Chapter 2
Stylized Facts

The name Stylized Facts refers to all non trivial statistical evidences which are observed throughout financial markets. Almost all price time series of financial stocks and indexes approximatively exhibit the same statistical properties (at least qualitatively). In addition it has been shown that Stylized Facts are robust on different timescales and in different stock markets [1].

The systematic study of Stylized Facts has begun in very recent time (approximately from '90) for two reasons: a technical and a cultural one. The former one is that the huge amount of empirical data produced by financial markets are now easily available in electronic format and can be massively studied thanks to the growth of computational power in the last two decades. In order to make a comparison with some traditional fields of Physics, a similar quantity of information is observed only in the output of a big particle accelerator. The latter instead is due to the fact that traditional approaches to economic systems neglect empirical data as candidates respect to which a theory must be compared differently from Physics. From this point of view standard Economics is not an observational science.

Turning now our attention to the experimental evidences of financial markets, the main Stylized Facts are

- the absence of simple arbitrage,
- the power law decay of the tails of the return distribution,
- the volatility clustering.

In the following sections we analyze them.

2.1 Absence of Simple Arbitrage

The absence of simple arbitrage in financial markets means that, given the price time series up to now, the sign of the next price variation is unpredictable on average. In other words it is impossible to make profit without dealing with a risky investment. This implies that the market can be seen as an open system which continuously reacts to the interaction with the world (i.e. trading activity, flux of information, etc) and

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self-organizes in order to quickly eliminate arbitrage opportunities. This property is also called arbitrage efficiency.

This condition is usually equivalent to the informational efficiency expressed in economic literature saying that the process described by the price p_t is a martingale that is

$$E[p_t|p_s] = p_s \tag{2.1}$$

where $t > s$. Here we are assuming that the price is a synthetic variable which reflects all the information available at time t. If this is not true the conditioning quantity is the available information I_s at time s and not only the price p_s.

However, the condition of martingale is uneasy from a practical point of view and the two-point autocorrelation function of returns is usually assumed as a good measure of the market efficiency

$$\rho(\tau, t) = \frac{E[r_t r_{t+\tau}] - E[r_t]E[r_{t+\tau}]}{E[r_t^2] - E[r_t]^2}. \tag{2.2}$$

If the process $\{r_t\}$ is at least weakly stationary then Eq. 2.2 simply becomes $\rho(\tau) = (E[r_t r_{t+\tau}] - \mu_r^2)/\sigma_r^2$ where $\mu_r = E[r_t]$ and $\sigma_r = E[r_t^2] - E[r_t]^2$. If the autocorrelation function of returns is always zero we can conclude that the market is efficient.

In real markets the autocorrelation function is indeed always zero (see Fig. 2.1) except for very short times (from few seconds to some minutes) where the correlation is negative (see inset of Fig. 2.1). The origin of this small anti-correlation is well-known and due to the so-called *bid ask bounce*. This is a technical reason deriving from the double auction system which rules the order book dynamics (see [2] for further details).

In the end we want to stress that the efficiency is a property that holds on average: locally some arbitrage opportunities can appear but, as they have been exploited, the efficiency is restored [3, 4].

2.2 Fat-Tailed Distribution of Returns

The distribution of price variations (called returns) is not a Gaussian and prices do not follow a simple random walk. In details very large fluctuations are much more likely in stock market with respect to a random walk and dramatic crashes are approximately observed every 5–10 years on average. These large events cannot be explained by gaussian returns. Therefore to characterize the probability of these events we introduce the complementary cumulative distribution function $F(x)$

$$F(x) = 1 - \text{Prob}(X < x) \tag{2.3}$$

which describes the tail behavior of the distribution $P(x)$ of returns.

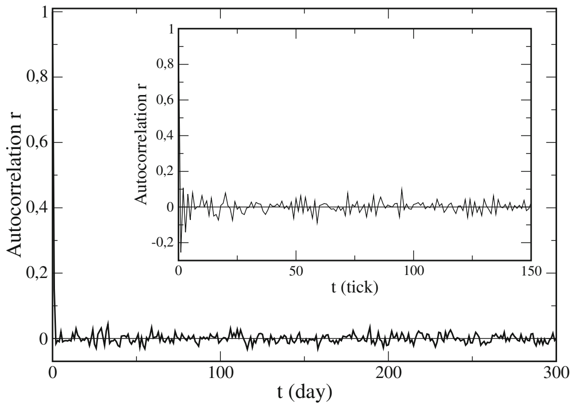

Fig. 2.1 We report the autocorrelation function of returns for two time series. The series of the main plot is the return series of a stock of New York Stock Exchange (NYSE) from 1966 to 1998 while the series of the inset is the return series of a day of trading of a stock of London Stock Exchange (LSE). As we can see the sign of prices are unpredictable that is the correlation of returns is zero everywhere. The time unit of the inset is the tick, this means that we are studying the time series in event time and not in physical time

The complementary cumulative distribution function $F(x)$ of real returns is found to be approximately a power law $F(x) \sim x^{-\alpha}$ with exponent in the range 2–4 [5], i.e. the tails of the probability density function (pdf) decay with an exponent $\alpha + 1$. Since the decay is much slower than a gaussian this evidence is called Fat or Heavy Tails. Sometimes a distribution with power law tails is called a Pareto distribution. The right tail (positive returns) is usually characterized by a different exponent with respect to the left tail (negative returns). This implies that the distribution is asymmetric in respect of the mean that is the left tail is heavier than the right one ($\alpha^+ > \alpha^-$).

Moreover the return pdf is a function characterized by positive excess kurtosis, a Gaussian being characterized by zero excess kurtosis. In Fig. 2.2 we report the complementary cumulative distribution function $F(x)$ of real returns compared with a pure power law decay with exponent $\alpha = 4$ and with a gaussian with the same variance.

When the tail behavior of the return distribution is studied varying the time lag at which returns are performed [1], a transition to a gaussian shape is observed for yearly returns. However it is unclear if this transition is genuine or due to a lack of statistics or to the non stationary return time series.

2.3 Volatility Clustering

In the lower panel of Fig. 2.3 we report the return time series of a NYSE stock (returns are here defined as $\log(p_{t+1}/p_t)$). As we can see the behavior of returns appears to be intermittent in the sense that periods of large fluctuations tend to be followed by

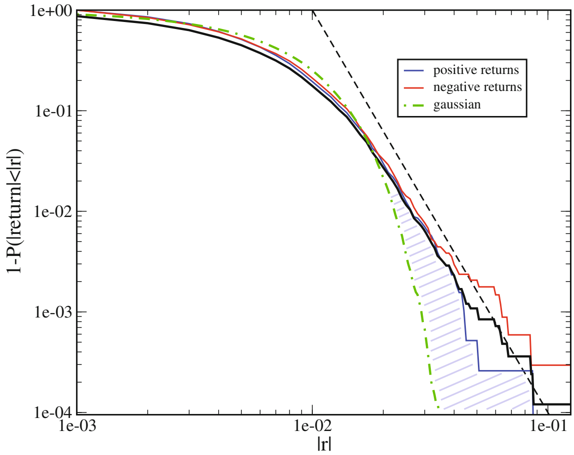

Fig. 2.2 We report the complementary cumulative distribution function of the absolute value of returns (*solid black line*). The *green dashed line* (·—) is the complementary cumulative distribution function of a gaussian with the same variance of the real return distribution. The *dashed black line* is a pure power law decay with exponent $\alpha = 4$. The *blue* and *red lines* are instead the complementary cumulative distribution functions for positive and negative returns respectively. We can see that *red curve* has a slower decay with respect to the *blue* one. This asymmetry between positive and negative returns is the origin of the non zero skewness of the probability density function of returns

large fluctuations regardless of the sign and the same behavior happens for small ones.

In Economics the magnitude of price fluctuations is usually called volatility. It is worth noticing that a clustered volatility does not deny the fact that returns are uncorrelated (i.e. arbitrage efficiency). Therefore the magnitude of the next price fluctuations is correlated with the present one while the sign is still unpredictable. In other words stock prices define a stochastic process where the increments are uncorrelated but not independent.

Different proxies for the volatility can be adopted: widespread measures are the absolute value and the square of returns. As a consequence of the previous considerations about the clustering of volatility, the autocorrelation function of absolute (or square) returns is non zero. We also find that the autocorrelation is well-described by a power law decay with exponent ranging from -1 to 0 as reported in Fig. 2.4. The very slow decay means that volatility is correlated on very long time scales from minutes to several months/years. The exponent of the autocorrelation function is not universal as the one of fat tails but it is typically around 0.2–0.3. The volatility clustering was observed the first time by Mandelbrot in 1963 [6].

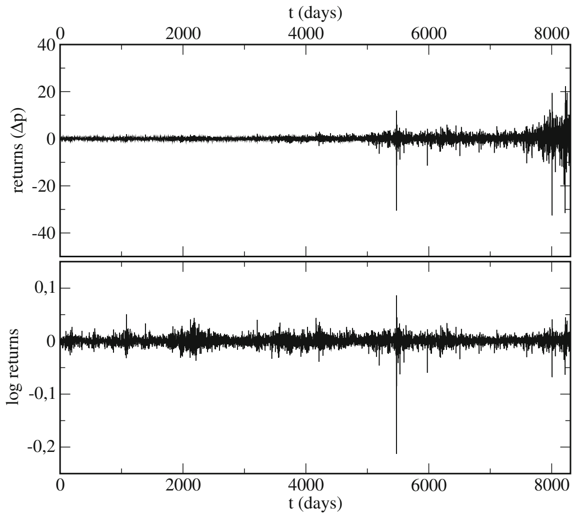

Fig. 2.3 Return time series of a stock of NYSE from 1966 to 1998. The two figures represent the same price pattern but returns are differently computed. In the *top* figure returns are calculated as simple difference, i.e. $r_t = p_t - p_{t-\Delta t}$ while in the *bottom* one returns are log returns that is $r_t = \log p_t - \log p_{t-\Delta t}$. From the lower plot we can see that volatility appears to be clustered and therefore large fluctuations tend to be followed by large ones and vice versa. The visual impression that the return time series appears to be stationary for log returns suggests the idea that real prices follow a multiplicative stochastic process rather than a linear process

2.4 Other Stylized Facts

Beyond these Stylized Facts we can state other relevant effects which are widespread in financial markets such as

- the gain/loss asymmetry, i.e. one observes large drawdowns in stock prices and stock index values but not equally large upward movements. This is linked to the asymmetry of the return pdf.
- leverage effect: the volatility of an asset are negatively correlated with the returns of that asset.
- trading volume and volatility are correlated.

See also [1, 3–5, 7, 8] for more details about Stylized Facts and their analysis.

2.5 Stationarity and Time-Scales

Before turning our attention to the analysis of the models which try to interpret Stylized Facts, we want to discuss a final question: the stationarity and the time scales of the observation of financial markets.

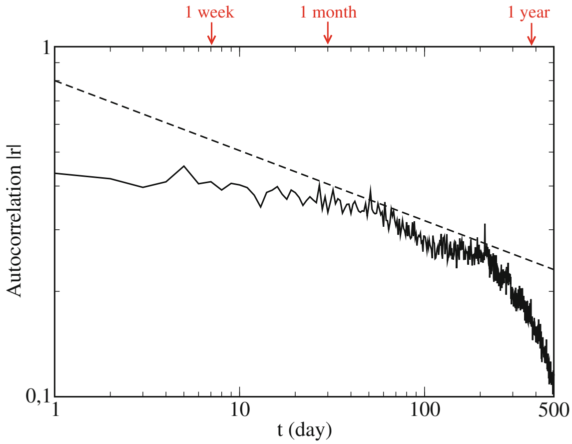

Fig. 2.4 Autocorrelation function of volatility measured as the absolute value of returns. We find that the function can be approximately described by a power law. We report a pure power law decay with exponent 0.2 for comparison. The return time series used in this analysis is the previous NYSE one from 1966 to 1988. It is worth noticing that the volatility is significantly correlated and then clustered on time scale longer than one year

The hypothesis of stationarity is usually invoked because, if satisfied, the statistical properties of the phenomenon under consideration become invariant under temporal translation. In the case of financial markets it is not clear whether the return time series verifies this condition: intraday activities, seasonality, weekends, holidays, Economy growth are elements that can *a priori* make the returns not stationary. However, it can be argued that the process is observed on the wrong scale (see [9]) and on a larger time scale the process may become stationary.

Last but not least it has been hypothesized that the non stationarity of financial series derives from the fact that they are studied in physical time units. On this account it has been proposed to define a time rescaling such that the transformation which makes stationary the financial data is the correct one. However, the choice of which elements should be involved in this transformation is arbitrary and ranges form seasonality of the calendar to the volumes of trading activity. The question is still open (see [10–13] for further references).

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