

## Chapter 2

# Preliminary Information About Waves and Materials

This chapter is divided on two parts. One part is devoted to waves and the other to materials. The first part includes preliminary general information about waves. It includes observation of waves in the world around us, scientific definitions of waves, some facts from the history of the study of waves, and some frequently used classifications of waves. This information can be found in many fundamental books on the theory of waves and in articles from Wikipedia and Scholarpedia. See the references on waves in different areas of science (40 titles) [1–40] and on elastic waves (24 titles) [41–64] at the end of the chapter.

In the second part, basic facts on materials are presented. First some important definitions and classifications are described, including the states of aggregation and phase of the substance, general view on materials, and division of mechanics on macromechanics, mesomechanics, micromechanics, and nanomechanics. Further, the modern structural mechanics of materials is briefly outlined. Here the procedures of continualization and homogenization are discussed and the concepts of material continuum and body are formulated, and some facts from structural nanomechanics of composite materials are given. This information can be found in many fundamental books on materials. See the list of works on materials (27 titles) [65–91] in the end of this chapter.

## 2.1 About Waves

### 2.1.1 *Observations and Definitions*

In this book, waves in the scientific sense (on Wikipedia: Wave) are discussed. Thus waves can be meant here as a subclass of motion in general. In science, motion is understood as one of the forms, in which matter exists. The second widespread scientific maxim states that in fact the entire world is in a state of motion. If this is true, then waves should be observed everywhere and by everybody. In fact, this is

the case: waves occur in nature and the world around us very frequently. As a result of this observation, a description of the wave phenomenon is, as a rule, becoming well known. It is considered sometimes that descriptive characteristics do not require a theoretical conception. Though the last statement has always to give rise to some doubt. The fact is that in such a description of waves, some criterion of distinguishing of wave motions from other motions is given whether deliberately or not. Practically everyone has seen waves on water, sand, or elsewhere. And it going seems that it is not very difficult to determine purely based on a description that one is observing waves.

Waves are very varied in their manifestations (see books in reference section on waves in different areas of science). Besides the well-known waves on water or in air, one can visually observe shock waves, explosion waves, seismic waves, optic waves, electromagnetic waves, magnetoactive waves, interferential waves, radio waves, high-flood waves, and rolling waves in rivers, waves in glaciers, waves in transportation streams in tunnels, chemical metabolic waves, and waves in processes of river and see sediments deposition, epidemic waves, and population waves.

Such different manifestation of wave motion in the world around us have resulted in the statement (see “Wave” in Wikipedia) that “a single, all-encompassing definition for the term *wave* is not straightforward. . .an attempt to define the necessary and sufficient characteristics that qualify a [phenomenon](#) to be called a *wave* results in a fuzzy border line.”

What follows are some definitions of waves from various sources.

*Encyclopedia Britannica*: speaking generally, we may say that it denotes a process in which a particular state is continually handed on without change or with only gradual change, from one part of a medium to another.

*Webster's New World College Dictionary*: a periodic motion or disturbance consisting of a series of many oscillations that propagate through a medium or space, as in the propagation of sound or light: the medium does not travel outward from the source with the wave but only vibrates as it passes.

*Encarta® World English Dictionary*: an oscillation that travels through a medium by transferring energy from one particle or point to another without causing any permanent displacement of the medium.

*Chambers Dictionary of Science and Technology*: a time-varying quantity which is also a function of position.

The well-known *Whitham's* book: a wave is any recognizable signal that is transferred from one part of the medium to another with a recognizable velocity of propagation.

Nevertheless, for the most part of aforementioned waves of diverse nature (including the mechanical waves), some common attributes can be identified:

**an observed disturbance in a certain location must propagate with a finite velocity to some other location of that space; as a rule, the process must be close to oscillatory, if it is observed in time.**

Note that a motion is assumed to be oscillatory, when it takes place in the neighborhood of some fixed state, is restricted in its variation from this state, and is repeated in most cases.

It is universally recognized that any wave observation that extends beyond the limits of daily terrestrial experience must be associated with a theoretical scheme. First of all, this scheme confers certain properties on the space, in which waves propagate. For example, traditional physical schemes are based on the continuum concept, whereby a set of scalar, vector, and tensor quantities is associated with each geometric point in the actual space and deals with so-called physical fields. In selecting the fields, the physical medium (acoustic, elastic, electromagnetic, and so forth), on which the motion is mathematically described using equations with partial derivatives—equations of mathematical physics—is fixed by this very same thing.

Thus, in contrast to the descriptive approach to wave phenomena, which requires the knowledge of wave attributes only,

**in the so-called scientific-cognitive approach,  
some initial theoretical scheme is always presented and used.**

Every theoretical wave description has at least two independent parameters—time and space coordinates. Continuum physical schemes establish the relations between fields depending on these parameters. As a result, the differential equations are derived, among solutions of which must be such solutions that describe waves.

### ***2.1.2 Classifications of Waves***

Waves can be classified by different attributes, and actually the different classifications exist in parallel.

For example, a characteristic of the solution of wave equations such as its smoothness turned out to be critical in theoretical wave analysis. Knowledge of the solution smoothness is equivalent to knowledge of its continuity or discontinuity, and also of this solution's quantitative estimates (types of discontinuities, order of continuity, etc.). The practice of separately studying waves corresponding to discontinuous and continuous solutions was established long ago. The delimitations occurred as a result of the differences in the physical interpretation of the mechanisms of the excitation of waves and process of wave motion. Thus, there exist two branches of science devoted to the study of the same physical phenomenon.

The branch of study associated with discontinuous solutions treats a wave as a singular surface motion relative to some given smooth physical field. That is to say, wave motion is understood as motion in the space of a field jump on a given surface.

The second branch is associated with continuous solutions describing a continuous motion. Two classes of waves are considered here.

**Hyperbolic waves** are obtained as solutions of differential equations of hyperbolic or ultrahyperbolic types and, consequently, are clearly defined by the type of equation.

It is also possible to speak of another type—**dispersive waves**. This type is defined by the form of solution.

It is claimed that a medium, in which the wave propagates, is **dispersive and the wave itself is dispersive**, if the wave is mathematically represented in the form of a familiar function  $F$  of the phase  $\varphi = kx - \omega t$  ( $x$  is the spatial coordinate,  $k$  is the wave number,  $\omega$  is the frequency, and  $t$  is time), and if the phase velocity  $v = \omega/k$  of the wave depends nonlinearly on frequency. Very often, the dispersivity is fixed in the form of nonlinear function  $\omega = W(k)$ .

Solutions of type  $u = F(kx - \omega t)$  are admitted not only in hyperbolic differential equations, but in parabolic ones, and some integral equations.

The criteria of hyperbolic and dispersive waves are not mutually exclusive; hyperbolic and dispersive waves are therefore encountered simultaneously. Among other things, the majority of waves in materials with microstructure discussed in this book are precisely these kinds of waves.

Let us fix here the classification standard in physics, which differ from the aforementioned **hyperbolic–dispersive wave** by a kinematic attribute. It consists of four types:

1. **Solitary waves or pulses, which are sufficiently short in duration and locally irregular given in space disturbances;**
2. **Periodic (most often, harmonic or monochromatic) waves, which are characterized by disturbances in the entire space;**
3. **Wave pockets, which are locally regular given in space disturbances;**
4. **Trains of waves—harmonic wave pockets.**

It is worth noting here that only the second type wave in the foregoing classification will be analyzed in this book, and all the nonlinear wave motions studied subsequently will be hyperbolic ones.

### 2.1.3 *From History of Studying the Waves*

Following are some important facts from the history of study of waves.

They can be found in book on history of science [6] and on Scholarpedia [15].

“The study of waves can be traced back to antiquity where philosophers, such as Pythagoras (560–480 BC), studied the relation of pitch and length of string in musical instruments. However, it was not until the work of Giovanni Benedetti (1530–1590), Isaac Beeckman (1588–1637), and Galileo (1564–1642) that the relationship between pitch and frequency was discovered. This started the science

of *acoustics*, a term coined by Joseph Sauveur (1653–1716) who showed that strings can vibrate simultaneously at a *fundamental frequency* and at *integral multiples* that he called *harmonics*. Isaac Newton (1642–1727) was the first to calculate the speed of sound in his *Principia*. However, he assumed isothermal conditions, so his value was too low compared with measured values. This discrepancy was resolved by Pierre Simon Laplace (1749–1827) when he included adiabatic heating and cooling effects. The first analytical solution for a vibrating string was given by Brook Taylor (1685–1731). After this, advances were made by Daniel Bernoulli (1700–1782), Leonard Euler (1707–1783), and Jean d’Alembert (1717–1783) who found the first solution to the linear wave equation. While others had shown that a wave can be represented as a sum of simple harmonic *oscillations*, it was Joseph Fourier (1768–1830) who conjectured that arbitrary functions can be represented by the superposition of an infinite sum of sines and cosines—now known as the Fourier series. However, while his conjecture was controversial and not widely accepted at the time, Lejeune Dirichlet (1805–1859) subsequently provided a proof, in 1838, that all functions satisfying *Dirichlet’s conditions* could be represented by a convergent Fourier series. Finally, the subject of *classical acoustics* was laid down and presented as a coherent whole by John William Strutt (Lord Rayleigh, 1832–1901) in his treatise *Theory of Sound*. The science of modern acoustics has now moved into such diverse areas as sonar, auditoria, electronic amplifiers, etc.

The study of hydrostatics and hydrodynamics was being pursued in parallel with the study of acoustics. Everyone is familiar with Archimedes (287–212 BC) *eureka* moment; however, he also discovered many principles of hydrostatics and can be considered to be the father of this subject. The theory of fluids in motion began in the seventeenth century with the help of practical experiments of flow from reservoirs and aqueducts, most notably by Galileo’s student Benedetto Castelli. Newton also made contributions in the *Principia* with regard to resistance to motion and also that the minimum cross section of a stream issuing from a hole in a reservoir is reached just outside the wall (the *vena contracta*). Rapid developments using advanced calculus methods by Siméon-Denis Poisson (1781–1840), Claude Louis Marie Henri Navier (1785–1836), Augustin Louis Cauchy (1789–1857), Sir George Gabriel Stokes (1819–1903), Sir George Biddell Airy (1801–1892), and others established a rigorous basis for hydrodynamics, including vortices and water waves. This subject now goes under the name of fluid *dynamics* and has many branches such as multi-phase flow, turbulent flow, inviscid flow, aerodynamics, and meteorology.

The study of *electromagnetism* was again started in antiquity, but very few advances were made until a proper scientific basis was finally initiated by William Gilbert (1544–1603) in his *De Magnete*. However, it was only late in the 18th century that real progress was achieved when Franz Ulrich Theodor Aepinus (1724–1802), Henry Cavendish (1731–1810), Charles-Augustin de Coulomb (1736–1806), and Alessandro Volta (1745–1827) introduced the concepts of charge, capacity, and potential. Additional discoveries by Hans Christian Ørsted (1777–1851), André-Marie Ampère (1775–1836), and Michael Faraday (1791–

1867) found the connection between electricity and [magnetism](#), and a full unified theory in rigorous mathematical terms was finally set out by James Clerk Maxwell (1831–1879) in his *Treatise on Electricity and Magnetism*. It was in this work that all electromagnetic phenomena and all optical phenomena were first accounted for, including waves. It also included the first theoretical prediction for the speed of light.

The fundamental significance for solid mechanics of determination of mechanical properties of materials was realized with development of industry as early as eighteenth century [2]. The studies carried out by Ernst Florence Chladni in 1787 on longitudinal vibrations of rods defined one of the main stimuli of development of solid mechanics. Seemingly, the study of waves in solids was started with determination of sound velocity in materials, which was associated with above-mentioned progress in the determination of mechanical (first of all, elastic) properties of materials. For example, Thomas Young deeply made sure that the elasticity of materials can be determined by the sound velocity.

One of the first determinations of the sound velocity in a solid was carried out by Jean Baptiste Biot, who observed in Paris the construction of the water-pipe made of cast iron pipes and published the results in 1809. Next important step in this direction was made by Guillaume Wertheim (for example, in 1851), who specified many moments in previous results on one-dimensional propagation of waves in elastic materials. Other important step is associated with determination of the sound velocity in a rubber, where the results of Franz Exner of 1874 should be mentioned.”

In the twentieth century, the theoretical study of waves in solids (materials) concentrated principally on three directions: linear waves (mainly, elastic waves), nonlinear waves (mainly, elastic waves, as well), shock waves (mainly, elastic and plastic waves).

The first (classical) direction of the study of waves is very close to the topics of this book and is reflected in many beautiful books that appear in this chapter’s reference list on elastic waves. Among these books, Achenbach’s classical text [1] and Royer’s and Dieulesaint’s relatively new textbook [18] can be recommended.

The second direction will be discussed in all subsequent chapters, whereas the third direction is beyond the scope of this book.

## 2.2 About Materials

### 2.2.1 General Definitions and Classifications

Let us start with general definitions (Wikipedia: Material). “**The material** is anything made of matter, constituted of one or more [substances](#). Wood, cement, hydrogen, air, and water are all examples of materials. Sometimes the term **material** is used more narrowly to refer to substances or components with certain

physical properties that are used as inputs to [production](#) or [manufacturing](#). In this sense, materials are the parts required to make something else, from [buildings](#) and [art](#) to [stars](#) and computers.”

**A physical substance** is usually defined as the aggregate of discrete formations that have a rest mass (atoms, molecules, and their more complicate formations).

*Two states of the substances* are distinguished: the state of aggregation and the state of phase.

*Four states of aggregation* are known: **gaseous, plasma, liquid, and solid.**

**The gaseous state** is characterized by translatory, rotational, and oscillatory motions of molecules. Distances between molecules are large, that is, the molecule packing density is not high.

**The plasma state** differs from the gaseous state in that it is an atomized gas with equal concentrations of positive and negative charges. It singles out only for this purpose that, as many people believe, the substance in the Universe consists of just plasma.

**The solid state** is characterized by only oscillatory motions of molecules near immovable centers of equilibrium with frequencies of  $10^{13} \div 10^{14}$  oscillations per second. Translatory and rotational motions are absent and distances between molecules are small, that is, the packing density is high.

**The liquid state** is close to the solid state in terms of packing, but it is close to the gaseous state in terms of molecular motions.

*The phase states* are distinguished by their order in the reciprocal placement of molecules. There are three such states: crystalline, liquid, gaseous.

**The crystalline phase state** is characterized by a “far” order in the placement of molecules, where the order is kept at distances, exceeding the molecule dimensions by  $10^2 \div 10^3$  times.

**The liquid phase state** is characterized by a “near” order in the placement of molecules, when the putting in order is observed only in immediate “nearness”, that is, at distances of a few molecules. At larger distances the placement is unpredictable. Often, this state is termed **amorphous**. Solid amorphous substances are called **glasslike**. The glasslike state differs essentially from the liquid amorphous state and is sometimes known as an isolated state.

The gaseous state of aggregation and the gaseous phase state coincide practically.

**The solid state of aggregation corresponds to two different phase states: crystalline and glasslike.**

**Materials** are defined as substances in a solid state of aggregation. Materials traditionally include, for example, machine-building and building materials and polymer and composite materials. Recently materials are divided into five types:

**1. Metals and alloys. 2. Polymers. 3. Ceramics and glasses. 4. Composites. 5. Natural materials: wood, leather, cotton/wool/silk, bone.**

The solidity mentioned in preceding definitions is treated in mechanics as the property whereby a body has a preference for a given configuration. A change in the body's shape relative to the configuration is measured by a deformation. Within the framework of an axiomatic procedure in constructing the mechanics of materials, these two notions (configuration, deformation) are defined exactly. This accuracy is reached within the framework of the thermodynamics of material continua.

Classical physics thinks of a solid body as a system of a large number of coupled and interacting particles, called discrete formations. It turns out that the description of the changing form of a body taking into account the motion of each particle is a too complicated problem. In addition, this description is inexpedient in classical macromechanics because knowledge of the individual motion of a particle (number of particles in  $1 \text{ cm}^3$  has the order  $10^{22}$ ) gives a picture of the micro- or nanoscopic motion, whereas in many cases changes in the form of a body can be studied successfully as a manifestation of the macroscopic motion.

The macro-description of materials was predominant in the mechanics of materials up to the twentieth century, when meso- and micro-descriptions were proposed and developed (the first one owing mainly to the in-depth analysis of metals; the second one owing to the widespread fabrication and application of composite materials in the second half of the twentieth century). Both new descriptions are based on understanding materials as having the internal structure of meso- and microlevel substances. It is assumed also that this structure cannot be neglected in the mechanical processes studied in meso- and micromechanics. Recently, the thriving development of nanomechanics of materials is emerged.

### 2.2.2 On Structural Mechanics of Materials

The structural mechanics of materials is divided into macromechanics, mesomechanics, micromechanics, and nanomechanics.

The mechanics of materials as part of the physics of materials studies mechanical phenomena in materials and is concerned mainly with continuum models of materials. As it is well known, both classical and modern physics assume materials as having discrete structure system of formations. The transition from a discrete to a continuum system is accomplished using *the procedure of continualization*—replacement of the volume occupied by the discrete system on the same volume occupied by the continuum with certain continually distributed physical properties. In other words, continualization establishes some sort of correspondence between a piece of a real solid body (occupying the volume  $V$  and having a complex discrete internal structure and fuzzy external boundary) and a piece of a fictitious body of the same volume  $V \subset \mathbb{R}^3$  (and, of course, the same configuration with a now fixed external boundary), to each point of which the set of averaged physical characteristics is attributed.

The mass density  $\rho$  is the first of these characteristics. They form the fields and are therefore called **the fields of thermodynamical characteristics**.



The geometric area  $V \subset \mathbb{R}^3$  (finite or infinite), in which the field of mass density  $\rho(x, y, z)$  is given, is called in physics **the material continuum** or **the continuum**. The notion of **body** is defined as a material continuum in the regular area of a space. But the notion of material continuum only is not sufficient for description of the deformation process in solid bodies.

As a rule, **the continuum is equipping** (like a ship, the term is proposed by famous Moscow mechanic Alexey Il'ushin). This means that the scalar field of mass is complemented by three fields: a vector field of displacements and tensor fields of strains and stresses. In mechanics, these three notions (fields) are defined precisely.

Thus, the procedure of continualization of a discrete system gives a continuum description of a piece of material. This piece can be considered separately. It can be treated as a homogeneous (its physical properties are identical at all points) or inhomogeneous (its physical properties change from point to point). If the material consists also of many continuum pieces (e.g., a granular composite material consists of a matrix with embedded granules), then the discrete system is modeled as a piece-wise homogeneous material. Two basic approaches can then be used: the exact approach, based on an application of the equations of continuum mechanics to each separate homogeneous piece, and the approximate approach, based on the procedure of averaging of the mechanical parameters of the entire piece-wise composition.

*The procedure of homogenization (averaging)* consists in choosing, in the space occupied by a body (material), a cube, whose dimensions are essentially smaller, than those of an inhomogeneous body. This cube must include a sufficiently large number of pieces (otherwise, the averaging procedure becomes false). A cube (volume) chosen in such a way is called **a representative cube (volume)**. The center of this cube is usually a point, to which all the averaged properties of the cube are attributed. As a result, a homogeneous material with continuum characteristics is now considered. The important role of **the characteristic size of inhomogeneities** of the material should be mentioned. This quantity with a length dimension is also called **the characteristic size of the internal structure**. There are two restrictions on this new parameter.

*Restriction 1.* For problems with varying surface loading, the characteristic size of the internal structure must be at least one order less than the characteristic length of variability.

*Restriction 2.* For wave problems, the characteristic size of the internal structure must be at least one order less than the wave length.

**Thus, in the theoretical analysis of waves in materials, the second restriction should always be taken into account.**

The foregoing restrictions can be considered as a concrete display of the following general requirement:

**The elementary volume should be a representative one.**

In other words, the characteristic size of the internal structure cannot be commensurate with the scale of averaging. This condition is called **the condition of effective homogenization**.

Finally, the ultimate goal of an averaging procedure is the effective description of material as a material continuum. Also, this procedure is the fundamental one in structural mechanics.

Let us now discuss one of the common features of continualization and homogenization procedures. Let us start with the fact that the main tool of these procedures is a representative volume. Most often it has the form of a cube with the side of essentially more less than the lesser sizes of a body, to which the procedure is applied.

This feature becomes noticeable in the cases, where the body has a finite extent or is semi-infinite (e.g., semispace). While a cube that is being moved continuously over a body, then the average over the cube value of some thermodynamical parameters is evaluated. This value is assumed to be the value at the cube center, and this value is assumed to be the value of the parameter at the corresponding to the cube center point of new (fictitious) continuum.

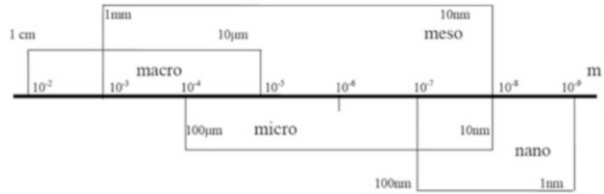
But when the cube goes to the boundary, it loses the property of representativeness: at least, starting with the distance to the boundary equal to half of the cube side. Thus, the continualization and homogenization procedures are not quite accurate at the near-the-surface areas. In wave theory, this means that surface waves in continua can be described by models that are not quite accurate. In these cases, more adequate models should be applied.

The structural mechanics of materials is considered a division of the mechanics of materials, where the basic relationships include the parameters of the internal structure of materials. Now, depending on the sizes of inclusions (granules, fibers, sheets) in the internal structure of materials, structural mechanics is divided into macromechanics, mesomechanics, micromechanics, and nanomechanics.

But sometimes such a division is sufficiently conditional, because the same material can be the object of study of different mechanical phenomena, which require the use of models that are different from those used in the four divisions of mechanics of materials of different scale levels mentioned earlier.

For example, when waves of kilohertz range frequencies are studied, it is usually expedient to consider these waves (the long waves) within the framework of macromechanics, whereas waves of megahertz range frequencies (the short waves) can be more adequately considered within the framework of micromechanics.

**Fig. 2.1** Classification of internal structure of materials by the attribute of admissible size of particles



### 2.2.3 A Few Words on Nanotechnology and Nanomechanics of Materials

Let us say here a few words about nanotechnology and nanomechanics, because they are very recent and attractive parts of the mechanics of materials.

To begin with, nanomechanics arose as a result of the formation and development of nanophysics and nanochemistry. *Nano-* (from the Greek word for *dwarf*) means one thousand millionth of a particular unit. The prefix “*nano*” in the words “*nanotechnology*” and “*nanomechanics*” indicates a length of 1 nm ( $1 \times 10^{-9}$  m).

A new classification of materials including nanomechanics is shown schematically in Fig. 2.1.

It is conventionally accepted that Richard Feynman was the first to predict the development of nanotechnology. In his well-known lecture *There's Plenty of Room at the Bottom*, delivered at a meeting of the American Physical Society in 1959, Feynman formulated the basic principle of nanotechnology: “*The principles of physics, as far as I can see, don't speak against the possibility of maneuvering things atom by atom*”.

At that time there were no tools to analyze the nanostructure of substances. Electronic microscopes, the main tool to deal with nanomaterials, were invented fairly recently. The first scanning electronic microscope was developed in 1942 and became available in the 1960s. The scanning tunneling microscope and the atomic force microscope, used to study nanomaterials, were created in the 1980s (the former by Binnig and Rohrer (IBM Zürich) in 1981 and the latter, by Binnig, Quate, and Gerber in 1986). The inventors of both microscopes were awarded the Nobel Prize in Physics in 1986). Through these microscopes, the surface of a material can be seen on the nanometer scale. That is what favored the success of many experiments on nanomaterials.

Eric Drexler is reckoned the second predecessor of nanotechnology. He once organized a new division of technology and wrote that nanotechnology was the principle of manipulating atoms by controlling the structure of matter at the molecular level and that “this road leads toward a more general capability for molecular engineering which would allow us to structure matter *atom by atom*”.

*The atom-by-atom construction* is now called *molecular nanotechnology*.

Nanotechnology as a whole can be understood as research and technology development at the atomic, molecular, or macromolecular levels on a length scale of approximately 1–100 nm range, to provide a fundamental understanding of

phenomena and materials on a nanoscale, and to create and use structures, devices, and systems that have novel properties and functions because of their small or intermediate size.

In certain cases, the critical length scale may be under 1 nm or larger than 100 nm. The latter case includes the composites (matrixes with fillers from nanoformations) that have the unique feature at 200–300 nm owing to the local bridges or bonds between nanoformations and the polymer matrix.

The primary concept in the theoretical interpretation of nanomaterials includes the idea that all materials are composed of particles, which in turn consist of atoms. This concept coordinates well with the classical concept. The next statement—that particles may be visible or invisible to the naked eye, depending on their size—introduces something novel into the classical understanding the materials. The structural mechanics of materials admits the size of granules from nanometers to centimeters and so forth (in rock mechanics, for example).

The point of view that nanomaterials as materials whose internal structure has nanoscale dimensions are something new to science is not true. It was relatively recently realized that some formations of oxides, metals, ceramics, and other substances are nanomaterials. For example, ordinary (black) carbon was discovered at the beginning of 1900. Fumed silica powder, a component of silicon rubber, is also a nanomaterial. It came into commercial use in 1940. However, only recently it has become clear that the particles constituting these two substances have nanoscale dimensions.

Size is not the only characteristic of nanoparticles, nanocrystals, or nanomaterials. A very important and specific property of many nanoformations is that the majority of their atoms localize on the surface of a formation, in contrast to ordinary materials where atoms are distributed over the volume of a particle.

Consider now carbon nanoparticles. Science has long been aware of three forms of carbon: amorphous carbon, graphite, and diamond. The highly symmetric molecule of carbon  $C_{60}$  was discovered in 1985. It has a spherical form, resembling a football, with carbon atoms on the surface and contains 60 atoms in five-atom rings separated by six-atom rings. These molecules were named *fullerenes* and have come been studied fruitfully. The number of kinds of fullerenes discovered has increased considerably, reaching many thousands to date.

Fullerene molecules form carbon nanotubes, which may be considered relatives of graphite. Nanotubes can be thought of as graphite lattices rolled up into a tube; they are molecules with a very large number of atoms  $C_{10,000} - C_{1,000,000}$ . Nanotubes differ in length, diameter, and the way they are rolled. The internal cavities may also be different, and tubes may have more than one sheet. Atoms at the ends of a fullerene molecule form the “hemi-spherical caps.” Sheets may be rolled differently, forming *zigzag*, *chiral*, and *armchair* structures. Two types of nanotubes have been distinguished: *single-wall* and *multi-wall* nanotubes.

Thus, it can be said that the property uniting all known nanoparticles is their dimensions. Their internal structure may vary considerably. The aforementioned high level of surface localization and the various features in the chemical–physical structure of nanoformations (owing their intermediate position between the macro-

and atomic worlds) manifest themselves as their particular mechanical properties. Their mechanical characteristics exceed considerably those of traditional materials. Today's study of the mechanical behaviour of nanoparticles, nanoformations, and nanomaterials is in the early stage. Only the external manifestations of mechanical phenomena have been detected, but their mechanisms have been insufficiently studied.

In closing this short introduction into nanomechanics, it seems pertinent to recall a discussion on mechanical properties of new materials organized in the UK on 6–7 June 1963 and published in the *Proceedings of the Royal Society* in 1964. In his concluding remarks, Professor Bernal, one of the organizers, said the following:

*Here we must reconsider our objectives. We are talking about new materials but ultimately we are interested, not so much in materials themselves, but in the structures in which they have to function.*

The nanomechanics faces the same challenges that micromechanics did 40 years ago and that John Bernal described so eloquently.

### 2.2.4 To Structural Nanomechanics of Composite Materials

Let us return now to the structural mechanics and consider the basic elements of the theory of composite materials as that theory of materials that exerts a great influence on structural mechanics.

Classical mechanics of materials was used to divide materials into two classes: homogeneous and heterogeneous ones.

**Homogeneous** materials are understood as materials with an internal structure of atomic-molecular character (with the characteristic size of the structure close to that of atoms or molecules). This means that such materials have a discrete molecular structure, which is changed mainly by applying the procedure of continualization to the model representation by the homogeneous continuum.

**Heterogeneous** materials are understood as materials with an internal structure that is essentially larger than molecular-kinetic sizes (sizes of molecules, crystal lattice, and so on). This means that these materials consist of components (phases) and have a macroscopically inhomogeneous internal structure. As a rule heterogeneous materials are modeled by a piecewise homogeneous continuum, which assumes that each component of the internal structure is also modeled by a homogeneous continuum. Thus, as it was mentioned earlier, the procedure of continualization is applied in this case not to the material as a whole but to the separate components of the material.

**Composite materials** are the typical representatives of heterogeneous materials, which can be separated into natural and artificial composites. They are conventionally defined as consisting of a few components (phases) with differing physical properties. As a rule, these components alternate many times in the space. The way they alternate, the conditions at the interface, the geometrical form, and

the physical properties of the components define the internal structure of the composite.

In real composites, the internal structure is at best close to periodic. The most difficult processes in continuum description are those that take place at an interface. Macro-, meso-, and micromechanics consider these processes practically from the same position based on general physics conceptions. Nanomechanics introduces into this problem the new features associated with the intermediate states of interface processes between general physics laws and quantum physics laws.

In continuum modelling, all problems of composite interfaces are reflected in formulations of boundary conditions between matrix and filler. Thus, the novel problem in the nanomechanics of composites that distinguishes this branch from the older branches (macro-, meso-, micromechanics of composites) consists in the adequate formulation of boundary conditions.

The next important distinctive feature of the nanomechanics of composites consists in the very high values of main mechanical properties of nanofillers, which are novel for the mechanics of materials (e.g., extremely high Young modulus values).

**The most important similarity of all four branches of structural mechanics of materials is the applicability of common for all branches continuum models.**

The mechanics of composites is concerned largely with specially designed materials. As a rule, the internal structure of composite materials assumes a jumping (stepwise) change in the properties of components (phases) on interfaces and the presence of the soft and stiff components. A stiff component is considered to be an arming or reinforced one and is usually called a filler, whereas a soft component is conditionally called a matrix (binder). The difference in some mechanical properties (e.g., Young modulus) of composite components can be as much as 100–1,000 and more times.

The most commonly known and used composites are the granular (granules as reinforcing fillers), fibrous (fibers as reinforcing fillers), and layered (thin layer sheets as reinforcing fillers) composites.

The complexities associated with an analytical description of mechanical phenomena in composite materials have resulted in the creation of approximate continuum models. These models, on the one hand, retain the main physical properties of a system and, on the other hand, these models are quite simple and assume analytical solutions for different mechanical problems including the problems of wave propagation.

At present, many different approximate models are proposed and quite advanced. They take into account the internal structure of materials, determine the necessary mechanical parameters, and solve practically all important problems. These models can be divided into the structural models of different orders. The basic model (structural model of the first order) is based on the assumption that the material is a homogeneous continuum, whose mechanical properties should be determined on the basis of standard tests. The internal structure of a composite is revealed here in the same way as is done for engineering and building materials

(steel, iron, wood, or plastics). The properties, which are found using an averaging procedure, depend on the basic parameters of the internal structure. They are given mainly in the form of algebraic relationships.

This circumstance makes it possible to foresee, at the design stage, the averaged properties of the composite material. These abilities of the model together with the technological possibilities for designing engineering composites constitute one of the main directions in the development of the mechanics of composites. In most cases when dealing with the averaging properties, it is understood that one is working within the framework of a classical continuum model of elasticity.

The last comment is associated with the mechanical properties of materials. Ideally, each material should have as though the certificate with its fixed physical properties. For a long time, such data on materials were provided by experimental mechanics. Here the direct tests are highly valuable, because indirect tests require recalculation by use of some theoretical formula that may not be entirely for a given test. Nowadays, the practice of using indirect tests and theoretical calculations of mechanical properties of new materials is very popular, especially in nanomechanics. Therefore, the results obtained in this way and reported in scientific literature data should be estimated with some critical and skeptical eye.

This comment seems appropriate because the analysis of waves in materials requires a knowledge of the physical properties of materials and treats the data on properties as the established fact.

## Exercises

1. Identify those areas of science, where the wave motion is observed (e.g., economic or sociology) and compare the mathematical description with the traditional areas ones.
2. Look for the additional to six shown in the chapter definitions of waves and formulate distinctions between the found definitions and the shown ones.
3. Come up with a new classification of waves and compare the criteria of the new classification and the ones in the chapter.
4. Review the most frequently used classification of materials into five types and try to come up with other classifications. Formulate the difference between them.
5. Explain the differences between procedures of continualization and homogenization.
6. The different leading in science countries have established different ranges for macro-, meso-, micro-, and nanolevels of the structure of materials. Find and compare these standards. Try to explain the difference between them.
7. Nanotechnology is understanding in very distinguishing ways. Try to come up with a definition, that is the most suitable for mechanics.

8. Write list of currently existing nanotubes (carbon, silica, and so forth). Verify whether carbon nanotubes are the leading type of nanotube in applications and scientific investigations.
9. Among the classical classes (granular, fibrous, layered) of composites, identify the intermediate classes and try to construct a new refined classification of composites.
10. List the direct and indirect experiments for determination of the elastic properties of materials (you can start by consulting the book [67] from the references on mechanics of materials or the book [9] from the references on waves in different areas of science). Compare the numbers of existing direct and indirect experiments.

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