practically certain that the Jesuit had in his hands the *Gouguyi [The right gougu]* by Xu Guangqi, which included exactly that number.[212] Therefore, he had not actually studied what the Chinese had really written, some time before, about the fundamental property of right-angled triangles: the *Zhoubi*. On the contrary, he knew the reduction of the *gougu* [right-angled triangle] to the Chinese translation of *Elements*, written by Euclid, but now reinterpreted here by Ricci and his collaborators converted to Christianity.[213]

> They divide the zodiac into 28 constellations. They call the heart of Scorpio the Dragon's heart, and the tail of Scorpio the Dragon's tail; they call the Canicula [the Dog Star, Sirius] the Wolf, and Capricorn the Ox. They call the last star of Ursa Minor [the Pole Star] the King, because it was once immobile, close to the Pole; all the others were worshipped. A

[212] See above, Sect. 10.2; Tonietti 2006a, Chap. IV, pp. 158–170.

[213] Kepler's editors failed to identify the books implicitly quoted in the passage. Kepler 1993, v. XI 2, p. 513.

few days ago, I took a book containing all the method for the calculation of eclipses. It has not been copied yet. After I have studied it, and understood it, I will be glad to communicate it to Europe. In their calculation, they are not wrong by more than two or three quarters. What an amazing thing that it did not change more in 4,000 years. .... 1623.[214]

As it deals with all the same problems, the Jesuit's letter was almost a summary of another one written by Sabatino De Ursis (1575–1620) to a third brother, with the same aims: to correct the Chinese calendar, to calculate eclipses better, and to obtain pertinent books to be translated. This had been written in Portuguese more than 10 years earlier, in 1612, carefully explaining the differences between the Chinese calendar and the European one, which had recently become the Gregorian one of 1582. For Chinese culture, its importance lay in the marking of actions, forms of behaviour and ceremonies. If the days are not correct, "... todo o Reino fica emgannado ..." ["... all the kingdom is deceived ..."]. For this reason, the Office of the Calendar counted on the "... maiores da Corte ..." ["... leading members of the Court ..."] and was under the direct control of the Ministry of Rites and the Emperor himself.

For a long time, since before the foundation of the Empire, as we learn from the *Zhoubi [The gnomon of the Zhou]*, of which we have already spoken, Chinese functionaries were able to establish the solstices and the equinoxes by measuring the shadow of the sun projected by the graduated *bi* [the pole, the Greek gnomon]. Thus, they had also discovered that it varied from 1 year to another: the Western precession of the equinoxes, called *suicha* [difference of the year] in Chinese. Observing the phenomena of the earth, united to the sky, through the gnomon, and accepting their continuous transformations, with their *lifa* [rules of the calendar] the Chinese imperial functionaries re-established the calendar every year, measuring again whether the winter solstice arrived at the same moment, or not. They had already corrected it several times.

For the reasons that we already know well, some Jesuits had learnt the astronomy and mathematics that were current at that time in Europe, in order to take part in the corrections, and propose their new calendar. Since the time of the Yuan [the Mongols of the thirteenth and fourteenth centuries], the Chinese emperors had maintained a second Office of Astronomy, which carried out its calculations following Arabic traditions. Why should it not be possible, reflected De Ursis, with the help of the converts to Christianity like Xu Guangqi, to replace the Chinese tables and calculations with their own? That is to say, in other words "... como fizerão com as dos mouros ..." ["... to do with us as they did with the Mahomettans ..."]? The eternal theories and the "absolute" truths to which these Jesuits made reference were, at that time, those of Christophorus Clavius and Antonio Magini, and thus took their inspiration from Ptolemy, or perhaps partly from Tycho Brahe, but definitely not from Copernicus.

Like Matteo Ricci, who had recently died, also Sabatino De Ursis relied on his greater precision in calculating the exact moment of the eclipse. Consequently, he

[214] Kepler 1993, v. XI 2, pp. 300–301.

had started to find some support in the imperial Office, for the new corrections to be introduced following the theories of the Extreme West. But we can understand from the letter that obstacles were already blocking the success of the operation. While a first petition had received approval, the emperor's authorisation was slow to arrive. "A este 2° memorial n*ã*o deu despacho nigum até agora el Rei. Dicem, porque no mesmo memorial se pedia desse maior dignidade aos dous mandarins que se havi*ã*o de applicar a isto;" ["This second memorial has not been sent yet by the King. They say that the reason for this is to be sought in the event that in the same memorial, a promotion in position was requested for the two mandarins who were to take charge of this;"]. We do not know whether the diplomatic excuse stemmed only from the Chinese, or also from the Jesuits. However, we have already written that everything ended up in the general expulsion of the Jesuits from China. They must have forgotten that the astronomy of the Arabs had penetrated into the Country-at-the-Centre, because Kubilai Kan had become emperor. Thus they should first have converted the Ming emperor to Christianity, in order to change the basic calendar of Rites, and not vice versa.

De Ursis thus had to leave Beijing, and took refuge at Macao, where he died a few years later. We also know the reasons why the Jesuits were re-admitted to the Empire-at-the-Centre. It is clear that they continued to search for the necessary instruments to succeed in their conquest. The novelties from Europe were now called Galileo Galilei and Johannes Kepler. Everybody desired to use the former's "cannon", or "eye-glass", or "telescope" to use the new name coined by the Accademia dei Lincei. These *wangyuanjing* ["lenses to see far away"] were presented to the Emperor in 1634. "With an instrument of the kind, it is not only possible to observe the heavens, but also to see objects at a distance several *li*, as if they were in front of the observer's eyes. It is extremely useful to observe the enemy within the range of cannon-shots." And thus we again encounter the war between the Ming and the Qing invaders, to which western sciences owed so much of their penetration into China.[215]

Schreck's letter was to take more than 4 years to arrive in Europe, and more time was necessary for it to arrive in Kepler's hands. He was able to reply only in 1627, commenting on it, sentence by sentence, with brief considerations. Among these, we find the following:

["... if the Chinese use astronomic tables in the form of a calendar, I ask, what kind of tables have they used so far, and who was the author of these? 14 years ago, more or less, I read the itinerary of a man of the church, through the land of the Tartars and the kingdom of the Mongols [...] to Cathai or, as I believe, China itself. This author says that for the Chinese, there are two tribunals [the Offices] that are representative of astronomers, who have the power, like priests, of fixing the solemnities: the greater authority is that of the tribunal from which the most reliable forecasts of eclipses come. The author judges that the others use the doctrine of

[215] De Ursis 1612; entirely quoted and translated in D'Elia 1947, pp. 74–117, on pp. 105, 109, 107, 86, 62.

Hipparchus. But there are two other works containing tables, which it would seem to be useful to mention to Father Terrentius, so that he can investigate whether there is any indication of these among the Chinese. The first is the calculation of the Turks, based on the years from the Hijra, for which see Scaliger [Julius, 1484–1558] and others. The Saracens will fill up the whole of the Orient with their inventions and teachings. [...]

The other work is the Persian tables, which use the Jesdagirdis years [which began in 632 A.D. with the Persian king, Jesdegerd III], for which see Scaliger and Christmannus [Jacob, 1554–1613]. These appear to have been only tables for the movements of the Sun and the Moon, and for eclipses. If I am not mistaken, these were copied by Arzachel [al-Zarqali, 1030–1100], and consequently, in the end, knowledge of them was incorporated into the Alfonsine tables, since these also recalled the Persian order of the months. [...] If the kingdom of the Persians thus arrives as far as the Indus, and the Indies border on China, it must be determined whether this knowledge of the tables ever penetrated into the tribunals of the Chinese. .... [....] .....

Perhaps something similar to these approaches may have been appreciated in that period by the Chinese, and accepted, in order to prepare some image of their year. [...] If the shortest cycle of the eclipses is then null, also the longest one will be useless for the periods of the State; it remains [to be understood] how the Chinese use any table for the movements of the Sun and the Moon, and how they try to correct them. [...] Irritated by the many epicycles of Tycho, ...., he will be able to teach the Chinese these eccentrics [of the Moon]; [...] however, if we observe the nature of things, they are not perfect, but degenerate into an elliptic trajectory .... if the Chinese also desired to know for what reasons the Moon abandons its perfect eccentric [circle] in the remaining intermediate positions, Father Terrentius could teach them from my *Commentaries on Mars* and from the *Epitome Astronomiae Copernicanae [Summary of Copernican Astronomy].* There is no circle in the intention of the heavenly mover, but [the ellipse] arises from a material, that is to say, geometrical, necessity; consequently, the trajectory becomes elliptic as a result of the convergence of two causes of the movements, one of which attracts the Moon away in a circle from its setting to its rising, and the other, maintains it in equilibrium in straight lines from the Earth and towards the Earth. The first of these causes has its origin in the non-material species of the Earth. [...] The Moon, in truth, is not fixed to the world with nails, but swims in it, almost freely. This convergence, I say, of two movements gives rise to the elliptic trajectory, which can be demonstrated by a suitable geometrical method. This is the reason why, in the title in bronze of the *Tabulae Rudolphinae [Rudolphine Tables]*, the word "Magnetica" is added on the right-hand side of "Geometria". If the Chinese were instructed in a more complete manner ... then they could be directed towards the other figure on the right, that is to say, the Stathmica [steelyard, from the Greek for "to weigh" "] (Fig. 8.7).[216]

[216]Kepler 1969, v. X, pp. 7 and 21–22. The allegory of Magnetism is at the top on the right, holding in its hand also the needle of the magnetic compass. The one next to it shows the steelyard, which

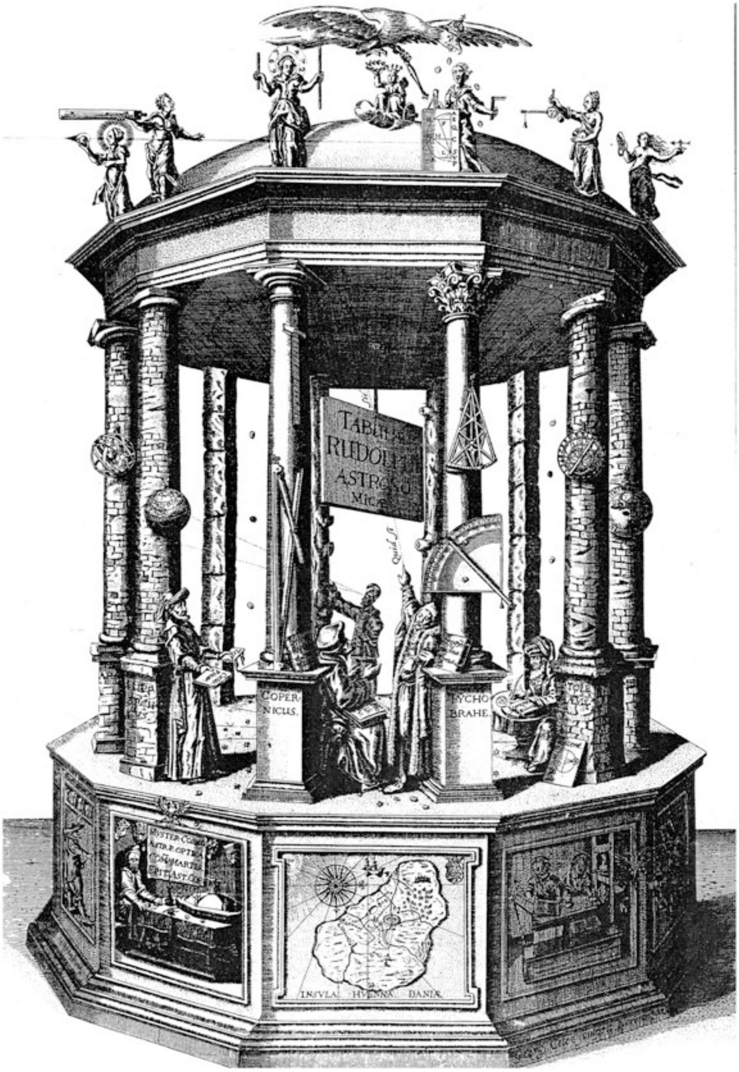


**Fig. 8.7** Title page of *Tabulae Rudolphinae* with the allegories imagined by Kepler (Kepler, Opera Omnia vol. X 1969 title page)

Of the *Hypparchus*, Kepler could only quote the effects contained in the *Tabulae Rudolphinae*. Actually, they had not yet been [and in reality, never were] published, "… if you desire the demonstrations. But there will be a forthcoming edition, if

symbolises the way in that the sun has a stronger influence on the near planets, but less on the farther ones. This is followed, in order, by the Geometry of triangles, the numbers of Logarithms, the Telescope and lastly, the Physics of light.

God grants me life. [. . . ] The reason for the sexagesimal cycle appears to be simply arithmetic elegance, not heavenly movements. [. . . ]" Our European astronomer doubted that the Chinese had kep eclipses for 4,000 years, or that they had used fractions for a long time. "Fractions for the quantities of a year cannot have been stably known ever since the times of observations began: these fractions need to be learnt many centuries earlier. In the meantime, if they have become accustomed to using fractions of years, good; then they will appreciate the fractions of the *Tabulae Rudolphinae*. [. . . ] Already, 1,900 years ago,Euclid made a collection of problems or of theorems known to the ancients. Thus it is confirmed that the Chinese spring from the Ionians, doubtlessly transported there through the Red Sea or the Persian Sea to the islands of the Orient, and from there to the nearby continent."

Kepler commented on what Terrentius said about the theorem of Pythagoras in Chinese, in accordance with the current Western tradition. ". . . that Greek had received the grace to excogitate it from his star, therefore he offered a hecatomb of oxen." The imperial astronomer traced the 28 Chinese constellations back to the Arabs, who would take the number of days and houses from the Moon. ["It would be surprising if all the Chinese doctrine of the stars were not taken from the Arabs, who were the most recent".] He accused Terrentius of believing an idiocy, if he thought that the Pole Star was closer than it is now to the pole of the world. [". . . but if this is the teaching of the Chinese, this proves it is false that this information would be very ancient among them. [. . . ] . . . is it not true perhaps that the Chinese boast, if Terrentius himself makes reference to their experience of certain eclipses? Rather, let them first demonstrate that their stories and their observations of a nature dating back to 4,000 years ago are true. In fact I expressed the suspicion above that those eclipses derived from a retroactive calculation. [. . . ]

Let Father Terrentius communicate to us the height of the pole of that place, or the exact height of the Sun precisely at midday [the latitude] . . . let him also communicate the customs of the people, and what have that is fixed and established in ordering the days of the year, and what they allow to be movable and variable; which beginning of the year they observe, whether the day of the year is certain, or whether the beginning changes, like the Saracens. And then we will help to accommodate it in a kind of year which both corresponds to the sky and is acceptable for the Chinese. Lastly, we will help to convince people, not so much of the laws of our calendar, but of all the gentle yoke of Christianity. . . . Ratisbona [Regensburg], December 1627."]

On the occasion of its publication, on January 15th 1630, the imperial mathematician on loan to Wallenstein also added a short appendix, in which he predicted the eclipse of the sun which was to take place on June 10th [1630][217] of the Gregorian calendar. With the dedication to the great General, the little "commentatiuncula" were at last printed at Sagan. ". . . the farthest limit of Asia, the kingdom of the Chinese, presents a request to the European mathematicians; the mathematicians ask me . . . for advice about how to regulate times in the Orient, and to reply to

[217]An eclipse of the sun actually took place on June 10th, 1630; Espenak 2008.

the letters from China …”. The military victories of his valorous soldier and host had opened up routes towards the seas and oceans, also conquering some of the Baltic regions. “The wars have opened up the Ocean to you, peace will be suitable to maintain its use … For the former, you needed the labours of Hercules … or the rivals of Saturn’s courses, you will be able to arrive at the latter by pouring over that dirt a pleasant thunderstorm, with the nearby size of Jupiter, with justice, I mean, with equity, and friendship, by means of alliances, with clemency, …”.[218]

Thus, in spite of the event that they had generally condemned various Copernicans (except Jesuitically, perhaps, in the silence of their minds), and in particular Galileo Galilei, certain Jesuits did not refrain from exploiting their competence when it was convenient to do so. But the Tuscan astronomer, who was less Jesuitic, would not be pleased, and did not answer the letter.[219] If they wanted his precise calculations for eclipses, they would have to take, together with them, his absolute truth for them. Otherwise, nothing doing. Thus, through the German Jesuit Fathers, the request was passed to Kepler, who was less susceptible, and answered. Together with the *Tabulae*, however, he too offered his model of elliptic orbits, his books and his own theory.

His letters also allow us to understand better to what extent Kepler defended his absolute truth, not only against the rival European truths, but also against the Oriental ones. Thus, as a good astronomer and a militant Western mathematician, he imagined that the Chinese, too, must have derived their astronomical and mathematical doctrines from the Greek cradle, via the Arabs and the Indians. Also fractions, which were so much more convenient, not only for calculations in the field of astronomy, could not have been invented by the Chinese in ancient times. Sure enough, the European imperial mathematician believed that it was necessary, in any case, to go through the same inexorable march, which had taken centuries here in the West: from Greek geometrical proportions to Arabic numerical fractions, which finally passed into Europe. Even the Chinese could not possible have learnt them in any other way than from Pythagoras and Euclid.

In this case, the absolute of scientific truths had been transferred into history, as well, which thus became a compulsory course, without options. Clearly, how could the stars have influenced and led the Europeans and not the Chinese? Here, the idea of a single, universal progress mingled with a weak version of astrology. And, *natürlich*, Kepler modelled all these convictions on his Christian religious faith, equally suited for China, because it was equally universal.

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[218]Kepler 1993, v. XI 2, pp. 299–314. Kepler 1959, v. XVIII, n. 1055 pp. 309 and 534–535, n. 1122 pp. 416–417 and 562. Cf. Caspar 1993, pp. 331 and 350.

[219]D’Elia 1947, pp. 39–48. Pasquale D’Elia S.I., however, underestimated the persecution suffered by Galileo Galilei, and attempted to find in the Society of Jesus some manifestations of sympathy for the Copernicans (though hidden, and ambiguous expressions, such as “… Dominum Galileum mathematicum tradat.” [“… Mr. Galileo is to be treated like a mathematician.”]). He covered up the expulsion of the Jesuits from China in 1616 in an appendix, even calling it a “pause”, pp. 71–72. Cf. Schuppener 1997. The *Rudolphine Tables*, which were a list of numbers, actually arrived in China. Szczesniak 1949. Hammer 1950.

From Terrentius, we learn now that, in spite of the disastrous expulsion of 1616, and undoubtedly thanks to the new merits acquired by means of cannons on the battlefield, the Jesuits continued to hope that they might impose their new calendar, also in China. To beat the Chinese astronomers, they now tried to use the most famous Copernicans' results for eclipses. But the Jesuit, who returned to China, should have written that the Chinese were formidable adversaries, who had been capable of calculating with relative precision for a long time. Both he and Kepler understood that the European times would have to adapt to some Chinese customs. Was it an astute way to attain that Higher purpose declared by both of them? In the face of Chinese scientific culture, the differences between the Ptolemaic and Copernican astronomies had now become secondary, with respect to the main task.

With Galileo Galilei and Johannes Kepler, European scientific values were consolidated around dualistic polarities, as we saw in Chap. II, which were foreign to that faraway Oriental country, predominantly complex and immanent. Even more than Galileo Galilei, thanks to his religious faith spelled out clearly everywhere, and practised with a profound conviction, Kepler justified his scientific laws with the transcendent values that he had found in the Greek neo-Platonic mathematical tradition. Although the imperial mathematician also knew the results obtained by the Arabs, he continually made reference to his favourite world of classical Greece. He expressed his laws in communion with God, by means of the "unique and eternal" geometry of Euclid, reinvigorating it in the comments of Proclus.[220]

He did not love algebra. "Objiciat hic mihi aliquis doctrinam Analyticam, ab Arabe Gebri denominatam Algebram, Italico vocabulo Cossam: videntur enim in ea determinari posse omnis generis Polygonorum latera." ["Here, somebody could oppose to me the Analytical art called Algebra after the Arab Geber, its Italian name being Cossa [thing, unknown]: for in this art, the sides of all kinds of Polygon seem to be determinable."] For example, this was ably done by his friend "Instrument maker [*mechanicus*] to the Emperor", Jost Bürgi (1552–1632), an expert on clocks, and a great calculator of logarithms. Of him, he reported the calculation for the regular polygon of seven sides, which wanted to be excluded, because it was not constructible, and not 'knowable' according to his criteria. He changed Bürgi's symbols, which derived from the Italian tradition, using others "commodius" ["more convenient"], and obtained, in the end, the expression equal to "nihil" [zero].

$$7 - 14ij[x^2] + 7iiij[x^4] - 1vj[x^6].$$ [221]

But the procedure that moved through equations prompted the criticism of our natural philosopher. He underlined the differences and the limitations compared with his beloved geometry. "... plane nihil commune habere cum Definitionibus

[220] Caspar 1993, p. 380.

[221] Kepler 1619 [1969], p. 36. Kepler 1997, pp. 66–69.

nostris … […] … Haec omnia erant factu & possibilia, & faciliora quam dictu, ut norunt qui circinos tractant. Quidne facilius quam rectum angulum … facere, & .... scribere circulum …? .... […] … & miser Calculator, destitutus omnibus Geometriae praesidijs, haerens inter spineta Numerorum, frustra cossam suam respectat. Hoc unum est discrimen inter Cossicas & inter Geometricas determinationes." Algebra [" has absolutely nothing in common with the Definitions we gave above, … All this is possible and easier to do than to explain in words, as anyone knows who is used to handling compasses. For what is easier than to construct a right angle … [or to] draw the circle … ? … the unhappy Calculator, robbed of all Geometrical defences, held fast in the thorny thicket of Numbers, looks in vain to his *cossa* [algebra]. This one is the distinction between Algebraic and Geometrical determination."]

Kepler listed other elements that discriminated between algebra in the Italian version and his Greek geometry, underlining, above all, the discrete, numerical aspects of the former, compared with the continuous quantities of the tradition of the latter. He did not omit to point out that with irrational numbers, the algebraic procedure could only be approximate and inexact. "Concludimus igitur, Analyses istas Cossicas alienas esse a praesenti contemplatione; nec ullum constituere gradum scientiae, cum ijs comparabilem, quos explicavimus in superioribus." ["So we conclude that these Algebraic Analyses make no contribution to our present concerns; nor do they set up any degree of knowledge that can be compared with what we discussed earlier."][222]

The other "Arabic art" opposed by Kepler was the deterministic, predictive conception of astrology, because it was practised, above all in accordance with the style of that other culture. "… this Arabic art amounts to nothing … one must separate the precious stones from the dung, one must glorify the honour of God, …". Among the reasons for his continuous wanderings as in exile, we must also include the Turks, who belligerent were pressing on the borders of the Holy Roman Empire of the Habsburgs. Whether it was out of fear of an invasion, or of the religious differences between Islam and Christianity, Kepler knew, and also undoubtedly used the results of the Arabs in the fields of optics and astronomy, but tended to underestimate them too much.[223] He probably only considered them to be a stage in the course of the absolute truth from Greece to himself.

Faithful to the harmony of music, Kepler also placed on the scale pan of the balance the explanations that connected phenomena together, like the influence of the moon on the sea, or of planetary conjunctions on the weather. Thanks to that, he had encountered some success with the public, and scraped together some money. Together with these, however, the procedures that separated them were increasingly gaining importance. "I am a Lutheran astrologer, throwing out the chaff and keeping the grain."[224] He wanted, where necessary, to distinguish the precious pearls from

[222]Kepler 1619 [1969], I pp. 34–40; Kepler 1997, pp. 66–79.

[223]Caspar 1993, pp. 59, 75, 144, 157, 161, 329, 339.

[224]Caspar 1993, pp. 45 and 183. Kepler 1997, p. xxi.

the acorns, even if in reading the works considered here, I have not come across the famous Latin saying, *Nolite proicere margaritas ante porcos* [Do not cast pearls before swine]. Now, however, he was the one who was doing to the Arabs what we have seen the Arabs had done to the Indians. With the other subsequent characters who were exiting the scientific revolution in Europe, the balance would soon shift towards the art of separating, distinguishing and subtracting, in order to be able to invent transcendent laws more easily. In this, the contribution of the Arabic scientific culture would not be ignored.

We have seen that the whole *Harmonices mundi libri quinque* were based on the distinctions between constructible and non-constructible, knowable and non-knowable polygons, between consonances and dissonances, between hard and soft, male or female intervals, between soul and body, *und so weiter*. Kepler had transformed possible mathematical procedures into transcendent, eternal, universal laws. Chance was excluded. Would the dinner prepared by his wife, Barbara, be so tasty by chance? Would the ingredients mingle together by themselves? "…if pewter dishes, leaves of lettuce, grains of salt, drops of water, vinegar, oil and slices of eggs had been flying about in the air for all eternity, it might at last happen by chance that there would come a salad."[225] If his beloved first wife had prepared that dinner, was it not probable that conjunctions between planets, new or old stars, and astral movements, with the relative propitious or inauspicious consequences, likewise had an author?

In the abundance of secondary literature about Kepler, the curious reader can discover by himself both other interpretations close to my own, and those that are more distant. More frequently, he will find contemptuous judgements of modern university professors, whose salary is guaranteed, inflicted from the heights of their chairs in physics, astronomy and the history of the sciences, on poor Kepler. Just a few examples. For Owen Gingerich, the correspondence between the notes of the scale and the velocities of planets is a "shambles", because these were approximate, and in any case, by adjusting the numbers, it would be possible to find some kind of correspondence with any note "if you hammer away hard enough". Consequently, the whole construction is "… the edifice of a madman…", "… part fantasy and chimera …".[226] But everybody knows that the academic world of today carefully eliminates those who, for any reason, are considered madmen or heretics, preferring to offer work to the thousands of Gingeriches.

Bruce Stephenson moves along the same lines, albeit with tones that are at times apparently softer, given the popularising nature of his work. "… Kepler's harmonic theories were fanciful, even for their time." "… poetic metaphor, but not as scientific truth …". He considers still valid the judgement passed on Kepler in that period by the Jesuit, Giovan Battista Riccioli (1598–1671): "… the evidence … to establish those harmonies seems poetical or oratorial rather than philosophical." Thus, for Stephenson, the *Harmonices mundi libri quinque* were a "… valiant but misguided

[225] Gingerich 1970, p. 297.

[226] Gingerich 1991, pp. 60 and 63.

application of reason. … We judge, with the clarity [sic] of nearly four centuries' wisdom that Kepler was wrong."[227]

I am afraid that, with the "clarity" acquired through faith in the astronomic manuals of the twentieth century, Stephenson has, at this point, lost sight of the roots that Kepler placed in God for his truths. Without this, the truth invented by our imperial mathematician is downgraded, *of course*, to a "… mildly interesting substratum of coincidence[s]."[228] The severe God of Calvin cannot have assigned the same 'grace' to Kepler and to Stephenson.

The historian should be made wary by the event that Stephenson tried to present to us a Kepler who was hesitant about the meaning to assign to his Third Law. "… trying furiously [sic] to assimilate the new discovery into his harmonic speculations and physical theories, [Kepler] was not altogether certain whether the harmonic law was itself one of the Creator's archetypal principles … reveals his uncertainty whether it, too, is a small-eccentricity approximation."[229]

But how could our devout German astronomer have had any doubts, that the sesquialtera ratio 2:3, between the powers of planetary distances and times, might not have been established by God as a universal archetype? In a book where 2:3 was the consonant interval of the fifth between notes, did it perhaps not coincide with the music to which the planets were likewise made to dance?[230] And why, then, should our short-sighted imperial mathematician have worried that not all the numbers were perfectly correct, amid imperfect telescopes, corrections of every kind, boring approximate logarithmic calculations, carried out, undoubtedly, by sinners? Anyway, no trace can be found of the problem speculated about by Stephenson in the abundant relevant literature.

On the contrary, it is fairly easy to understand the reason for the efforts made by Stephenson: separating "… one of his most profound discoveries …" from the remaining dung, to use Kepler's language, which had become such as a result of the evolution of the sciences. But we all know, *pace* all these anachronistic historians, that from manure flowers spring up, but from a faith in reason, only bad philosophies with their relative a posteriori reconstructions.

As if this were not enough, after confessing to a "… relative ignorance of Renaissance musical theory …", our modern-day writer- astronomer compared

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[227]Stephenson 1994, pp. 8, 11, 249 and 251.

[228]Stephenson 1994, pp. 126–127.

[229]Stephenson 1994, p. 141.

[230]In the *Epitome astronomiae copernicanae*, the angle at which the Sun was observed from the Earth was 720 parts of 360°, that is to say, half a degree; this was the same whole number 720, the smallest, with which musical ratios were to be distributed into two octaves. 720 was also the same fraction of 360° at which the Moon could be seen. This allowed the copy of God on Earth, the measurer that is to say man, to observe a total eclipse of the Sun, and to invent astronomy. Kepler 1953, v. VII pp. 277–279. Kepler 1619 [1969], III p. 45–46; Kepler 1997, pp. 192–193. Cf. Koyré 1966, p. 295. In Caspar's edition , Kepler made reference to "Book III", Chap. VI, of the *Harmonices mundi* …, whereas "720" is found in Chap. VII. The printing error is remaining for a long time.

Kepler's distinction between "durus" and "mollis" to the subsequent major and minor tonalities. Thus he was able to discover, again, that our poor natural philosopher based his approach "... on a transitional and incomplete theory of the two types of harmony."[231]

We already have enough indications to stop and think that Stephenson would have not helped us to understand Kepler better, leaving him in his historical context. On the contrary, he would have selected for us what he preferred, in accordance with his own personal (limited?) knowledge of astronomy, physics, logic and music, as it is written today, also ignoring the equable temperament. Perhaps he did not fully realise either that he could speculate as much as he liked on whether Kepler had written "Book V" of the *Harmonices mundi* ..., thus abandoning Ptolemy's edition, only after discovering the Third Law. But then the imperial astronomer would undoubtedly have imagined it consistent in every aspect with his harmony of the world, and not 'hard to assimilate' to the rest.[232] What else would have stopped Kepler from printing the Third Law separately? Does self-contradiction or otherwise, logically speaking, depend on the great difference between historians of the context and historians of *advertising*?

Rightly, also this English-speaking scholar noted that in general, Kepler's harmonic theory had been underestimated. He correctly traced its beginnings back to the letters written in 1599 to Edmund Bruce, Herwart and Mästlin.[233] Here, in fact, we can already see some of the ideas that are included, with adjustments, in the big book of 1619. But Stephenson presented it, as if he had believed that "... equally long strings with tensions in the proportion 3:4 would make the interval of a fourth, as between Saturn and Jupiter; with tensions in the proportion 4:8, they would make the interval of an octave, as between Jupiter and Mars; and so on."[234] If he had really written this, Kepler would have ignored one of the several defects for the ear propagated by the Pythagoreans. As we have seen above, and as is well known, this was the trap into which the musician Vincenzio Galilei and his son Galileo did not fall: the ratios for the tension of strings must increase with their squares, and not linearly.

In his letter to Bruce, instead, Kepler had sustained something different from what Stephenson attributes to him. The imperial astronomer had written to him because he wanted to get the opinion of Italians, and Bruce was then at Padua with Galileo Galilei. "Velim tamen ex aliquo excellenti Musico quibus abundat Italia, discere artificiosam et Geometricam tensionem totius clavichordij, aut si solo aurium judicio feruntur, quaero ex ipsis, an non alicubj in Organis et instrumentis duplex F, duplex A etc. fiat: Et an omnino idem sonet cantio ex G in D transposita. Velim ad jam subjectam formam tendj Instrumentum, et probarj, an aurium quoque judicio satisfaciat. Incipitur a Γ, ex qua D sic tenditur ut consonet (consonet, id est

[231] Stephenson 1994, pp. 13 and 120.

[232] Stephenson 1994, pp. 117 and 141.

[233] Kepler 1949, v. XIV, n. 128 pp. 7–16; n. 130, pp. 21–41; n. 132, pp. 43–59.

[234] Stephenson 1994, pp. 90–91.

sonum eundem teneat) cum 2/3 parte de Γ, C vero sic, ut cum 3/4 parte de Γ consonet, … Hae sunt voces indubiae. Quae sequuntur, dubitationem habent, cum bifariam possint tendj. …" ["However, I would like to learn from some excellent musician, of whom there are plenty in Italy, the fine, geometrical tension [tuning] of the whole clavichord, or if they are guided only by their ears; and I ask the same people if they do not calculate twice F [fa], twice A [la] etc. in organs, and in instruments in some places: [I would also like to know] whether, transposed from G [sol] to D [re], the melody sounds exactly the same. Then I would like the instrument to be stretched [tuned] in the form indicated below, and the question to be asked whether it satisfies also the judgement of the ears. Start from Γ [sol], from which D [re] is stretched [tuned] so that it is consonant (it is consonant, that is to say, it maintains the same sound) with 2/3 of the part of Γ, and thus also C [do], so that it is consonant with 3/4 of the part of Γ … These sounds are certain. Those that follow have an uncertainty when they can be stretched [tuned] on the double value."]

As musicians well know (and here we see that also Kepler knew), but perhaps not all astronomers, the Pythagorean scale does not allow transpositions. That is to say, in this scale, the intervals of the fifth, the fourth, … are not all equal to 2:3, 3:4, … and depend on the note from which you begin. This could be heard perfectly in organs and in harpsichords if they were tuned in the Pythagorean manner. "Sic aures non ablegantur in tensione ad octavas quintas quartas tertias, ubi decipj [sic! decepj] possunt, sed ad unisonum, quem citra errorem dijudicant." ["Thus in the tension [tuning], the ears are not relegated to octaves, fifths, fourths, thirds, in which they may be deceived, but to unison, which they judge without error."] To obtain a precise tuning, our German natural philosopher then described a theoretical instrument that took its inspiration from the cithara, called by him "directorium", with which it was possible to verify in unison the other instruments used in practice. All the numerical proportions referred here to the lengths, and he made no mention of the tension of the strings. For that, he would have had to deal with relative weights.

Stephenson's error was due not only to his admitted ignorance of the musical problems that afflicted people at the time, above all for fixed-tuning instruments like organs and harpsichords, on which it would not have been possible to modulate. He also suffered (his?) bad translation. In this case, the Latin *tensio*, which derives from *tendo*, is not to be translated into English as *tension*, but in general as *tuning*. After all, how could you stretch an organ pipe? This Bruce should therefore have read also Vincenzio Galilei and have addressed those Italians whose help the imperial astronomer sought in that period. At the end of his letter, through his English intermediary, Kepler asked Galileo Galilei a question about the local magnetic declination for the compass, referring to Mercator. It is thus clear that the letter would be shown by Bruce to the Italian astronomer. In actual fact, it ended up among Galileo's manuscripts kept in the Biblioteca Nazionale in Florence.[235] This, then, is likely to be the route by which our German natural philosopher may have

[235] Kepler 1949, v. XIV, pp. 13–15 and 459. Dickreiter 1973, pp. 76–77 and 130.

obtained an indication of the book *Dialogo della musica antica et moderna* of the Italian musician Vincenzio Galilei. On it, he subsequently meditated at the time of his mother's trial and his drawing-up the complete version of the *Harmonices mundi libri quinque.* But unfortunately, this same route would not be taken by Stephenson, who dedicated himself too much to astronomy, and too little to music, with the unhappy consequences that we see.

In the letter to Herwart, the question briefly cropped up again. "… exercitatus Organista posset solo aurium judicio de clavj judicare, vel nunquam alias audito illo Organo. Videor olim Stuccardiae aliquid hujuscemodj inaudijsse de duodecim Italorum tonis, et de geminatione quarundam atrarum clavium." ["… the expert organist might decide on the keys with the sole judgement of his ears, or otherwise, having listened to the organ, never. I think I heard something of this kind at Stuttgart, about the 12 notes of the Italians, and about the doubling of certain black keys."] On this subject , Kepler wrote that he would have been glad to ask Roland de Lassus, if he had still been alive, but this renowned Flemish polyphonist had died shortly before, in 1594, at Munich.[236] May he perhaps have heard that the Italians, including Vincenzio Galilei, tuned their organs and harpsichords by ear, dividing the octave into 12 equal intervals? But he probably had not yet read the book in which this musician had invented rules, which were not Pythagorean, to tune strings and wind instruments. At the time, Kepler instead had started to think about a way of explaining, by means of geometry, why, out of all the infinite possible proportions, music did not use more than seven. Here he wrote them as

$$\frac{1.1}{2} \quad \frac{1.2}{3} \quad \frac{1.3}{4} \quad \frac{1.4}{5} \quad \frac{1.5}{6} \quad \frac{2.3}{5} \quad \frac{3.5}{8}$$

Or, perhaps, our imperial mathematician had heard about Heinrich Loriti, known as Glareanus (1488–1563), who had theorised 12 musical modes, instead of the ecclesiastic 8? This Swiss theoretician was to be quoted in his subsequent letter to Mästlin. Here, he also proposed an analogy with the colours of the rainbow, again classified on the basis of their proportions, sustaining that the same could be done with tastes and smells.[237]

The same approach, of judging Kepler's musical theories by means of modern sciences, almost completely ignoring the historical behaviour of the imperial mathematician, was adopted also by Francis Warrain. However, this French scholar arrived at conclusions at variance with those of the two preceding severe critics. This study, relatively scrupulous in its attention to the *Harmonices mundi* …, aimed to show that the results obtained by Kepler in that period not only should not be disputed, but could also incorporate the movements of the asteroids and the new planets discovered subsequently. Even the luminous frequencies of atoms were compared to the musical notes of the planets. Thus it is not so much a question

[236]Kepler 1949, v. XIV, pp. 29. Dickreiter 1973, p. 130.

[237]Kepler 1949, v. XIV, pp. 50–51. Dickreiter 1973, pp. 126–127.

of denying the harmonies, but rather of extending them to chromaticisms and enharmonic relationships.[238]

Equally well-disposed to follow Kepler in his metaphysical and musical arguments, though enhancing the historical events of his life, Alexander Koyré wrote about him. It is interesting that also this para-French philosopher and historian presented his efforts as aiming at the truth. And for Koyré, as for our imperial astronomer, this truth was to be reached in a relative independence from observational astronomy. For this reason, he should take a greater interest than others in the *Harmonices mundi* … , albeit, unfortunately, only in his "Book V": because this was the one which contained the Third Law. And yet, even Koyré ignored the event that Kepler had more faith in God than in human reason. Our para-French scholar even mistook him for a "faithful follower" of Pythagoras, because he left out his geometrical theory of music. In the end, this philosopher of science dropped his apparent tolerance, and sentenced that the tortuous route towards the truth, from the closed spheres of Kepler, would end up in the Infinite space of Newton. Here, "the thought of the divine Mathematician" would prove to be "completely devoid of any musical inspiration".[239] In Sect. 10.2, on the contrary, (naturally?), we shall see the musical model invented by the English scholar for that gravitational space so dear to the physicists of today.

Without the usual undisguised simplifications and aims, Gerald Holton sums up the complexity of Kepler quite well, were it not for the event that the reduced him excessively to the Pythagoreans, and completely eliminated music from his mathematical harmonies.[240] Arthur Koestler told us the novel of the life and works experienced by the imperial mathematician, trying to depict him as irrational, a fool, "… a mad spokesman of the Holy Spirit …", a "… mad architect …". He had decided to follow him along the course, full of shifts and second thoughts, of his discoveries, and through the unfathomable meanders of his explicit confessions. But our wandering Hungarian polyglot, essayist and writer concluded that his work was a "… Pythagorean synthesis of mysticism and science …", confusing him with Fludd and the Rosicrucians. And yet, for him, Kepler succeeded (only in some of his works?) in separating astronomy from theology, to unite it with physics (that of Aristotle?). In short, he obtained, like a sleep-walker, those customary inedible results that he did not understand himself, which were destined to come to light with their (rational?) realisation only with Newton.

Koestler succeeded in the task of giving an effective summary of the *Astronomia nova*, with all its mistakes, which however led our sleep-walker towards the truth of the first two laws. But as regards the context for discovering the Third Law, he could only allude to "… long, patient endeavours …".[241] However, that chatterbox of Kepler, who loved digressing on every subject, did not tell us them as such in

[238] Warrain 1942.

[239] Koyré 1966, p. 308.

[240] Holton 1993, pp. 3–23.

[241] Koestler 1991.

this case, whereas in the other he had considered it in the reader's interest to do so. May this therefore not have happened because the story had gone differently? Our Hungarian nomad, who wrote in English, was likewise ignorant of music, and therefore he could not understand that on the contrary, a "sesquialtera" ratio could be intuitive, and 'the object of a dream' in a book about harmony. Astronomy had led Kepler to discard a linear ratio between the times of revolution T and the distances R of planets from the Sun ($T^1$ proportional to $R^1$; 1:1, unison). They would become too fast, and Saturn, for example, would have to complete its revolution in just 9 years, since it is about 9 times farther from the Sun than the Earth. A double ratio ($T^1$ proportional to $R^2$; 1:2, octave) would not work, either, because the planets would move too slowly, and Saturn would have to take as long as 81 years to complete one revolution. The intermediate proportion between the two was the "sesquialtera" ($T^2$ proportional to $R^3$; 2:3, fifth), which became easy to imagine by analogy, and spontaneous when writing a book about musical intervals in astronomy, because it corresponded precisely to one of the most consonant ratios. At present, Saturn completes one revolution in about 30 years.

The musical arguments of our German natural philosopher have been of interest almost exclusively for historians of the relative theories and musicologists. Among them, Daniel Pickering Walker would appear to be the one most deeply struck. However, he suffers from the defect of judging Kepler's arguments by the ensuing justifications of consonances through harmonic tones. Therefore, as these would revitalise, by means of modern-day acoustics, some numbers of the Pythagoreans, the geometric theory of our Baroque character "... does not work very well ...", above all for thirds and sixths. It is clear that Kepler derived his judgements (and prejudices) from a desire to submit music to (and discipline it by?) his philosophical-scientific convictions. We have already seen proof of this in the chapter about Vincenzio Galilei. Here we might add a couple of his observations about the Aristoxenians, who are considered "... opposed to our tradition ...", and about the superparticular ratios of the Pythagoreans, seen as verified "... empirically on the monochord ...". When, on the contrary, the best empiricists, thanks to their ear, were to be the former, who were also present in the Western tradition with Vincenzio Galilei.[242]

Told by Michael Dickreiter, the story of how much music Kepler must know, and how he took part in the musical life of his time, is extremely interesting. We must remember the importance given to music in the Lutheran liturgy. At school, he sang every day, and received lessons of musical theory. We have seen that our imperial mathematician quoted some of these hymns in his *Harmonices mundi* .... At Graz, he also would have encountered music in the Italian style, like that of Andrea Gabrieli (c. 1520–1586) and Orazio Vecchi (1550–1605). Nor should we forget Roland de Lassus and Adrian Willaert (c. 1490–1562). At the court in

[242]Walker 1967, pp. 238–239; Walker 1989a, pp. 72–73. Perhaps for the same reason, our English musicologist treated Vincenzio Galilei as a Pythagorean, while on the contrary, this Tuscan lutenist was rather on the side of Aristoxenus, as we have seen above in Sect. 6.7 of Part I.

Prague, our imperial astronomer may have met, among the most renowned artists, Philippe de Monte (1521–1603), Hans Leo Hassler (1564–1612) and Jacob Hassler (1569–1622). Here, they played various styles of music, from the kind with several choruses of the late Venetian Renaissance to the early accompanied monodies. The religious persecutor at whose hands Kepler suffered at Linz, Daniel Hitzler, took pains to correct also the hymn-books of that church. Would discussions take place between him and our heretic also about Luther's music, or what the correct hymns for a church should be? Kepler once wrote about a concert that he heard in the house of a noble.

Dickreiter concluded: "… wir bei der Untersuchung seiner Musiktheorie eine Kenntnis der Musik an der Wende vom 16. zum 17. Jahrhundert voraussetzen dürfen, die sowohl theoretischer Lektüre als auch in praktischer Übung erworben worden war." ["… together with the research on his musical theory, we may assume a knowledge of music about the turn of the sixteenth and seventeenth centuries,which he had acquired both in his theoretical readings and in practical exercises."][243] But our natural philosopher admitted that he did not practise the art of composing. "Artem vero componendi cantus, quae praxis est Musica, nequaquam profiteor." ["However, I do not in any way profess the art of harmonizing the singing, which is the practice of music."][244] Before returning to his Venice, Andrea Gabrieli had passed through Bavaria and Bohemia, where Kepler had lived. With his madrigals for the theatre, Orazio Vecchi must have been appreciated more highly at the court receptions attended by our German astronomer. Though his ideas of the solar system attracted him more towards the vocal polyphony of the period, it cannot be excluded that with a certain licence, he might permit himself also to enjoy some hidden Mediterranean melodies here and there.

Like every good musician and musicologist, Dickreiter noted the limitation of the choices made by Kepler for the ratios between the notes, compared with the transpositions to the different levels of the scale, and with modulations.

Without doubt, Kepler also described the equable termperament, which would make them possible without any difficulty, as we have already seen above. "Kepler ist damit der erste deutsche Musiktheoriker, der die gleichschwebende Stimmung exact beschreibt, soweit das mit den Mitteln der damaligen Mathematik möglich war." ["Thus Kepler is the first German theoretician of music who described the equable temperament with precision, as far as possible with the means of mathematics available in that period."] But one line above, our German-speaking musicologist had written that the ratio 18:17, taken as a semitone by Vincenzio Galilei, was only an approximation for this temperament.[245] Was it possible he did not know that Kepler should have calculated $\sqrt[12]{2}$, the exact value of the tempered

[243]Kepler 1955, v. XVII p. 52. Dickreiter 1973, pp. 123–138.

[244]Kepler 1619 [1969], III p. 85; 1997, p. 254. Dickreiter 1973, pp. 145–146.

[245]Dickreiter 1973, p. 158.

semitone, perfectly well?[246] Thus, he could not do it, because otherwise, he would have had to abandon his musical-astronomic construction based on polygons. However, for the imperial mathematician, sounds had become vibrations, and the string half as long, that is to say, tuned to the higher octave, produced twice as many.[247]

How the *Harmonices mundi* … was interpreted and evaluated by Marin Mersenne, John Wallis and Isaac Newton, we shall see in the following chapters. Athanasius Kircher refuted it in his *Musurgia Universalis*. Besides being Catholic, and sustaining the astronomical model of Tycho Brahe, the German Jesuit who had migrated to Rome found Kepler's harmony "… obscuram, & mysticis verborum involucris intricatam … […] … Sed hoc nequaquam prudenti astronomo satisfacere potest, cum nobis non constet utrum proportio motuum ex natura rei ita se habeat; cuius tamen diversitas observationum, prorsus contrarium ostendat, & proinde nihil certum & solidum ex hisce circa harmoniam coelorum concludi possit; …". ["… obscure and couched in mystical coverings of words ... .... But this cannot satisfy a prudent astronomer at all, if there is no proof for us from the nature of the thing, whether the proportions of movements are exactly like that; however, the diversity of his observations shows exactly the opposite, and therefore from these nothing certain or solid can be concluded about the harmony of the heavens; …"].[248]

Less orthodox and conventional were the comments on the *Harmonices mundi* … made by Andreas Werckmeister (1645–1706), one of the first musicians and musicologists to sustain the validity of the equable temperament. Even if he did not always seem to understand all the aspects perfectly, especially the mathematical ones, he described it in the following terms. "Wie er dann an unterschiedlichen Orten das Temperament approbiert, und unter andern des Vincentii Galilaei Fleiß so er in Verfassung der Temperaturen angewendet sehr lobt." ["As he [Kepler] then approves of the temperament in various places, and among others the diligence of Vincenzio Galilei, so he expresses great praises in the composition applied to the temperaments."] But then, in his last (posthumous) theoretical work, he must have had doubts that in this way, he might have misunderstood Kepler. In reality, the imperial mathematician had tolerated the temperament of Vincenzio Galilei only for instruments with a fixed tuning, because he only considered the numbers of Pythagoras, Plato and Euclid to be truly harmonious. "… so saget er es sey ein höchst schädlich und verfürisch Ding wenn man durch die temperirten harmonischen Zahlen in rebus Musicis ein Judicium anstellen wolte das rechte Wesen des Gesanges dadurch zuerlangen." ["… Then he [Kepler] states that it is an extremely harmful and seductive [misleading] thing if by means of tempered

[246]The approximation of Vincenzio Galilei appears to be very good, because 18/17 equals 1.05882, compared with the root 1.05946 that is obtained from Table 8.3 of Zhu Zaiyu.

[247]Kepler 1940, v. VI, p. 388. Dickreiter 1973, p. 159.

[248]Kircher 1650, pp. 376 and 378. Dickreiter 1973, pp. 193–194.

harmonious numbers, one wanted to formulate a judgement about musical matters, to achieve in this way the correct essence of singing."][249]

It is therefore a nice paradox that Werckmeister had learnt how Vincenzio Galilei tempered his notes through our natural philosopher, who sustained something completely different. As in certain scientific questions, also in music Kepler had obtained the opposite effect to the one that he desired, in a world that was about to celebrate the splendour of the accompanied monodies, leaving aside his beloved polyphony *à la* Roland de Lassus.

In an admirable extract of fewer than ten pages, to which our modest character was reduced, the search for music would be vain. Rather, we would find "mechanicism", a profound philosophy for which the reader will judge by himself the degree of agreement with Kepler's *anima mundi*. Here we also find the sparkle of a beautiful pearl. In the Olympic contest disputed between Galileo and Kepler about who arrived first among modern scientists (in reality those of the 1960's), this general philosophical-scientific history in many volumes assigned the victor's palm to Galileo: "... Galileo occupa una posizione in un certo senso superiore a quella di Keplero." ["... Galileo occupies a position which in a certain sense is superior to that of Kepler."][250] The German's diversity had again been transformed into inferiority, with the aggravating circumstances of those intolerable nationalistic veins from which we particularly desire to stay at a good distance.

On the contrary, Silvia Tangherlini dedicated all the required space to underlining the Platonic and Pythagorean ideas present in the *Harmonices mundi* .... To find some of these roots also in the Renaissance of Marsilio Ficino,[251] this Italian scholar highlighted his effort to reconcile the neo-Platonism of Proclus with Christianity. Inhabiting in the Sun, depended on his "rational" God Kepler's decision to remain as far away as possible from any immanent form of the infinite. It was reduced, also by Kepler (hiding it from us), to the incommensurables of geometry, capable of avoiding numerical irrationals, as in the tradition of Euclid.[252] It is from these convictions that we can understand why the search for the truth in transcendence, away from sensible world, was to remain so often present and characteristic of Western sciences. Refusing the infinite for the universe left the Sun in the central position. Not using it in the ratios between notes made it possible to select those seven rational ones, excluding the equable temperament.

The fine illustrations reproduced in the popularising work written by Anna Maria Lombardi about Kepler do not exempt us from the thankless task of indicating errors, omissions and sloppiness. The complicated course towards the first two laws, and the results obtained in the field of optics are presented with interesting details ignored by us. But at a certain point in the text, the Third Law is enunciated in a nonsensical manner. In another table, certain ratios for notes are exchanged, with

[249] Dickreiter 1973, pp. 197–198.

[250] Geymonat 1970, v. II pp. 105–107 and 506–514.

[251] See Sect. 6.4 of Part I.

[252] Tangherlini 1974.

the addition of a debatable comment. Above all, the faith of our German astronomer in his variant of astrology is totally distorted, completely overlooking "Book IV" of the *Harmonices mundi* .... I will leave it to the reader to decide how serious this was, due more to the chief editor of the journal than to the authoress. A blur thus descends over the merits of this Italian graduate in physics and lover of the violin, in enhancing the essential function of music in the overall work of Kepler, which partly inspired the famous 3/2 of his last law. Unfortunately, also on this subject, she expresses anachronistic judgements, and ignores the equable temperament, perhaps because she is less sensitive to it, not playing the piano, but the violin. It is more serious that she cancelled the general polyphonic sense from the general structure of the *Harmonices mundi* ..., and condemned the work as a whole as "Attualmente ... non solo privo di senso, ma anche scorretto ..." ["At present ... not only without sense, but also incorrect ..."]. What is more, (perhaps she had not read Dickreiter?), she presents us with a Kepler "... rimasto isolato dalle novità musicali che si stavano sviluppando in Italia ..." ["... who was isolated from the musical novelties that were developing in Italy"]. Above, we have just seen the opposite, thus he knew them, and for clear reasons, he did not agree with them.[253]

To those who still believe in the exclusive resolving force of experiments, I would like to propose the following one. To verify whether the planets in their revolutions emit music that can be heard by our ears, and what this music is, the solar system would need to be filled with an adequate terrestrial air. Kepler wrote, "Jam soni in coelo nulli existunt nec tam turbulentus est motus ut ex attritu aurae coelestis eliciatur stridor." ["... No sounds exist in heaven, and the motion is not so turbulent that a whistling is produced by friction with the heavenly air."][254] The experiment would be difficult to perform, not so much because it would be a bit expensive, but because nobody knows how to divert military expenses for this purpose. In the era of the supercomputer, however, it might be possible to use it to simulate a solar system that revolved, not (unstably?) in an ethereal void, but in air. What notes would then be produced?

Not only colleagues and religious figures of several faiths have continued to persecute Kepler for centuries. Another war did so, 300 years later, harassing the person of Max Caspar. Under the bombing raids, he wrote the best biography of Kepler, while another Thirty-Years-War were finishing, the years that began in 1914, and were to come to an end (maybe) in 1945. Curious, isn't it? Was it Saturn again that was in opposition to Jupiter in Capricorn?

Recently, Kepler's hand-written comments on the *Dialogo della musica antica et moderna* printed by Vincenzio Galilei have been published. Among the most interesting observations, we can read the following. With the ratio "$\frac{\sqrt{72}}{9}$", the German astronomer could divide the Pythagorean tone 9:8 into two equal parts, even if he did not want to accept it. Sheet after sheet, he noted down the protagonists, ancient and modern, of the dispute. In the one between Zarlino and Vincenzio

[253]Lombardi 2000, *passim*.

[254]Kepler 1619 [1969], V p. 197; 1997, p. 423.

Galilei about the intervals of the third and the sixth taken from Ptolemy by the Venetian, he wrote, "Negat Galilaeus. Sto à Zarlino, ut in secundis teneam Galilaeum. ... pugnant pater et filius ridiculè, utri Keplero nomen." ["[Vincenzio] Galilei denies it. I am on the side of Zarlino, as I consider Galilei the second. ... father [Vincenzio] and son [Galileo] quarrel in a ridiculous manner, both the one and the other contend for the name with Kepler."] When he found Aristoxenus in the text, he generally criticised him.

As regards the main question of the choice between the ear and numbers, here Kepler expressed his side particularly clearly. "Auditus servus est ratio domina, incipit cognitio ab auditu, sed ratio consummat harmonices perceptionem, ... [...] ... Pythagoras numeros spectavit, praeterijt itaque consonantias imperfectas: non satis attendit sensum, indulsit speculationi. Aristoxenus attendit sensum, qui cum esset crassus, nimio plus indulsit sensu, et cum latitudine nimia assumpsit speculationum principia. Ptolemaeus obtinuit sensum cum ratione conjunctum, at in Aristoxeno habet commune quaedam discordantia, sed non consentiunt ... anima concordantibus cum Pythagoraeico vero contemplationes numerorum, ... Ego rursum ad Aristoxeni sensus accedo, sed illos instruo sensus ratione aliter quam Ptolemaeus: Hic vult ratione super numeris de parte dextra e propinquo audiri, sensum eminus et a parte sinistra astipulari, ita ratio illi seducit sensus quia est erronea. Ego verò sensui rationem addo informatorem et quasi paedagogum in qua discat sua vi uti et rationem non super numerorum arcanis, sed super communibus axiomatibus dialecticis impossibilitatis." ["The hearing is the servant, reason is the mistress; knowledge starts from the hearing, but reason perfects the perception of harmony, ... [...] ... Pythagoras examined numbers, and thus overlooked the imperfect consonances [third and sixth]: he did not observe the sense [of hearing] sufficiently, he took the liberty of speculating. Aristoxenus observed the sense [of hearing], but as this was unrefined, he took the liberty of listening too long, and assumed too many speculative principles in abundance. Ptolemy held sense fast together with reason, and yet he has in common with Aristoxenus some dissonances, but the considerations of numbers, together with the truth of the Pythagoreans, do not reconcile consonances with the soul. On the contrary, I am close to the senses of Aristoxenus, but I prepare the senses with a reason different from that of Ptolemy. On the right, he wants with his reason that the sense be listened very closely, more than the numbers, and also on the left, to fit in with them from far off; thus his reason pulls the senses to himself, because it is wrong. I, instead, add to the sense the reason that guides, a kind of educator, in which you learn to make use of its force: also, a reason, not over the arcana of numbers, but over the general axiomatic dialectics of impossibility."][255]

Let us stop here. We may observe that, with Galileo Galilei and Johannes Kepler, the journey of European natural philosophers towards transcendent (religious in part) and dualistic scientific truths, expressed by mathematics, was now well under way. They were starting to lose Latin as a common language, to speak their various

[255] Kepler 2009, pp. 21, 24, 28–29. Cf. Kepler 1619.

national languages (all alphabetic and linear, with that dualistic characteristic of being able to use small or capital letters): Galileo did this most. But wouldn't those local, contingent terms, subject to their relative history, have cast doubts on the great claims of constructing laws outside space and time, with a consequently universal and eternal validity? Without doubt, Kepler still thought that Latin should continue to be the perfect language, not only for Christianity, but also for sciences, as participants in the truths transmitted by God to the souls of men. How, then, were they to solve this problem? The following natural philosophers were to decide differently, inventing a special symbolism for mathematics, made independent of national languages and therefore suitable for that purpose.[256]

> They castrated the Poet, so that he would not be lewd,
> and after taking the testicles away, the life remained.
> Beware, Pythagoras, they say you think too much:
> they leave the life and first destroy the mind.
> JohannesKepler

> You also know that the sciences
> that I love and cultivate most
> are those of higher mathematics
> thanks to which I subtract from time
> and thanks to which I steal from fame
> the right and the task
> of producing new discoveries;
> for when in my tables
> I descry the novelties
> of the centuries that will come,
> I take from time the privilege
> of narrating what I have said.
> Calderon de la Barca, *Life is a dream*, I 612–623.

[256]For different points of view, see Gozza 2000.

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