Chapter 2
The Polish School of Mathematics

This chapter presents the philosophical views on mathematics and logic that appeared in the papers (and research practice) of the representatives of two main mathematical centres in interwar Poland, namely the Warsaw one and the Lvov one.

2.1 Warsaw School of Mathematics: Sierpiński, Janiszewski, Mazurkiewicz

Speaking about the philosophy of mathematics in the Warsaw School of Mathematics one must recall three figures: Wacław Sierpiński, Zygmunt Janiszewski and Stefan Mazurkiewicz. Their philosophical views on mathematics were expressed primarily in set theory.

However, let us begin with Sierpiński's habilitation procedure, which took place in 1908. His habilitation lecture, delivered during the meeting of the Council of the Faculty of Philosophy at the Jan Kazimierz University in Lvov, concerned a certain issue of the philosophy of mathematics. Its title was 'Pojęcie odpowiedniości w matematyce' [The Concept of Correspondence in Mathematics]. Then the lecture was published as a paper (bearing the same title) in *Przegląd Filozoficzny* [Philosophical Review] in the year 1909.

Sierpiński aimed at reflecting on the role and significance of the concept of correspondence in mathematics. He examined various disciplines that embraced this concept, paying special attention to the concept of equipollency of sets and cardinal numbers, operations, analytic geometry, complex numbers, geometry (in particular cartography, projective geometry and descriptive geometry), analysis and the concept of function. He concluded that the concept of correspondence was one of the most important mathematical concepts, writing:

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> It penetrates all areas of mathematical thought; it is the basis on which we build other fundamental concepts; it is the source of all the most wonderful ideas (1919, p. 8).[1]

Sierpiński justifies this fact by quoting Poincaré's statement from *La Science et l'hypothèse*:

> Mathematicians do not study objects, but the relations between objects; to them it is a matter of indifference if these objects are replaced by others, provided that the relations do not change (1905, p. 25).[2]

Finally, Sierpiński postulates:

> [. . .] the fact that the science—thus separated—which is mathematics, finds so many real applications can be explained by the existence of perfect correspondence between the domain of abstraction and the domain of true reality (1909, p. 19).[3]

It is a strong thesis regarding one of the fundamental issues of the philosophy of mathematics, namely the problem of relations between pure mathematics and applied mathematics as well as the problem of the mathematisation of the physical world. In fact, Sierpiński neither solves these problems nor justifies his thesis, but at this point, it is not the most important thing. What is essential is the fact that he chose a problem of the philosophy of mathematics as the theme of his habilitation lecture.

Several years later it was Zygmunt Janiszewski that made a similar choice. Although his *Habilitationsschrift* concerned topology, he decided to lecture on the problem of the dispute between realists and idealists in the philosophy of mathematics during the session of the Council of the Faculty of Philosophy of the Jan Kazimierz University in Lvov, held on 11 July 1913. The title of his lecture was 'O realizmie i idealizmie w matematyce' [On Realism and Idealism in Mathematics]. It was published, bearing the same title, in *Przegląd Filozoficzny* in 1916 (as was Sierpiński's lecture).

The debate between realism and idealism had been held in the philosophy of mathematics since the very beginning (cf. ontological concepts concerning mathematical objects, which were put forward by Plato, recognised as the father of idealism, and by Aristotle, seen as the father of realism). Its apogee fell at the turn of the twentieth century because of Cantor's set theory, especially after 1904 when Zermelo proved the well-ordering theorem, which turned mathematicians' attention

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[1] 'Przenika ono wszystkie dziedziny myśli matematycznej; jest podstawą, na której budujemy inne zasadnicze pojęcia; jest źródłem wszystkich najwspanialszych pomysłów.'

[2] 'Les mathématiciens n'étudient pas des objets, mais des relations entre les objets; il leur est donc indifférent de remplacer ces objets par d'autres, pourvu que les relations ne changent pas' (1902, p. 32).

[3] '[. . .] fakt, że nauka, tak oderwana, jaką jest matematyka, znajduje tyle zastosowań realnych, wytłumaczyć daje się istnieniem doskonałej odpowiedniości między dziedziną abstrakcji a dziedziną realnej rzeczywistości.'

to the (controversial) axiom of choice.[4] The debate comes down to the question, 'What does it mean "to exist" (in mathematics)?' Let us notice that both the axiom of choice and Zermelo's well-ordering theorem announce the existence of certain objects (the axiom of choice—the set of selectors; Zermelo—the relation of well-ordering) in a non-constructive way, i.e. they do not give any information about the postulated objects: how to construct them.

In his paper Janiszewski analyses the statements of realists and points to the difficulties they faced. He also discusses the necessary and sufficient conditions for existence in mathematics. Naturally, non-contradiction is a necessary condition. But is it sufficient? Idealists claim that it is. In their opinion, 'being' means 'being non-contradictory.' Realists say that the answer is negative, i.e. in mathematics what has 'a (good) definition' exists (1916, p. 163). Obviously, this leads to another problem: what does '*good* definition' mean?

Consequently, according to realists a set is defined when—if one cannot define all its elements individually—at least the law of construction of any element of the set is given (cf. 1916, p. 168). Whereas idealists claim that one can define a set without defining its individual elements. A set is defined when we have a membership criterion (Cantor accepted this principle).

Janiszewski concludes that in the philosophy of mathematics the debate between idealists and realists shows that:

> [. . .] contrary to the spread belief about the complete obviousness and certainty of mathematical argumentations here we can also encounter controversial problems (1916, p. 169).[5]

Such cases were numerous in mathematics. However, solutions were always found. Will this apply to the philosophy of mathematics? Janiszewski gives a pessimistic answer, stating:

> One should doubt it. Since the diversity of philosophical views, which is revealed in this dispute and which is its source, is this eternal difference that caused controversies between nominalists and Platonists throughout the Middle Ages, this dispute has lasted until today as the conflict between positivism and idealism (1916, p. 170).[6]

It is worth noting that Janiszewski does not support any party of the debate. He only presents various standpoints and arguments, which is—as will be seen—a typical attitude of the Warsaw mathematicians' environment.

Yet, let us return to the question posed while presenting the theme of Sierpiński's habilitation lecture. Let us ask why Sierpiński and Janiszewski chose

[4] The well-ordering theorem is equivalent to—on the basis of a proper axiomatic system of set theory—the axiom of choice. On the theme of these relationships as well as the history, status and meaning of the axiom of choice in mathematics see Murawski (1995), Appendix I.

[5] '[. . .] w przeciwieństwie do rozpowszechnionego mniemania o bezwzględnej oczywistości i pewności rozumowań matematycznych i tu spotykamy kwestie sporne.'

[6] 'O tym należy wątpić. Różnica bowiem filozoficznych poglądów, która się objawia w tym sporze, która jest jego źródłem—jest tą odwieczną różnicą, która powodowała przez średniowiecze ciągnący się spór między nominalistami a platończykami, który ciągnie się i dziś między pozytywizmem a idealizmem.'

issues concerning the philosophy of mathematics although they were 'purebred' mathematicians. Could the fact that their habilitation procedures were conducted before the Council of the Faculty of Philosophy, embracing mostly humanists and not mathematicians, have been a decisive factor? Could the scholars not have been interested in a strictly mathematical (more technical) theme? After all, Sierpiński and Janiszewski might have chosen some popular mathematical issue. The fact that they chose themes pertaining to the philosophy of mathematics shows that at the Lvov University the intellectual atmosphere was good as far as the foundations and philosophy of mathematics were concerned, and both mathematicians were interested in mathematics as such as well as its philosophical problems. Moreover, both were convinced that in Poland there was a need for some definite conception of growth in mathematics so that this discipline could be practised and developed. They wanted to define its methodological foundation, which should be set theory (as will be seen later).

Janiszewski showed interest in the philosophy of mathematics and was convinced about its significance much earlier, on the occasion of the publication of *Poradnik dla samouków* [A Guide for Autodidacts] in 1915. He was the soul of the whole undertaking and author of the biggest number of papers published in the guide. Besides the introduction and conclusion as well as the information chapter he wrote papers on differential, functional, difference and integral equations as well as on series, the foundations of geometry, logic and the philosophy of mathematics.[7]

The last two papers, i.e. 'Logistyka' [Logistics] (Janiszewski 1915a) and 'Zagadnienia filozoficzne matematyki' [Philosophical Problems of Mathematics] (Janiszewski 1915b), are the most important ones from our perspective.

The first paper, 'Logistyka', presents mathematical logic (called symbolic logic or—the special term used then—logistics). Janiszewski begins by explaining the reasons why this book, dedicated to mathematics, speaks about logic. He mentions four:

a) logistics is formulated as calculus (*algebra of logic*) whereas mathematics is regarded as the science concerning all calculuses;
b) it is the only science that can be applied to mathematics;
c) in some branches (e.g. the theory of relations) it examines the same objects as mathematics, but they are considered on a large scale;
d) logistic *calculus* has both logical and mathematical interpretations and thus it undeniably belongs to mathematics (namely, to set theory) (1915a, p. 449).[8]

[7] Besides Janiszewski the authors of the papers in *Poradnik* were: Stefan Kwietniewski, writing about analytic, synthetic, descriptive and differential geometry as well as the history of mathematics, Wacław Sierpiński, writing about arithmetic, number theory, higher algebra, set theory, real variable theory, differential and integral calculus, Stanisław Zaremba, writing about analytic function theory, partial differential equations, group theory and calculus of variations, and Stefan Mazurkiewicz who wrote about probability calculus. The introductory chapter 'O nauce' [About Science] was written by Jan Łukasiewicz.

[8] '(a) logistyka ujęta jest w postaci rachunku (*algiebra logiki*), matematykę zaś uważamy za naukę o wszelkich rachunkach;

(b) jest ona jedyną nauką, mogącą mieć w matematyce zastosowanie;

In a footnote Janiszewski adds that interpretation is also possible in number theory.

Furthermore, he characterises logistics, saying that it is '*formal logic* (i.e. the science of forms of pure thought) *using the mathematical method*; speaking more strictly: method, which so far only mathematics has applied on a large scale' (1915a, p. 449).

He regards the use of symbols in logistics as one of the characteristics that distinguish and differentiate it from other (also earlier) forms and branches of logic.[9]

In the discussed paper Janiszewski focuses on facts from the history of logistics and its most important achievements. He stresses that attacks on mathematical logic and undermining its significance are not supported by any serious arguments, and 'the funny, full of deeper thoughts, but mischievous chapters of the book *Science et méthode* by Poincaré, concerning discussing logistics, are rather satire than criticism' (1915a, p. 456).[10]

It is of interest to note his commentaries on the relations between logic and mathematics as well as his standpoint concerning the status of logic. Janiszewski is

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(c) w niektórych działach (np. teorii stosunków) traktuje o tych samych przedmiotach co i matematyka, tylko szerzej ujętych;

(d) *rachunek* logistyczny ma interpretację nie tylko logiczną, lecz i matematyczną, należy więc bezsprzecznie i do matematyki (mianowicie do teorii mnogości).'

[9] He even adds that 'it became the cause of unpopularity of logistics among philosophers' (1915a, p. 450).

[10] Poincaré wrote in *Science and Method* (1914, Book II, Chapter III: Mathematics and Logic, Paragraph VII; Pasigraphy): 'The essential element of this language consists in certain algebraical signs which represent the conjuctions: if, and, or, therefore. That these signs may be convenient is very possible, but that they should be destined to change the face of the whole of philosophy is quite another matter. It is difficult to admit that the word *if* acquires when written ɔ, a virtue it did not possess when written if.

This invention of Peano was first called *pasigraphy*, that it to say the art of writing a treatise on mathematics without using a single word of the ordinary language. This name defined its scope most exactly. Since then it has been elevated to a more exalted dignity, by having conferred upon it the title of *logistic*. The same word is used, it appears, in the *École de Guerre* to designate the art of the quartermaster, the art of moving and quartering troops. But no confusion need be feared, and we see at once that the new name implies the design of revolutionizing logic.'

Science et méthode (1908, Livre II, Chapitre III: Les Mathématiques et la Logique, VII. La pasigraphie): 'L'élément essentiel de ce langage, ce sont certains signes algébriques qui représentent les différentes conjonctions: si, et, ou, donc. Que ces signes soient commodes, c'est possible; mais qu'ils soient destinés à renouveler toute la philosophie, c'est une autre affaire. Il est difficile d'admettre que le mot *si* acquiert, quand on l'ecrit $\supset$, une vertu qu'il n'avait pas quand on l'écrivait si.

Cette invention de M. Peano s'est appelée d'abord la *pasigraphie*, c'est-à-dire l'art d'écrire un traité de mathématiques sans employer un seul mot de la langue usuelle. Ce nom en définissait très exactement la portée. Depuis, on l'a élevée à une dignité plus éminente, en lui conférant le titre de *logistique*. Ce mot est, paraît-il, employé à l'École de Guerre, pour désigner l'art du maréchal des logis, l'art de fair marcher et de cantonner les troupes; mais ici aucune confusion n'est à craindre et on voit tout de suite que ce nom nouveau implique le dessein de révolutionner la logique.'

aware of the fact that mathematical logic can be a convenient and useful tool for analysing language and arguments, that 'sometimes logistic calculus can have the significance of a method and can facilitate making conclusions' (1915a, n. 1, p. 456). He clearly declares:

> We recommend everyone to get to know some aspects of logistics; those who want to have an idea of the present day condition of logic, especially professional philosophers and in a way, mathematicians, too [. . .]. Particularly, it becomes indispensible for them if they want to work on the philosophy of mathematics (1915a, p. 455).[11]

Knaster writes that Janiszewski himself did his best 'to gain profound knowledge of mathematical logic, then known as logistics, and began applying it' (1960, p. 2). He used mathematical logic 'first of all to solve methodically mathematical problems using widely the specific symbolism of set theory' as well as 'to reveal deficiency and ambiguity in the structure of mathematical concepts, even such basic ones as line and surface' (1960, p. 2). However, in his paper Janiszewski clearly states that (mathematical) logic is an independent and autonomous mathematical discipline and not only a mathematical method or tool (cf. 1915a, p. 456); that 'in fact, it does not aim at (at least direct) practical benefits' (1915a, n. 1, p. 454). This should be emphasised, considering that Janiszewski studied in France, which was influenced by Poincaré (cf. above). This 'pro-logic' attitude and the emphasis on the significance of mathematical logic for mathematics itself, together with the decisive acceptance of its autonomy and independence, are very important and they characterise the Warsaw School (and undoubtedly, contributed to the development of the Warsaw School of Logic).

The other paper written by Janiszewski concerns philosophical problems of mathematics. The author discusses particular questions of a philosophical nature related to mathematics, especially the problem of deductive or inductive character of mathematics, the character of mathematical induction, the correctness of definition, the nature of objects of mathematics as well as the mode of their existence. He presents the debate between idealists and realists, discusses the role and importance of antinomies, philosophical issues concerning space and the related problem of the nature and character of geometrical theories as well as the sense of the question about their validity. Each of these problems contains references. At the end of the paper, there is a list (with commentaries) of general publications concerning the philosophy of mathematics, which proves Janiszewski's excellent knowledge of the current philosophical literature on the subject of mathematics. It is worth noting the subtle distinctions he drew when formulating problems. Another characteristic is that—like in other publications—he never formulates his own views but only presents (in a very competent way) other people's opinions. Thus he shows the complexity of problems. On the one hand, he stresses the independence of mathematicians' work from certain philosophical issues and on the other hand, he thinks

[11] 'Pewne zaznajomienie się z logistyką należy polecić każdemu, kto chce mieć pojęcie o dzisiejszym stanie logiki, szczególniej więc fachowym filozofom, a poniekąd i matematykom [. . .]. Staje się zaś ona dla nich niezbędna, jeśli zechcą się zająć filozofią matematyki.'

that there are controversial philosophical questions that do influence mathematicians' work. He writes:

> The problems discussed in the previous paragraphs are, so to say, outside the scope of mathematicians' work: whatever views concerning these problems they have, or if they have no views, that will not influence—at least directly—their work within mathematics and will not hinder them from reaching an agreement with other mathematicians in this sphere. Regardless of their definitions of natural numbers or mathematical induction all mathematicians will use them in the same way. However, there are disputable questions exerting a direct influence on current mathematical work. They concern the *importance* of certain mathematical argumentations and *objectivity* of certain mathematical concepts (1915b, p. 470).[12]

Among the latter he mentions the dispute concerning imaginary quantities, infinitesimal calculi, the summation of series or Poncelet's continuity principle, which are now of a historical character. He also analyses the questions of the accuracy of definitions, which he regards as valid (for instance, whether mathematics should allow impredicative definitions) or certain issues related to set theory.

Indeed, set theory played an important role in the Warsaw School. It all began with Sierpiński's discovery. In 1907, he stated that plane and line were composed of the same number of points. Soon he learnt[13] that 30 years earlier this fact had been discovered by Georg Cantor and that it had been the basic result of a new discipline, namely set theory. From that moment on, this theory became Sierpiński's main interest. As a professor of the Jan Kazimierz University in Lvov from 1910[14] he lectured on this subject there,[15] and he wrote the textbook entitled *Zarys teorii mnogości* [An Outline of Set Theory] (1912).

Interned at the beginning of World War I by the Russian authorities[16] (in Wiatka), he finally found himself in Moscow—thanks to his Russian colleagues'

[12] 'Zagadnienia, poruszone w poprzednich paragrafach, znajdują się, że tak powiemy, poza obrębem działalności matematyka: jakiekolwiek będzie on miał poglądy na nie, czy też nie będzie ich mieć wcale, to nie wywrze—przynajmniej bezpośrednio—wpływu na jego pracę w obrębie matematyki i w tym obrębie nie utrudni porozumienia z innymi matematykami. Bez względu na to, za co uważają liczby naturalne albo indukcję matematyczną, wszyscy matematycy będą się nimi posługiwać w jednakowy sposób. Istnieją jednak i takie kwestie sporne, które mają wpływ bezpośredni na aktualną pracę matematyczną. Dotyczą one *ważności* pewnych rozumowań matematycznych i *przedmiotowości* niektórych pojęć matematycznych.'

[13] Mostowski writes (1975, p. 9) that when Sierpiński made this discovery he asked his colleague Tadeusz Banachiewicz, who had studied in Göttingen and then became a professor of astronomy at the Jagiellonian University, whether he knew that conclusion. Banachiewicz answered by sending him a telegram containing only one word, 'Cantor.' Thus he turned Sierpiński's attention to Cantor's works. The former began studying them.

[14] He was the head of one of the two chairs of mathematics; the other one was directed by Józef Puzyna.

[15] The opinion, which is sometimes spread, that these were the first lectures on this new discipline conducted in the world is wrong. Earlier lectures on set theory were given by Ernst Zermelo (Göttingen in 1900–1901), Felix Hausdorff (Leipzig in 1901) and Edmund Landau (Berlin in 1902–1903, 1904–1905).

[16] When the war broke out Sierpiński was on holiday in Russia.

efforts—where he collaborated with Nikolai N. Luzin and where he got to know the theory of analytic sets, then being formulated and developed. In the future Sierpiński would be one of the most important figures who developed this part of set theory, i.e. descriptive set theory.

In Lvov Sierpiński made the young mathematicians Zygmunt Janiszewski, Stefan Mazurkiewicz and Stanisław Ruziewicz interested in set theory.

Several months after the Russian authorities had evacuated their university from Warsaw to Rostov-on-Don in 1915, Poles opened their university in Warsaw. Its first professors included Zygmunt Janiszewski and Stefan Mazurkiewicz. Towards the end of 1918 Wacław Sierpiński joined them, taking over the chair of mathematics. This place therefore gathered people who had the same research interests: they were dedicated to set theory.[17]

In 1917, responding to the appeal of the Mianowski Fund, Janiszewski wrote a paper entitled 'O potrzebach matematyki w Polsce' [About the Needs of Mathematics in Poland] (1917). This small (only six pages) paper became the programme of the whole generation of Polish mathematicians. Janiszewski postulated focusing on one branch of mathematics[18] and creating a new mathematical periodical. He wrote:

> According to the above-mentioned project one should create a strictly scientific periodical, entirely dedicated to one of these branches of mathematics in which we have outstanding, truly creative and numerous workers. This paper [. . .] would accept papers in any of the four languages that mathematics recognises as international [. . .]. The periodical would contain, besides original papers, bibliographies of this branch, summaries, and even reprints of important papers published somewhere else, in particular translations of valuable papers, published in 'non-international' languages, i.e. mainly Polish works that are wasted as unknown; finally, correspondence: answers to questions [. . .].
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> [. . .] let us return to mathematical creativity. Here dealing with common themes can create a suitable atmosphere. A researcher just needs co-workers. When he is alone he usually stops creating. The reasons are not only psychological, the lack of stimulus: being alone he *knows* much less than others who work jointly. What he gets is only results of research, fully developed and complete ideas, they are often published several years after they were formulated. Secluded, he did not see how and from what they originated; he did not experience this process with their creators. 'We are far from these forges or pots in which mathematics is created; we come late and there is no help, we must be behind,' I heard from some Russian mathematician in Göttingen, speaking about his fellow countrymen. How much more it applies to us!
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> Well, if we do not want to always 'fall behind' we must resort to radical means, reach the foundations of evil. We must create such a 'forge' at home! We can succeed only by gathering most of our mathematicians and making them work on one branch of mathematics. At present it is being done by itself; one must only help this movement. In fact, creating

[17] Ruziewicz was a professor of the University of Technology and at the Jan Kazimierz University. He was also Rector of the Academy of Foreign Trade in Lvov.

[18] The importance of this postulate can be testified by a story described by Marczewski (1948, pp. 17–18): 'When [. . .] in 1911 Puzyna, Sierpiński, Zaremba and Żorawski met as a group during the Congress of Natural Scientists and Medical Doctors in Cracow they could not find a common subject: their interests were so much divergent.'

> a special periodical for one branch of mathematics at our place will draw many to work in this domain.
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> But the periodical would help create this 'forge' in another way: then we would be a technical centre of mathematical publications concerning this branch. New works would be sent to us and relationships would be maintained with us (1917, pp. 15 and 18). [19]

Naturally, the field that was to draw the research efforts of Polish mathematicians was set theory and related disciplines: topology, the theory of real functions, etc.[20] It was the area of research of the Warsaw mathematicians, who had come from Lvov, and some Lvov mathematicians. In order to fulfil Janiszewski's second postulate the new periodical *Fundamenta Mathematicae* was called into being. The cover of its first volume[21] said that the periodical was dedicated to 'set theory and related issues (direct applications of set theory), Analysis Situs,[22] mathematical logic, axiomatic investigations.' The first volume appeared in 1920.[23]

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[19] 'W myśl powyższego projektu należałoby założyć u nas czasopismo ściśle naukowe, poświęcone wyłącznie jednej z tych gałęzi matematyki, w których mamy pracowników wybitnych, prawdziwie twórczych i licznych. Czasopismo to [. . .] przyjmowałoby artykuły w każdym z czterech języków uznanych w matematyce za międzynarodowe [. . .]. Pismo to zawierałoby, obok artykułów oryginalnych, bibliografie tej gałęzi, streszczenia, a nawet przedruki ważniejszych artykułów, drukowanych gdzie indziej, szczególnie zaś tłumaczenia artykułów wartościowych, drukowanych w językach nie "międzynarodowych", a więc przede wszystkim prac polskich, które marnują się nieznane; wreszcie korespondencje: odpowiedzi na zapytania [. . .].

[. . .] powróćmy do sprawy twórczości matematycznej. Tu atmosferę odpowiednią może wytworzyć dopiero zajmowanie się wspólnymi tematami. Konieczni prawie dla badacza są współpracownicy. Odosobniony najczęściej zamiera. Przyczyny tego są nie tylko psychiczne, brak pobudki: odosobniony *wie* o wiele mniej od tych, co pracują wspólnie. Do niego dochodzą tylko wyniki badań, idee już dojrzałe, wykończone, często w kilka lat po swym powstaniu, gdy ukażą się w druku. Odosobniony nie widział, jak i z czego one powstawały, nie przeżywał tego procesu razem z ich twórcami. "Jesteśmy z daleka od tych kuźni czy kotłów, w których wytwarza się matematyka, przychodzimy spóźnieni i, nie ma rady, musimy pozostać w tyle" mówił mi w Getyndze o swoich rodakach pewien uczony matematyk rosyjski. O ileż bardziej stosuje się to do nas!

Otóż, jeśli nie chcemy zawsze "pozostawać w tyle", musimy chwycić się środków radykalnych, sięgnąć do podstaw złego. Musimy stworzyć taką "kuźnię" u siebie! Osiągnąć zaś to możemy tylko przez skupienie większości naszych matematyków w pracy nad jedną gałęzią matematyki. Dokonywa się to obecnie samo przez się, trzeba tylko temu prądowi dopomóc. Otóż niewątpliwie utworzenie u nas specjalnego pisma dla jednej gałęzi matematyki pociągnie wielu do pracy w tej gałęzi.

Lecz jeszcze w inny sposób pismo dopomogłoby do wytworzenia się u nas tej "kuźni": bylibyśmy wtedy ośrodkiem technicznym publikacji matematycznych w tej gałęzi. Do nas przysyłano by rękopisy nowych prac i utrzymywano by z nami stosunki.'

[20] Let us note that in his paper Janiszewski does not speak clearly about any concrete discipline. It cannot be excluded that in those days the conflict with Zaremba was about to start—cf. Chap. 3. In fact, Zaremba wrote a paper about the needs of mathematics, which was published in the same volume of *Nauka Polska* [Polish Science] as Janiszewski's work.

[21] This phrase was repeated in every volume.

[22] Today called typology.

[23] Unfortunately, Janiszewski did not see the publication of this volume—he died on 3 January 1920 when the Spanish influenza struck again.

Janiszewski and other scientists saw and stressed the connection between set theory and other branches of mathematics (both the classical ones and the ones being developed). They did not see it a separate and single theory. In his paper 'À propos d'une nouvelle revue mathématique: *Fundamenta Mathematicae*' (1922), written on the occasion of the second volume of *Fundamenta*, Henri Lebesgue stated that 'set theory was removed beyond the sphere of mathematics by the great priests of the theory of analytic functions,' and if 'now this ostracism against set theory vanishes' it is thanks to the fact that 'set theory, which developed from the theory of analytic functions, could turn out to be useful for its elder sister and could show people of good will its values and riches.'

This conviction of the place and role of set theory in mathematics, shared by the creators of the Polish School of Mathematics, found its decisive expression in the above-mentioned *Poradnik dla samouków*. Stefan Mazurkiewicz wrote in the paper 'Teoria mnogości w stosunku do innych działów matematyki' [Set Theory vs. Other Branches of Mathematics], published in the third volume of *Poradnik* (as a supplement to the first volume):

> Reflecting on the table of 'the division of mathematics' made by Janiszewski (*Poradnik*, vol. 1, pp. 22/23) we can see that the position of set theory in the table was determined in a very special way. The table has two wings, which is in accordance with the traditional division of mathematics into two branches: on the left we have analysis (including arithmetic and algebra) and on the right—geometry. On the central line we have only two theories: set theory and group theory. Moreover, let us see that moving downwards the table we go from generally simpler branches, more primary and self-sufficient, to more complex ones, requiring external supporting means; thus we have a kind of pyramid of mathematical skills, pyramid obviously based on top. This top is set theory, which occupied the highest place in the table, having the foundations of arithmetic, the foundations of geometry and topology directly under it. Finally, we can see numerous 'lines of relation,' diverging (mostly centrifugally) from set theory in all directions.—Recapitulating, one can say that the table gives set theory the place that is almost prevailing in mathematics (being both basic and central); furthermore, it highlights its influence on other fields (1923, pp. 89–90).[24]

[24] 'Rozważając ułożoną przez Janiszewskiego tablicę "podziału matematyki" (*Poradnik*, t. 1, str. 22/23), dostrzegamy, że stanowisko teorii mnogości zostało w tablicy tej wyznaczone w sposób bardzo szczególny. Tablica jest dwuskrzydłowa, co jest zgodne z tradycyjnym podziałem matematyki na dwie gałęzie: po lewej stronie mamy analizę (łącznie z arytmetyką i algebrą), po prawej geometrię. Na linii środkowej znajdujemy dwie tylko teorie: teorię mnogości i teorię grup.—Zauważmy nadto, że przesuwając się w tablicy omawianej od góry ku dołowi, przechodzimy na ogół od działów prostszych, bardziej pierwotnych samowystarczalnych—do bardziej złożonych i wymagających z zewnątrz czerpanych środków pomocniczych, tym sposobem mamy tu rodzaj piramidy umiejętności matematycznych, opartej oczywiście na wierzchołku. Otóż tym wierzchołkiem jest teoria mnogości, która zajmuje w tablicy miejsce szczytowe, mając pod sobą bezpośrednio podstawy arytmetyki, podstawy geometrii i topologię.—Wreszcie widzimy liczne "linie związku", rozchodzące się (przeważnie odśrodkowo) od teorii mnogości we wszystkich kierunkach.—Reasumując, powiedzieć można, że tablica nadaje teorii mnogości stanowisko niemal dominujące w matematyce (gdyż zarazem podstawowe i centralne), ponadto zaś uwydatnia jej oddziaływanie na inne działy.'

Mazurkiewicz then discusses the significance and role of set theory within the theory of real functions, analysis, geometry and the foundations of mathematics. He stresses that the theory of functions of a real variable 'gave the first impulse to the creation of set theory, and today is predominantly a direct application of the latter' (1923, p. 90). He adds that 'in the theory of functions of real variable, set theory leads first of all to the systematisation of problems and provides a certain structure to the formless mass of details' (1923, p. 92). He also shows that investigations within functional calculus are essentially dependent on set theory, in particular the generalisations of the very concept of function are reliant on it. In geometry, set theory did not find—according to Mazurkiewicz—'a wider application and it is not going to find it' (1923, p. 97). Yet, we owe 'extreme enrichment of our knowledge of spatial forms' (1923, p. 97) to set theory.

This shows that the Warsaw School treated set theory as the basis of mathematics in the methodological sense and not the philosophical one (i.e. ontological and epistemological). Janiszewski's programme 'generated' the set-theoretic foundations of mathematics as a non-philosophical but mathematical direction. In the Warsaw School set theory was treated as a kind of auxiliary theory (though of fundamental significance) for mathematics. The team realised that set theory (like topology) was only developing and—as Mazurkiewicz put it in the quoted paper in *Poradnik*—was 'at the embryonic stage' (1923, p. 98). This fact 'very firmly counteracts the possibility of a wider application of set theory and topology to mathematics [...]' (1923, p. 98). However, 'as set theory moves forward its importance will undoubtedly increase' (1923, p. 98). Janiszewski expressed this belief in his conclusion in *Poradnik dla samouków*, focusing on the role of set theory as the new and universal language of mathematics, on the role of the axiomatic method as well as its affinity with logic. Whereas in his paper concerning set theory written for *Poradnik*, Sierpiński remarked:

> Despite the relatively short period (merely 40 years) set theory has managed to develop to an extraordinary extent and has occupied a first-rate position in mathematics. Today even a lecture on the foundations of higher mathematics cannot omit certain information from set theory (1915, p. 222).[25]

The treatment of set theory as the basis of mathematics in the methodological sense was expressed in the emphasis on its application in other branches of mathematics. For example, consider the fact that in *Fundamenta Mathematicae* there were relatively few papers dedicated to the 'internal' problems of set theory. Most papers showed the application of this theory to topology, the theory of functions or analysis.

Of special interest to our present discussion is the problem of the awareness of the relationships between set theory and logic and the foundations of mathematics

[25] 'Pomimo stosunkowo krótkiego okresu czasu (zaledwie 40-letniego) teoria mnogości zdążyła już nadzwyczajnie się rozwinąć i zająć pierwszorzędne stanowisko w matematyce. Dzisiaj już nawet wykład podstaw matematyki wyższej nie może się obyć bez pewnych wiadomości z teorii mnogości.'

as well as the philosophy of mathematics in the Warsaw School. In the paper 'Teoria mnogości w stosunku do innych działów matematyki' (1923), volume 3 of *Poradnik dla samouków*, Mazurkiewicz refers to Janiszewski's paper 'Zagadnienia filozoficzne matematyki' from volume 1 of *Poradnik* (cf. Janiszewski 1915b) and stresses:

> [. . .] revealing certain contradictions, i.e. antinomies, within set theory has become one of the motifs to review the principles of formal logic [. . .] [and that] on the basis of the concept of a set there was an attempt (made by Peano's school and then by Russell and Whitehead) to pack the whole mathematics within the framework of one uniform hypothetical-deductive system; although the attempt was defective it was extremely interesting because of the tendencies to synthesis which it contained (1915b, p. 98).[26]

In the paper which Mazurkiewicz quoted, Janiszewski discusses the philosophical problems of set theory from the standpoint of the debate between realists and idealists, and formulates the conclusion that set theory is necessary for reflections on the philosophy of mathematics. He writes:

> In order to study the philosophy of mathematics one should know *well* set theory, arithmetic, the foundations of geometry and the basic concepts of infinitesimal analysis; then the knowledge of logistics is necessary; finally, general education in philosophy is needed (1915b, p. 486).[27]

Finally, a word must be said about another important characteristic of the Warsaw School of Mathematics, which has been already mentioned and will be

[26] '[. . .] ujawnienie w łonie teorii mnogości pewnych sprzeczności, tj. antynomij, stało się jednym z motywów rewizji zasad logiki formalnej [. . .] [oraz że] na gruncie pojęcia zbioru podjęta została (przez szkołę Peany, a następnie przez Russella i Whiteheada) próba wtłoczenia całej matematyki w ramy jednolitego systemu hipotetyczno-dedukcyjnego, próba wprawdzie ułomna, jednak niezwykle interesująca z uwagi na tkwiące w niej tendencje do syntezy.'

[27] 'Do studiowania filozofii matematyki należy znać *dobrze* teorię mnogości, arytmetykę, podstawy geometrii i podstawowe pojęcia analizy nieskończonościowej; następnie konieczna jest znajomość logistyki; wreszcie potrzebne jest ogólne wykształcenie filozoficzne.'

It is worth quoting Janiszewski's further words concerning the necessary competences to exercise the philosophy of mathematics. He writes: 'It is not sufficient to be active in this field [philosophy of mathematics]; it is necessary to understand mathematics more profoundly, which can be expected only of those who have been creative in this field themselves. May the example of so many philosophers who, even having thorough mathematics education, have made mathematical mistakes in their works concerning the philosophy of mathematics and showed misunderstanding (though not ignorance!) of mathematics, be repellent here. Whereas the lack of philosophical education often makes mathematicians, dealing with these problems, misunderstand the philosophical aspects of these problems; they simply overlook numerous issues' (1915b, p. 486). ('Do czynnej jednak pracy na tym polu to nie wystarczy; koniecznym jest głębsze zrozumienie matematyki, czego można oczekiwać tylko od tych, którzy sami w tej dziedzinie pracowali w sposób twórczy. Niech przykład tylu filozofów, którzy, mając duże nawet wykształcenie matematyczne, popełnili w swych pracach nad filozofią matematyki błędy matematyczne i wykazali niezrozumienie (choć nie nieznajomość!) matematyki, działa tu odstraszająco. Brak znowu filozoficznego wykształcenia powoduje często u matematyków, zajmujących się temi zagadnieniami, niezrozumienie filozoficznej ich strony, przeoczenie po prostu całej masy zagadnień.')

referred further. The school did not favour any concrete philosophical doctrine within the philosophy of mathematics although the current concepts of the philosophy of mathematics were well-known there.[28] The only important thing was the correctness and fruitfulness of the methods applied. What was important was results and not concrete methods. It particular, this was expressed in research concerning the axiom of choice, which evoked numerous controversies. Some rejected it whereas others accepted it, recognising it as indispensible in mathematics. The Warsaw School took the stand that the mathematical implications of this axiom should be investigated and thus strict mathematical reflections should replace philosophical reflections. This attitude was clearly supported by Sierpiński, writing:

> Regardless of the fact whether we tend to accept Zermelo's axiom or not, we must take into account, in any case, its role in set theory and analysis. On the other hand, if Zermelo's axiom has been questioned by some mathematicians [. . .], it is important to know which theorems this axiom proves. (After all, if nobody questioned Zermelo's axiom it would be beneficial to analyse which proofs are based on this axiom—this is done, as one knows, for other axioms, too) (1923, p. 78).[29]

He repeated this opinion in his monograph, which was published a few decades later:

> Still, apart from our personal inclination to accept the axiom of choice, we must take into consideration, in any case, its role in set theory and in calculus. On the other hand, since the axiom of choice has been questioned by some mathematicians, it is important to know which theorems are proved with its aid and to realize the exact point at which the proof has been based on the axiom of choice; for it has frequently happened that various authors have made use of the axiom of choice in their proofs without being aware of it. And after all, even if no-one questioned the axiom of choice, it would not be without interest to investigate which proofs are based on it and which theorems are proved without its aid—this, as we know, is also done with regards to other axioms (1965, p. 95).

Therefore, the examination of the role of the axiom of choice in mathematics leads to solving the (philosophical) question of its validity. The axiom should not be assumed or rejected *a priori*; neither should its application be limited. But, putting aside personal philosophical convictions one should investigate (in a way impartially) which theorems, and how, depend on this controversial axiom. The same applies—*per analogiam*—to other axioms or hypotheses having a similar status (for example, the continuum hypothesis).

[28] Let us add that Janiszewski himself said that he was rather a philosopher than a mathematician, and '[. . .] he deals with mathematics in order to become convinced how far the human mind can go by its logical thinking' (Steinhaus 1921).

[29] 'Niezależnie od tego, czy jesteśmy osobiście skłonni przyjąć pewnik Zermelo, czy też nie, musimy w każdym razie liczyć się z jego rolą w teorii mnogości i analizie. Z drugiej zaś strony, skoro pewnik Zermelo był kwestionowany przez niektórych matematyków [. . .], jest ważną rzeczą wiedzieć, jakie twierdzenia są dowodzone przy pomocy tego pewnika. (Zresztą nawet, gdyby nikt nie kwestionował pewnika Zermelo, nie byłoby rzeczą pozbawioną interesu badanie, jakie dowody opierają się na tym pewniku—co też robi się, jak wiadomo, i dla innych pewników).'

2.2 Lvov School of Mathematics: Steinhaus, Banach, Żyliński, Chwistek

This section presents Hugo Steinhaus, Stefan Banach, Eustachy Żyliński and Leon Chwistek—representatives of the Lvov School of Mathematics. Adopting the fundamental ideas of the programme formulated by Janiszewski the school specialised in other mathematical branches than the Warsaw School. While the Warsaw mathematicians dealt with set theory, topology and mathematical logic, the prevailing domain in Lvov was functional analysis initiated by Stefan Banach (whom Steinhaus discovered for mathematics), and developed by such figures as Steinhaus, Stanisław Mazur, Władysław Orlicz, Juliusz Schauder, Stefan Kaczmarz, Stanisław Ulam and Władysław Nikliborc. This field did not require such profound studies of logic and the foundations of mathematics as the fields pursued in Warsaw. Hence it is comparatively difficult to find remarks on mathematics as such—which this book focuses on—in the works of the Lvov mathematicians. This may have been caused by the fact that logic was not developed in Lvov, although its atmosphere favoured this field as well as the foundations of mathematics. It was only in the year 1928 that a chair of logic was created there. The chair was given to Leon Chwistek. Earlier Eustachy Żyliński had been the only Lvov mathematician who dealt with mathematical logic. However, it should be added that the other mathematicians of this environment did not disqualify the foundations of mathematics and logic. In fact, they 'casually' dealt with it. Let us mention Banach and his joint work with Tarski concerning the paradoxical decomposition of the sphere (1924) or the Banach-Mazur results on constructive methods in mathematics and computable analysis (cf. Mazur 1963).

Speaking of Banach, it is worth saying that he did not stand aloof from the philosophical environment in Lvov. In particular, in his *Dziennik* [Diary] (1997) Kazimierz Twardowski wrote that Banach had participated (on 7 March 1921) in the inaugural session of the Section of Epistemology of the Polish Philosophical Society (cf. 1997, vol. 1, p. 201) as well as in the session of the Society, held on 26 March 1927, during which Zygmunt Zawirski had lectured on the relation between logic and mathematics. Banach also took the floor in the discussion after the lecture (cf. 1997, vol. 1, p. 300). At the First Congress of Polish Mathematicians, held in Lvov in 1927, Banach gave a talk 'O pojęciu granicy' [On the concept of limit] (on 7 September 1927) at the meeting of the section of mathematical logic (cf. 1997, vol. 1, p. 323). In January 1923, Banach delivered a paper concerning paradoxes in mathematics during the session of the Polish Philosophical Society in Lvov. He spoke about the paradoxes related to the concept of the equipollency of certain sets (e.g. the set of whole numbers and the set of even numbers) as well as the problems of the Banach-Tarski paradox. He showed that the cause of these paradoxes were infinite sets and the axiom of choice, which were not formally inconsistent with set theory. In Banach's opinion, solving these apparent paradoxes required constructing a logical system that 'evokes no objections.' This remark characterises to some extent the Lvov mathematicians' attitude towards logic.

Banach did not see anything wrong in the fact that mathematical practice lacked a good logical system. In the Lvov School the cultivation of mathematics did not have to be completed with additional research on logic and the foundations of mathematics.

The picture of mathematics adopted in Lvov can best be reconstructed on the basis of certain remarks included in the works aiming at popularising mathematics, especially in Steinhaus's popular publications. We will also pay attention to several (unfortunately, separate) remarks of another representative of the Lvov environment, namely Eustachy Żyliński, regarding mathematics as a science. These remarks (when we have no systematic and complete testimonies) can give us a certain image of his views.

Reflecting on Steinhaus's philosophical views on mathematics we must first of all mention his popular book *Czem jest a czem nie jest matematyka* [What Is and What Is Not Mathematics] (1923). He presents numerous issues, especially the definition of mathematics, its historical development, practical applications, the method of mathematics, differential calculus and integral calculus, computational mathematics, errors in mathematics as well as the relations between mathematics and life. From our perspective his reflections on the definition of mathematics as a science and on mathematical methods are most interesting.

Trying to define mathematics as a science, Steinhaus stresses that it grew from certain practical needs of man but, in fact, it is a theoretical science. He writes:

> We can see that here we are dealing with an old, developing science, growing out of the background of practice and connected with the world of real applications, but a *theoretical* science, which does not avoid the biggest efforts even when dealing with some issues devoid of any utilitarian character, e.g. the quadrature of the circle (1923, p. 25).[30]

Mathematics is characterised by the use of the deductive method, but 'its axioms and definitions have the feature of randomness to a large extent' (1923, p. 25). Another characteristic, which differentiates it at face value, is the use of symbols, which on the one hand is necessary but on the other hand can lead to the so-called symbolmania (cf. Twardowski's work 'Symbolomania i pragmatofobia' [Symbolmania and Pragmatophobia], 1927), i.e. 'the mania of the mechanical use of symbols,' which 'contradicts mathematical psychology' (1923, p. 27).

Although Steinhaus had sympathy for logic, he did not see it as an independent discipline with its own research problems and methods, but as a tool of deduction. He gave this picture of logic in the booklet in question. Moreover, he describes it relatively late—only in the second part of the booklet, reflecting on the method of mathematics. This is how Steinhaus characterises it:

> Mathematics aims at discovering absolutely true theorems. In order to do that it uses the method called *deductive*. In other words, it formulates new theorems from those that it has

[30] 'Widzimy, że mamy tu do czynienia z nauka starą, rozwijającą się, wyrosłą na podłożu praktyki i związaną ze światem zastosowań realnych, ale nauką *teoretyczną*, nie uchylającą się przed największymi wysiłkami nawet wtedy, gdy chodzi o zagadnienia zupełnie pozbawione utylitarnego charakteru, jak np. kwadratura koła.'

> made sure to be sufficient, using the *logical* way, i.e. correct deduction without references to observation, experiment, the testimony of the senses or spatial outlook as well as to vision, revelations or authority (1923, p. 74).[31]

The deductive method in some sense determines the object of mathematics. Steinhaus writes:

> Therefore, we can see that mathematics has its object determined only by the method and that every deductive theory is mathematics; that after all, this description of mathematics is just a framework that will be filled only after mathematical axioms are introduced, and they are—to some extent—arbitrary (1923, p. 78).[32]

And he adds:

> A characteristic feature of mathematics is its method. The mathematical method is deductive, synthetic and formal (1923, p. 80).[33]

The deductivity of the method of mathematics consists in the fact that 'the only means which the mathematical reasoning uses is deduction' (1923, p. 80). In Steinhaus's opinion the regularity of the method of mathematics is revealed in the choice of axioms, assuming that axioms can be both mathematical and logical. Choosing the latter 'is not done on the logical way but by virtue of the verdict of another instance, which some call "intuition" and others "feeling of certainty"' (1923, p. 81).

In mathematics definitions serve to shorten statements. However, 'the choice of definition determines the direction of our development of mathematics, i.e. which combinations of symbols we will recognise as important and worth separate shortening' (1923, p. 81).

The feature of formality is that in mathematical reasoning one can consider only such content of concepts that has been included in definitions. Steinhaus writes:

> The formalism of the mathematical method consists in that any content of the considered concepts is excluded in case someone wanted to assign them some nondefinitional content, and all that is contained in the very sound of words and is not clearly visible in the definitional agreement is as far as possible rejected from the definition (1923, p. 81).[34]

[31] 'Matematyka stawia sobie za cel wykrywanie teorematów absolutnie prawdziwych. Do tego celu używa metody zwanej *dedukcyjną*. Innymi słowy wysuwa ona z teorematów, co do których już upewniła się dostatecznie, nowe, drogą *logiczną*, tj. drogą poprawnego wnioskowania bez odwoływania się do obserwacji, do eksperymentu, do świadectwa zmysłów lub też oglądu przestrzennego, czy też do wizji, objawień albo autorytetu.'

[32] 'Widzimy więc, że matematyka ma swój przedmiot określony tylko przez metodę i że jest matematyką każda teoria dedukcyjna, że jednak to określenie matematyki jest tylko ramą, która zostaje wypełniona dopiero po wprowadzeniu pewników matematycznych, a one są—do pewnego stopnia—dowolne.'

[33] 'Charakterystyczną cechą matematyki jest jej metoda. Metoda matematyczna jest dedukcyjna, syntetyczna i formalna.'

[34] 'Formalizm metody matematycznej polega na tym, że wyklucza się z rozumowań matematyki wszelką treść pojęć rozważanych, o ile by ktoś chciał im przypisać jakąś treść pozadefinicyjną, a z definicji odrzuca się o ile możności wszystko, co mieści się w samym dźwięku wyrazów a nie jest wyraźnie uwidocznione w umowie definicyjnej.'

The only utilitarian and 'instrumental' character of logic towards mathematics is stressed in the following statement of Steinhaus:

> The teaching of formal logic finds in mathematics the most beautiful field for exercises and examples (1923, p. 169).[35]

Apart from the above-mentioned characteristics the aesthetic element plays an important role in the development of mathematics.[36] In Steinhaus's opinion beautiful is 'what is understandable, what is sufficiently general to be applied to the known, and not *ad hoc* formulated examples, and at the same time not so general to be trivial' (1958, p. 43). In fact, there are no absolute criteria of beauty but the sense of beauty and drive for beauty 'influence the direction of mathematical investigations more strongly than the principle of perfect strictness' (1958, p. 44). In the paper 'Drogi matematyki stosowanej' [Ways of applied mathematics] he wrote:

> In the mathematician's soul, like in any other man's soul, there are various beliefs and passions, aversions and cults, superstitions and inclinations. The strongest of these feelings and the most respectable one is sensitivity to the beauty of mathematics. Not everyone can see the beauty of the mountains. Not everyone has been moved by the view of the sea, and the stars do not appeal to all people; it cannot be explained but it is even more difficult to explain what the beauty of a function of complex variable or of synthetic geometry is (1949, p. 11).[37]

Steinhaus valued applied mathematics and the applications of mathematics very much—in fact, he was fairly successful in this domain. He thought that the Platonic approach to mathematics interfered with the interest and involvement in its applications. In 'Drogi matematyki stosowanej' he wrote that this attitude 'is not only hostile to applied mathematics but also destroys all natural sciences' (1949, p. 11). Since he never defined clearly the connection of mathematical concepts and objects to reality experienced through the senses, we must content ourselves with his short aphoristic but beautiful and apt remark:

> Mathematics mediates between spirit and matter[38] (1980, p. 54).

[35] 'Nauka logiki formalnej znajduje w matematyce najpiękniejsze pole do ćwiczeń i przykładów.'

[36] Many authors writing about mathematics paid attention to this problem. Suffice it to mention Aristotle, Proclus or Poincaré. In particular, in his *Metaphysics* (book 3, 1078a52–1078b4) Aristotle writes that mathematics speaks, though not necessarily *explicite*, about beauty and reveals elements of beauty; moreover: beauty is one of the motive powers of this science. In the fifth century the Neo-Platonic philosopher Proclus Diadochus used similar words in *A Commentary on the First Book of Euclid's Elements*. And so did Henri Poincaré, living in the nineteenth century, in his work *Science et méthode*.

[37] 'W duszy matematyka, jak każdego człowieka, tkwią różne wierzenia i zamiłowania, awersje i kulty, przesądy i upodobania. Najsilniejszym z tych uczuć i najgodniejszym szacunku jest czułość na piękno matematyki. Nie każdy widzi piękno gór, nie każdy doznał wzruszenia na widok morza i nie do każdego przemawiają gwiazdy w nocy; tłumaczyć tego nie można, a jeszcze trudniej jest wyjaśnić, w czym tkwi piękno funkcji zmiennej zespolonej lub geometrii syntetycznej.'

[38] 'Między duchem i materią pośredniczy matematyka.' These words of Steinhaus were inscribed on his tombstone.

The importance which Steinhaus attributed to mathematics is also testified by the following remark made at the end of his booklet *Czem jest a czem nie jest matematyka*:

> No other science than mathematics strengthens so much our faith in the power of the human mind. The possibility to prove every theorem excludes all phraseology. In this autonomy from platitude, authority, in this independence of results from researchers' wishes and 'points of view' one can see both the scientific and pedagogical value of this science. If one can use the term 'mental health' mathematics can boast of playing the most positive role in 'mental hygiene' (1923, p. 169).[39]

Speaking of the philosophy of mathematics and logic in the context of the Lvov School of Mathematics it is worth mentioning—apart from Steinhaus—the figure of Eustachy Żyliński. He dealt mainly with the theory of numbers, but after 1919 his interests included algebra, logic and the foundations of mathematics. In particular, he proved (cf. 1925 and 1927) that in classical bivalent logic the only two-argument logical functions which are sufficient to define all the one and two-argument logical functions are binegation and disjunction (the Sheffer stroke). As for the problems related to the philosophy of logic and mathematics there are no separate works written by Żyliński. Although he wrote the large work *Formalizm Hilberta* [Hilbert's Formalism] (1935), he did not include any philosophical remarks regarding Hilbert's programme—contrary to what the title might suggest—but aimed at 'elaborating and giving a detailed presentation of certain formalism, on which the works of Hilbert and his school concerning the foundations of mathematics are based' (1935, Introduction, p. 1). Żyliński focuses on technical issues, especially set theory and the logic of sentences. He also announced that he would 'work on the extension of formalism H_1 embracing the foundations of arithmetic and mathematical concept of function in its applications' (1935, p. 2). However, this work has never been published. It is worth noting that the author understood the discussed set theory and the logic of propositions 'as separate disciplines' (1935, p. 2).

Żyliński's other works include several brief statements of a philosophical character. As they are just a few it is worth analysing them. In his own abstract of the lecture entitled 'O przedmiocie i metodach matematyki współczesnej' [On the Subject and Methods of Modern Mathematics], delivered on 21 May 1921 (1921–1922), he explained what mathematical theories were. He claimed that they could be identified with a set of consequences of the accepted axioms. We can read:

> A particular 'mathematical theory' can be recognised as a set of conclusions that 'can' be obtained through basic thoughts connected with the sense of certainty, applied to basic variety on the sub-varieties of which certain initial properties (axioms) are projected (1921–1922, p. 71a–71b).[40]

[39] 'Żadna nauka nie wzmacnia tak wiary w potęgę umysłu ludzkiego jak matematyka. Możność udowodnienia każdego teorematu wyklucza wszelką frazeologię. W tej niezależności od frazesu, od autorytetu, w tej niezawisłości rezultatu od życzenia badacza i od "punktu widzenia", upatruję nie tylko naukową, ale i pedagogiczną wartość tej nauki. Jeśli wolno użyć pojęcia "zdrowie umysłowe", to matematyce przypada najdodatniejsza rola w "umysłowej higienie".'

[40] 'Poszczególna "teoria matematyczna" uważana być może za zbiór wniosków, które "mogą" być otrzymane za pomocą podstawowych pomyśleń mnogościowych połączonych z uczuciem

One should note his rather imprecise understanding of logic referring to some subjective sense of obviousness and certainty rather than to the formally *a priori* rules of inference. Żyliński also allows an infinite set of consequences of accepted axioms while speaking of conclusions that *can* be drawn.

Referring to the mutual relation between logic and mathematics he states:

> From this point of view the relation between mathematics and extensional logic would present itself to some extent as a relation between special set theories and the general set theory (1921–1922, p. 71b).[41]

Assuming that the concept of object is 'the simplest natural concept' (1921–1922, p. 71b), he claims that mathematics is a natural science of objects. He strengthens his thesis, stressing that '[in] investigations of particular mathematical theories (e.g. number theory) we use observation and even experiment' (1921–1922, p. 71b).

In his work 'Z zagadnień matematyki. II. O podstawach matematyki' [Mathematical Problems. II. About the Foundations of Mathematics] (1928), Żyliński discusses the role of intuition in mathematics. He emphasises that intuition can help construct a proof but in no way can the proof itself refer to intuition:

> In mathematics intuition can direct a proof successfully but in no case can it be its ingredient (1928, p. 51).[42]

Consequently, we are dealing with a clear differentiation between the context of discovery and the context of justification. The former allows intuition and the latter does not.

Żyliński's works also embrace several statements on the role and significance of mathematics for other sciences and, broadly, the world of culture. In his abstract 'O przedmiocie i metodach matematyki współczesnej' he claims that 'a strictly synthetic exposition of every science consists in a certain mathematical theory, the theorems of which are binding in this science' (1921–1922, p. 71b). In the quoted publication 'Z zagadnień matematyki. II. O podstawach matematyki' he writes:

> The birth of mathematics is at the same time the birth of mankind's culture. [. . .] Together with the development of intellectual culture, geometry and arithmetic, apart from a purely practical life meaning, begin drawing minds thanks to the simple and distinct laws occurring in their area (1928, p. 42).[43]

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pewności, stosowanych do mnogości podstawowej, na której podmnogości nałożone są pewne własności początkowe (aksjomaty).'

[41] 'Z tego punktu widzenia stosunek matematyki do logiki zakresowej przedstawiałby się w pewnym stopniu jako stosunek specjalnych teorii mnogości do ogólnej.'

[42] 'Intuicja w matematyce może z pożytkiem kierować dowodem, lecz w żadnym razie nie może być jego częścią składową.'

[43] 'Narodziny matematyki są jednoczesne z narodzinami kultury ludzkości. [. . .] Wraz z rozwojem kultury intelektualnej geometria i arytmetyka, poza swym czysto praktycznym życiowym znaczeniem, zaczynają pociągać umysły dzięki wyjątkowo prostym i wyraźnym prawom występującym na ich terenie.'

On the other hand, in the memorial signed by Żyliński, Steinhaus, Ruziewicz and Banach on 14 April 1924 we can read:

> Today's mathematics is nothing else but a general theory of exact thinking connected with the feeling of certainty. [...] However, being the most general science of relations existing between objects mathematics is applied to every scientific and practical field, exceeding sufficiently enough the framework of descriptiveness, simple inductions or literary-artistic methods (Żyliński et al. 1924, p. 1).[44]

Therefore, it is an explicit statement concerning the object of mathematics and—being its consequence—explanation of the applicability of mathematics in other fields.

This chapter also presents Leon Chwistek's views on the philosophy of mathematics and logic. As explained in the introduction, he began his scientific career in Cracow but from 1930 he was a professor of mathematical logic at the Jan Kazimierz University in Lvov. There he developed his concepts and tried to create a school. No wonder, then, that his philosophical views on mathematics and logic are worth discussing in this chapter.

Although Chwistek's first works concerned experimental psychology, he is predominantly known for his treatises on logic. Chwistek, like some Polish logicians (e.g. Leśniewski, cf. Sect. 3.4 in Chap. 3), expressed his philosophical views while building and interpreting logical theories. Moreover, his research on logic was motivated to a large extent by his philosophical views. By creating semantics he wanted to overcome philosophical idealism and opposed the conception of absolute truth. He was not satisfied with solving concrete fragmentary problems (neither was Leśniewski) but he strove to formulate a system embracing mathematics as a whole.

Chwistek's interest in logic began during his studies in Göttingen, especially after he had listened to Poincaré's lecture delivered in the spring of 1909. Chwistek decided to combine the ideas of Russell and Poincaré and to reform the theory of logical types by omitting impredicative definitions. He began by criticising the system of the ramified theory of types formulated by Whitehead and Russell in *Principia Mathematica* (1910–1913). It mainly concerned the principle of reducibility: every sentential function has an equivalent sentential function of the same type and the first order (the so-called quantifier-free), which allowed the elimination of impredicative definitions. However, this principle is of a non-constructive character and thus it introduces—according to Chwistek—ideal objects. It constitutes a typical axiom of existence and, in Chwistek's opinion, in the deductive system one should not make any other presumptions than the principle of sense and deductive rules.

[44] 'Matematyka dzisiejsza jest niczym innym jak ogólną teorią ścisłego myślenia połączonego z uczuciem pewności. [. . .] Będąc jednak najogólniejszą nauką o relacjach zachodzących między przedmiotami, matematyka znajduje zastosowania w każdej dziedzinie naukowej i praktycznej, wychodzącej w dostatecznej mierze poza ramy opisowości, prostych indukcji lub metod literacko-artystycznych.'

Therefore, Chwistek attempted to reconstruct the system of Whitehead-Russell, and he did so in the nominalistic spirit. He formulated a certain version of the simple (simplified) theory of types.[45] He presented its foundations in the works: 'Antynomie logiki formalnej' [The Antinomies of Formal Logic] (1921a), 'Zasady czystej teorii typów' [The Principles of the Pure Theory of Types] (1922a) and 'Über die Antinomien der Prinzipien der Mathematik' (1922b). In the simple theory of types one can distinguish types of functions but not their orders. This theory allows one to eliminate only logical antinomies (the set-theoretic ones)—and so did the ramified theory of types formulated by Whitehead-Russell—but it does not remove semantic antimonies in the style of Richard's antinomy. Then Chwistek formulated a pure theory of logical types—theory of constructive types (cf. 1924 and 1925). Among other things it rejects the axiom of reducibility. Yet, it leads to certain big formal complications of logical systems (especially the theory of classes and the theory of cardinal numbers) resulting from the necessity of considering not only the types but also the orders of sentential functions (which now cannot be reduced to the lowest order). Thus it removes non-constructible objects at the cost of increasing the degree of the formal complication of the system.

The described investigations led Chwistek to create a complete theory of expressions and to rational metamathematics that was based on it. It was to be a more basic system than logic and it would make it possible to reconstruct the classical logical calculus and the whole of Cantor's set theory. It would also meet the nominalist assumptions, and therefore it would be free from existential axioms, mainly the axiom of reducibility and the axiom of choice. Chwistek's system was based on the assumption that its theorems and consequently, the theorems of classical logic and set theory reconstructed in it, refer only to the inscriptions which can be obtained in a finite number of steps with the help of a pre-established rule of construction, and not to what these inscriptions mean. At the same time, these inscriptions are understood as physical objects. Realising his programme, Chwistek approached the version of nominalism, which can be seen in the early works of Willard Van Orman Quine and Nelson Goodman.

We will return to Chwistek's nominalism further in the book. Now let us just state that his conceptions neither won widespread recognition nor played a big role in the development of logic. One can see the reasons for that in his symbolism, which was complicated, unclear and difficult to decipher, as well as in his rather

[45] This theory did not reach international logicians, and independently from Chwistek it was again formulated by F.P. Ramsey in 1925. In his introduction to the second edition of *Principia Mathematica* Russell paid tribute to Chwistek's conceptions. At the same time he paid attention to the costs they involved—they lead to the necessity of abandoning many important parts of mathematics. He wrote (cf. Whitehead, Russell 1925–1927, vol. 1, p. XIV): 'Dr Leon Chwistek [in his *Theory of Constructive Types*—Russell and Whitehead's footnote] took the heroic course of dispensing with the axiom without adopting any substitute; from his work, it is clear that this course compels us to sacrifice a great deal of ordinary mathematics.' Cf. the correspondence between Chwistek and Russell in Jadacki (1986). The reasons for the poor reception of Chwistek's works and results will be discussed later.

illegible and careless way of presenting results, in particular the lack of proper examples, especially at places that can raise the most serious doubts, which Chwistek replaced with such phrases as ‘it is easy to see that’—all of that made it difficult to understand his proposals and evaluate their worth. In his dissertations he often referred to other works spread in various periodicals and thus difficult to access. Another obstacle could have been the fact that Chwistek treated his results concerning the foundations of mathematics as an argument in favour of his diverse philosophical questions.

One can see a certain interest in the philosopher’s logical works after the year 1945, when there was greater curiosity in nominalism in the philosophy of mathematics.[46] His system of rational metamathematics was not sufficiently elaborated. Moreover, his collaborators (Jan Herzberg, Władysław Hetper and Jan Skarżyński) and disciples (Wolf Ascherdorf, Celina Gildner, Kamila Kopelman, Abraham Melamid, Józef Pepis and Kamila Waltuch) could not do it because all of them lost their lives during World War II. Chwistek went his own way and his research on logic did not follow the main trend of the historical development of logic. Like Leśniewski, he worked with just a few people and, for example, he did not collaborate with mathematicians. He did not have any close scientific contacts with the Lvov philosophers, either.

Let us present in detail Chwistek’s philosophical views related to logic and mathematics. We will focus on his judgements regarding the methodology of deductive sciences, which he put forward in *Granice nauki* [The Limits of Science] (1935), bearing the subtitle *Zarys logiki i metodologii nauk ścisłych* [Outline of Logic and Methodology of Exact Sciences].

According to Chwistek, human knowledge is neither complete nor absolute. It cannot be complete because the theorems concerning all objects lead to contradiction. It cannot be absolute since there is no one absolute reality. He wrote in *Granice nauki*:

> It follows from these considerations that the principle of contradiction does not permit complete knowledge, i.e. knowledge which includes the answer to all questions. The attempt to secure such knowledge will sooner or later conflict with sound reason (1948, p. 42).[47]

In his opinion, sound reason—besides acknowledging experience as a fundamental source of knowledge and besides the necessity to schematise cognised objects or phenomena—is a common factor of the whole correct process of cognition. It lies in the rejection of all assumptions which are not experimentally

[46] In the years 1950–1951 J.R. Myhill published a series of papers dedicated to the search for possibilities for using Chwistek’s systems of rational metamathematics in the proof of the consistency of set theory as presented by Bourbaki. Cf. Myhill (1950, 1951a, b). Let us add that ‘The Theory of Constructive Types’ by Chwistek was reprinted by the University of Michigan in the series *The Michigan Historical Reprints Series*—cf. Chwistek (1988).

[47] ‘Z rozważań tych wynika, że zasada sprzeczności wyklucza wiedzę pełną, dającą odpowiedź na wszystkie pytania. Dążenie do takiej wiedzy musi—czy prędzej, czy później—doprowadzić do kolizji ze zdrowym rozsądkiem’ (1935, p. 20; see also 1963, p. 17).

verifiable or are inconsistent with experience or are not based on reliable theorems concerning simple facts or cannot be reduced logically to such theorems. Both empirical and deductive knowledge is relative. The former is relative because there are various types of experience, matching various realities, and the latter—because it depends on the accepted system of concepts. Here Chwistek speaks about rational relativism.

He assumed the principle of rationalism of cognition and firmly opposed irrationalism. Rationalism lies in there being only two sources of knowledge, namely experience and exact reasoning. This concerns mathematics and exact sciences as well as empirical sciences and philosophy. He wrote in *Granice nauki*:

> [...] the point of departure in constructing our world view should not be a confused metaphysics, but simple and clear truths based on experience and exact reasoning (1948, p. 3).[48]

Therefore, he opposes irrationalism, metaphysics and idealism in philosophy and mathematics.[49] He roundly criticises Plato, Hegel, Husserl and Bergson. Seeing the errors of positivism he values its epistemological concepts. Let us add that Chwistek very much appreciated dialectical materialism, ignoring the fundamental contrasts between dialectical materialism and positivism. He describes his views concerning cognition as critical rationalism and contrasts it with dogmatic rationalism.[50]

Formal logic, and in particular Chwistek's rational metamathematics, is to be both a solution to the difficulties caused by irrationalism and a weapon to struggle with it. Chwistek begins his introduction to *Granice nauki* with the phrase 'Having experienced a period of unparalleled growth of irrationalism' (1935, p. III), and ends with the words, 'History teaches that the final victory has always been shared by nations that followed the principles of exact reasoning.' He also writes:

> When this new system [i.e. the system of rational metamathematics—remark is mine] is completely worked out, we will be able to say, that we have at our disposal an infallible apparatus which sets off exact thought from other forms of thought (1948, p. 22).[51]

Chwistek's epistemological views are close to neo-positivism. He claims that the object of scientific cognition can be only what is or can be given in experience, i.e. what we can see through our senses, possibly supported by tools:

[48] '[...] punktem wyjścia budowy naszego poglądu na świat nie powinny być męty metafizyczne, ale prawdy proste i jasne, oparte na doświadczeniu i ścisłym rozumowaniu' (1935, p. V).

[49] Chwistek rejects irrationalism and idealism not only as false philosophical theories but also because they are, in his opinion, the sources of human sufferings, social injustice, cruelty and wars.

[50] It is of interest to note that a certain difficulty in interpreting Chwistek's views is the fact that he often uses classical philosophical terms, giving them specific meanings, which he does not explain at all or explains insufficiently.

[51] 'Z chwilą, kiedy system ten zostanie wykończony, będzie nam wolno twierdzić, że rozporządzamy niezawodnym aparatem, oddzielającym myślenie ścisłe od innych form myślenia' (1935, p. XXIV).

> [. . .] in speaking about reality we have in mind not some ideal object but the patterns which must be employed in dealing with a given case (1948, p. 261).[52]

Chwistek recommended using the constructive method in both science and philosophy. He formulated it in ‘Zastosowanie metody konstrukcyjnej do teorii poznania’ [The Application of the Constructive Method to Epistemology] (1923). Although one can refer this way mainly to deductive sciences it is also applied in empirical sciences and philosophy. The analysis of intuitional concepts in a given discipline lies in the foundations of the constructive method. It allows the separation of primitive concepts, the meaning of which is characterised in axioms. Then using the laws of logic, (formal) theorems are formulated in the axioms. Later Chwistek concluded that constructing deductive systems on the basis of philosophy is pointless—such a system cannot be built due to the degree of the complexity of philosophical investigations.

As aforementioned, in Chwistek’s opinion the object of knowledge can be only what is given in experience. However, we are dealing with various kinds of experience. Thus we reach Chwistek’s most known and original philosophical concept, namely his theory of plurality of realities.[53] He first formulated it in the paper ‘Trzy odczyty odnoszące się do pojęcia istnienia’ [Three Lectures Concerning the Concept of Existence] (1917), stating that ‘intuitive faith in one reality seems a prejudice’ (1917, p. 145) and seeing the concept of plurality of realities in Pascal and Mach (cf. 1917, pp. 149–150). He developed his theory in the book *Wielość rzeczywistości* [The Plurality of Realities] (1921b), its final version being in *Granice nauki* (1935). He framed its foundations again in the English edition—*The Limits of Science*—published in 1948, i.e. after his death, but this version does not include anything new.

In his first period (before 1925), Chwistek differentiates between the meaning of the term ‘reality’ and the meaning of ‘existence.’ In his opinion the latter is of a more general character since it can concern both the objects of reality and abstract objects, such as the objects of mathematics:

> Assuming that all that exists is real, we had to regard mathematical relations and elements of experience as real (1917, p. 145).[54]

In ‘Trzy odczyty odnoszące się do pojęcia istnieni’ (1917), Chwistek identified three standpoints relating to existence: nominalism, realism and hyperrealism. According to him, nominalists ‘demand verbal definitions, excluding contradiction’ whereas realists ‘go without verbal definitions but exclude contradictory objects’ and hyperrealists ‘go without verbal definitions and do not exclude contradictory objects’ (1917, p. 126).

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[52] ‘[. . .] jeśli mówimy o rzeczywistości, to nie mamy na myśli jakiegoś idealnego obiektu, tylko te schematy, z jakimi w danym wypadku mamy do czynienia’ (1935, p. 229; see also 1963, p. 205).

[53] This theory is sometimes compared and juxtaposed with Popper’s conception of three worlds.

[54] ‘Gdybyśmy założyli, że wszystko, co istnieje, jest rzeczywiste, to musielibyśmy uznać za rzeczywiste stosunki matematyczne wraz z elementami doświadczenia.’

At first, Chwistek accepted only two realities and tried to formalise his own theory. In *Granice nauki* he gives up his attempts to formalise it and accepts four kinds of reality respectively to possible kinds of experience. Thus we have the reality of sensations, the reality of images, the reality of things (reality of everyday life) and physical reality (constructed in exact sciences). Simultaneously, he attributes separate existence and full theoretical equality to the particular realities.

Having briefly discussed Chwistek's general methodological and ontological conceptions we can proceed to his views concerning the philosophy of mathematics (although we have already mentioned some of his views on mathematics, which lie at the source of his logical concepts). His firm nominalistic standpoint came to prominence here.

Therefore, in Chwistek's opinion the object of deductive sciences, including mathematics, is expressions constructed in these sciences in accordance with their rules of construction. Consequently, the object of mathematics is not ideal objects, such as points, straight lines, numbers or sets. Here the expressions being the object of mathematics are physical objects that we are given in experience. They can be transformed according to the adopted rules. Any system approves rules and certain expressions that play the role of axioms from which theorems are deduced out. The rules of transformation and axioms are chosen in such a way that the expressions could be interpreted as descriptions of the analysed state of affairs. In order to be able to apply deductive theories to specific sciences and more generally, to perceive concrete areas of reality, the elements of the latter should be schematised.

According to Chwistek, geometry is an experimental science. In Chap. VIII of *Granice nauki* he writes:

> Geometry is an experimental science. It depends upon the measurement of segments, angles, and areas. The Egyptians conceived it in this way and it has remained essentially the same up to this very day. Today what is generally regarded as geometry, i.e. what is included in textbooks, is the peculiar mixture of experimental geometry and the geometrical metaphysics which was inherited from the Greeks as Euclid's *Elements* (1948, p. 170).[55]

The rise of the systems of non-Euclidean geometry of Bolyai, Gauss and Lobachevsky in the nineteenth century, which Chwistek regards as the most important achievement in exact sciences, abolished—in his opinion—Kant's idealism.[56] These geometries showed that, for example, the concept of a straight line is

[55] 'Geometria jest nauką doświadczalną. Polega ona na mierzeniu odcinków, kątów i powierzchni. Tak pojmowali ją Egipcjanie i taką pozostała w istocie swojej do dzisiaj. To, co uważa się powszechnie za geometrię za naszych czasów, tj. to, o czym pisze się w podręcznikach, jest osobliwą mieszaniną geometrii doświadczalnej i metafizyki geometrycznej, którą pozostawili nam w spadku Grecy pod postacią *Elementów* Euklidesa' (1935, p. 190; see also 1963, p. 170).

[56] The thesis that non-Euclidean geometries refuted Kant's philosophy of geometry seems not to be fully justified. Now, if we take into consideration that Kant distinguished between postulating the existence of an object and its construction this thesis is not valid, since postulating existence requires only the inner consistency of a given concept, and construction assumes a certain structure of perceptual space. So one can postulate the existence of a five-dimensional sphere since the

not of an objective character, but depends on the accepted axioms. It may suggest that conventionalism is the proper philosophy for geometry. Indeed, in his first works, e.g. the quoted paper 'Trzy odczyty odnoszące się do pojęcia istnienia' (1917), he states that the existence of systems of non-Euclidean geometry, which are consistent, refutes the thesis of the *a priori* character of geometry. It seems that he would tend to accept conventionalism, although he does not state this explicitly:

> Both systems [of Euclidean geometry and non-Euclidean geometry—remark is mine] are free from contradiction since they can be reduced to analytic geometry; thus they do not show any fundamental differences from the theoretical standpoint. Intuition reconciles easily with Lobachevsky's theorems, which seem paradoxical only at first sight [. . .]. Therefore, we reach the conclusion that both geometries are equally true since each refers to different straight lines; only the differences between both kinds of these lines cannot be formulated with the help of experimental and intuitional means so that a segment of a straight line, which we will draw or think of, can serve as an illustration of the first or the second kind, depending on our will (1917, pp. 144–145).[57]

However, in *Granice nauki* Chwistek categorically rejected conventionalism, stating that geometry—like all other fundamental experimental sciences –should be based on the theory of expressions. This is because conventionalism introduces hypothetical entities, as was the case in John Stuart Mill's works or later Poincaré's, a promoter of this direction.[58] Chwistek wrote:

> It seems that it is impossible to attain a general concept of geometry without using formulae. It is therefore clear that the conception of geometry as the science of ideal spatial constructions must be nullified. [. . .] To speak of different four-dimensional space-times it is necessary to employ five-dimensional spacetime. It is clear that all this has only as much meaning as do mathematical formulae (1948, pp. 186–187).[59]

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concept is consistent but it cannot be constructed since the perceptual space is three-dimensional. Kant stated nothing to contradict the possibility of constructing consistent systems of geometries other than the Euclidean one.

[57] 'Obydwa systemy [tzn. system geometrii euklidesowej i systemy geometrii nieeuklidesowej] są wolne od sprzeczności, można je bowiem sprowadzić do geometrii analitycznej, nie wykazują więc zasadniczych różnic z punktu widzenia teoretycznego. Intuicja godzi się z łatwością z twierdzeniami Łobaczewskiego, które tylko na pierwszy rzut oka wydają się paradoksalne [. . .]. Dochodzimy więc do wniosku, że obydwie geometrie są w równym stopniu prawdziwe, każda z nich bowiem odnosi się do innych linii prostych; tylko różnice pomiędzy obydwoma gatunkami tych linii prostych nie dadzą się uchwycić przy pomocy środków doświadczalnych ani intuicyjnych, tak że kawałek linii prostej, który narysujemy lub pomyślimy sobie, może służyć za ilustrację jednego lub drugiego gatunku zależnie od naszej woli.'

[58] Let us add that in Chwistek's opinion conventionalism also became a source of reactionary social views, reducing truth and truthfulness to effectiveness, thus leading to the strengthening of the ruling classes: 'It is good to see that idealism dressed in the feathers of conventionalism has become a tool in the hands of reactionary elements that is even more dangerous than the old dogmatic idealism' (1935, p. 186).

[59] 'Okazuje się, że dotarcie do ogólnego pojęcia geometrii bez formuł jest niemożliwe. Jasne jest, że idąc tą drogą, musimy dojść do unicestwienia geometrii jako nauki o idealnych utworach przestrzennych. [. . .] Żeby mówić o różnych czterowymiarowych czasoprzestrzeniach, musimy się odwołać do czasoprzestrzeni pięciowymiarowej. Jest jasne, że wszystko to ma tyle sensu, ile zawierają go formuły matematyczne' (1935, pp. 186–187).

According to Chwistek, arithmetic, mathematical analysis and other mathematical theories should be treated like geometry, thus consequently obtaining their nominalistic interpretations.

Chwistek's philosophical conceptions shared the fate of his logical theories (as mentioned earlier). Chwistek was alone in his search. His ideas were often criticised bitterly, as he himself wrote in *Zagadnienia kultury duchowej w Polsce* [Issues of Spiritual Culture in Poland]:

> [. . .] the spheres of professional philosophers reacted to the idea of the plurality of realities either with disrespect or with unparalleled indignation, verging on fierce rage (1933; cf. 1961, p. 203).[60]

What were the reasons for these reactions? Chwistek's philosophical investigations were not of a systematic character and he seemed not to treat them with full responsibility (as Pasenkiewicz wrote in *Przedmowa* [Foreword] to Chwistek's selected works, cf. 1961, p. VII). He did not explain many of the terms he used, and his conceptions 'had been announced before they were verified' (1961, p. VII).

Finally, despite the described circumstances there have been references to Chwistek and citations from his works. For example, the Australian philosopher Richard Sylvan refers to Chwistek's pluralism in his book *Transcendental Metaphysics* (1997).

[60] '[. . .] sfery zawodowych filozofów zareagowały na ideę wielości rzeczywistości już to jej lekceważeniem, już to bezprzykładnym oburzeniem, graniczącym z dziką wściekłością.'

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