Preface

In 1849 G.G. Stokes published a classical paper [Sto] in which he modeled the response of a viscous incompressible fluid by a constitutive equation that has, ever since, been known as the Stokes law. When combined with the results in the earlier paper of Navier [Na], the Stokes law leads to the Navier–Stokes system of nonlinear partial differential equations; this system, subject to the incompressibility constraint on admissible velocity fields, has served as the basic mathematical model for studying the motions of incompressible viscous fluids for over a century and a half. The success of the Navier–Stokes model, in both the incompressible as well as the compressible case, has been far ranging and is unquestioned, in spite of various fundamental problems which still actively engage the attention of fluid dynamicists and mathematicians all around the globe; these problems include the well-known issues of showing that turbulent flow is a consequence of the Stokes law and discovering an adequate existence and uniqueness theory for the case of three-dimensional flows. It is widely acknowledged that there also exists a large variety of common substances (such as blood, motor oils, molasses, etc.) which exhibit fluid-like behavior but can not be adequately described using the Stokes law; such fluids are termed non-Newtonian and a significant number of constitutive relations have been proposed to describe either individual non-Newtonian fluids or entire classes of them.

This is not a book about the behavior of fluids which are universally recognized as exhibiting distinctly non-Newtonian behavior. Rather, this volume addresses the following question: what kind of model results if, in the process which leads to the formulation of the Stokes constitutive relation, we do not, a priori, impose the dual restrictions (1) that the relationship between the components of the reduced stress tensor and the rate of deformation tensor is strictly linear and (2) that the reduced stress tensor depends only on the first-order gradients of the velocity field. Thus, unlike models of non-Newtonian fluid flow, in which some ad-hoc nonlinear relation is assumed between the stress and rate of deformation tensors, and unlike some of the efforts, which will be described in this book, to regularize the Navier–Stokes model in three space dimensions by adding onto the equations terms involving higher-order spatial derivatives of the velocity field, the incompressible, nonlinear, bipolar fluid flow model treated in this volume is simply a consequence of not rigidly imposing, a priori, the two key assumptions which lead to the Stokes law; the philosophy underlying this approach has been clearly spelled out in the book by Shinbrot [Sh].

The first rigorous development of the theory of viscous, multipolar fluid flow is to be found in the fundamental paper of Nečas and Šilhavý [NS1]; the follow-up paper by Bellout et al. [BBN1] focused on elaborating the model in the incompressible, nonlinear bipolar case. With respect to the work in both [NS1] and [BBN1], it is essential to note that the development of the constitutive equations proceeds in such a manner as to render the resulting theory entirely consistent with the basic principles of material frame-indifference and the second law of thermodynamics (in the form of the Clausius-Duhem inequality). Also, as concerns the higher-order boundary conditions, which must be formulated for fluid flow problems in which spatial derivatives of the velocity of order higher than two appear, these are a rigorous consequence of the principle of virtual work coupled with some fundamental results due to Heron [HB] on the traces of divergence free vector fields; as such, these boundary conditions, which are an essential part of the theory of incompressible, nonlinear, bipolar fluid flow, stand out in stark contrast to the ad hoc types of higher-order boundary conditions which have been employed by those authors who have studied the regularizing effects of adding higher-order spatial derivatives to the Navier–Stokes equations. Since the original development of the multipolar fluid models, for both compressible and incompressible flow, a number of research groups, primarily in the United States, Eastern Europe, and China, have explored the consequences of these models; their efforts, which will be described in this book, have focused on the solution of problems in the context of specific geometries, on the existence of weak and classical solutions, and on such dynamical systems aspects of the theory as the existence of compact global attractors and inertial manifolds. The present volume is devoted exclusively to the task of elucidating some of the results which have been obtained, thus far, for the case of incompressible, nonlinear, bipolar fluid flow.

We now offer a description of the contents of this volume in the order in which the material is developed. Chapter 1 develops the theory of incompressible multipolar fluid dynamics with an emphasis on the nonlinear bipolar model. We begin in Sect. 1.1 by reviewing the hypotheses which lead to the Stokes constitutive law for viscous fluid flow and the Navier–Stokes equations which are a direct consequence of that law. In Sect. 1.2 we review the development of the general multipolar fluid model as presented in the fundamental paper of Nečas and Šilhavý [NS1]; the specialization to the case of linear bipolar fluid response appears in Sect. 1.3. Section 1.4 presents the development of the system of partial differential equations governing flow of an incompressible, nonlinear, bipolar fluid and is based, primarily, on the analysis presented in [BBN1]; a key feature of this section nis the derivation of the higher-order boundary conditions from the principle of virtual work coupled with the analysis in Heron [HB]. Elementary examples of incompressible nonlinear bipolar flows, i.e., steady plane Poiseuille flow, steady Poiseuille flow in a circular cylinder, and plane Couette flow are analyzed in Sect. 1.5. In Sect. 1.6 we describe some of the other extensions and generalizations of the standard Navier–Stokes model for incompressible viscous fluid flow which have one or more attributes in common with the bipolar fluid model; these include the non-Newtonian models of Ladyzhenskaya type [La1, 2], [DuG], [Lio1], multipolar fluids of grade 3 [BNR], dipolar fluids [BG], the extended incompressible viscous flow models of Green and Naghdi [GN1, 2], and the nonlinear dispersive Navier–Stokes alpha (NS-α) model of incompressible viscous flow, also known as the viscous Camassa-Holm equations (VCHE), treated in [CFH1, 2, 3] and [FHT1, 2]. Examples of particular flows associated with the models introduced in Sect. 1.6 are then studied in Sect. 1.7. Chapter 1 concludes by presenting, as further motivation for construction of the non-Newtonian model studied in this monograph, a catalog of experimental results which are inconsistent with the Stokes' hypothesis.

Chapter 2 is devoted to the problem of plane Poiseuille flow of incompressible bipolar viscous fluids between parallel plates; general results concerning existence, uniqueness, and continuous dependence on the constitutive parameters are elaborated in Sect. 2.2. Then, in Sect. 2.3, we obtain sharp estimates for the velocity field associated with the bipolar fluid in terms of the velocity fields associated with specializations of the bipolar constitutive theory that result from setting one or more of the key constitutive parameters equal to zero. Uniqueness of the steady Poiseuille flow within a general class of equilibrium flows in the parallel-wall domain is proven in Sect. 2.4. Finally, in Sect. 2.5 we consider the problems of existence and asymptotic stability for time-dependent plane Poiseuille flow of an incompressible, nonlinear, bipolar fluid. The work in this chapter is based, primarily, on the analysis in [BBN1] and [BB1, 2, 3].

In Chap. 3 we turn to a study of a variety of incompressible bipolar flows in special geometries and types of domains. Building on the work in [BH2], we formulate the classical problem of flow between rotating concentric cylinders for an incompressible, nonlinear, bipolar fluid in Sect. 3.2 and prove results concerning the existence, uniqueness, and continuous dependence (on constitutive parameters) for such flows. Section 3.3 is devoted to an analysis of bubble stability in a non-Newtonian viscous fluid of the type that the nonlinear bipolar model reduces to when the higher-order viscosity is set equal to zero; using the analysis presented in [Bl6] we elaborate upon the dynamics of a spherical bubble cavity in such a fluid employing both linearized dynamics and Lyapunov theory to analyze the stability of the cavity. In Sect. 3.4 we examine the problem of (steady) exterior flow of an incompressible, nonlinear, bipolar viscous fluid in the plane. Following the approach employed by Bellout and Nečas [BN] we first study the exterior problem in a truncated domain containing an obstacle and, then, proceed to obtain the solution in the original unbounded domain by implementing a limit process; it is also shown that the solution predicts the existence of a drag force on the obstacle in the direction of the velocity field at infinity. Finally, in Sect. 3.4 we study, following the analysis in [BW], the problem of flow of an incompressible, nonlinear, bipolar fluid over a non-smooth boundary, focusing on flows in polygonal domains. More specifically, the work in Sect. 3.4 analyzes the stability of solutions with respect to perturbations of the boundary of the domain and examines the regularity of solutions for problems defined on polygonal domains providing, in particular, a description of the asymptotic behavior of solutions near corners on the boundary of the domain. It is shown, in contrast with similar results based on use of the Navier–Stokes system (which are at variance with experimental data) that solutions of the bipolar initial-boundary value problem are not stable with respect to perturbations of the boundary of the domain by Lipschitz curves. Indeed, as we explicitly point out in Sect. 3.5, it has been known since the work of Nikuradze in the 1930s (e.g., [ScG]) that at high Reynolds numbers the presence of even very small protrusions on the surface of a bounding wall for a viscous flow substantially affects the flow.

Chapter 4 is devoted to proving general existence and uniqueness theorems for incompressible bipolar flow as well as for those non-Newtonian flows which result from setting the higher-order viscosity equal to zero; results are established for problems in both bounded and unbounded domains as well as for problems with periodic boundary conditions. The analysis begins in Sect. 4.2, where a Galerkin argument is used to prove the existence and uniqueness of weak solutions for the initial-boundary value problem associated with incompressible, nonlinear, bipolar flow in a bounded domain of R^n, $n = 2, 3$, with sufficiently smooth boundary. We also study the regularity of the solution and prove some estimates which establish the asymptotic stability of solutions of the initial-boundary value problem; the work in this section is based, for the most part, on the results obtained in [BBN4]. Section 4.3 establishes the existence of weak and measure-valued solutions for incompressible non-Newtonian fluids which are generated as a special case of the bipolar model with vanishing higher-order viscosity. Employing an a priori restriction to consideration of the relevant space-periodic problems, the concept of a Young measure-valued solution is first defined. Then, for a certain range of the order of the nonlinearity associated with the non-Newtonian model in space dimension $n = 2$, the Young measures are shown to be Dirac measures, while for another range the Young measures are proven to be Dirac and the associated weak solutions are shown to be regular solutions; a similar set of results is generated for the case of space dimension $n = 3$. The discussion in Sect. 4.3 is based, in large measure, on the work in [BBN2], [BBN3], and [MNN]. In Sect. 4.4 we again consider the problem of flow of an incompressible, nonlinear, bipolar fluid in an unbounded, parallel-wall channel. To prove existence of solutions for this problem, we first establish the existence of approximate solutions in bounded subdomains of the channel by using a Galerkin approach; then it is shown that there exists a subsequence of such approximate solutions whose limit is a unique weak solution of the initial-boundary value problem; the bulk of the analysis in this section first appeared in [BH4]. Finally, in Sect. 4.5 we summarize some of the most important extant results on existence and uniqueness for solutions of the Navier–Stokes equations and recall a few of the unresolved problems in three space dimensions. We also discuss related work on existence and uniqueness theorems for some of the generalizations of the Navier–Stokes model that were described in Sect. 1.6, including the non-Newtonian Ladyzhenskaya type models, viscous flow models with artificial viscosity, the multipolar fluid model of grade 3, and the viscous Camassa-Holm equations.

Chapter 5 focuses on the existence of maximal compact attractors for incompressible bipolar and non-Newtonian flows in bounded domains and for space periodic problems. Proving the existence of a maximal compact attractor involves (1) proving the existence of absorbing sets in order to deduce the uniform compactness, for large time, of the relevant solution operator, (2) establishing the uniform differentiability of the solution operator on the attractor, and (3) proving the uniform boundedness of an associated linearized operator; this operator, which is associated with the linearization of the nonlinear, incompressible, bipolar equations about an equilibrium solution, is introduced in Sect. 5.2 where we also establish the linearized stability of solutions of the incompressible bipolar equations. The results in Sect. 5.2 are based on the analysis in [Bl4]. Section 5.3 presents the results obtained in [BBN5] for incompressible bipolar initial-boundary value problems (and hold for the space periodic problems, also) in dimensions $n = 2, 3$; in Sect. 5.3 the existence of a maximal compact global attractor is proven and estimates are obtained for both the Hausdorff and fractal dimensions of the attractor. For a different range of the exponent controlling the nonlinearity in the bipolar model, it is shown in Sect. 5.4 that a maximal compact attractor exists for the space periodic problem, in $\dim n = 2$, for which both the Hausdorff and fractal dimensions are independent of the higher-order viscosity; it is also shown, independently, that the corresponding non-Newtonian space periodic problem admits a maximal compact attractor in space dimension $n = 2$; the results presented are based on the work in [Bl3]. Finally, it is shown in Sect. 5.5 that, as a consequence of the analysis in [Bl2, 3], the attractor for the bipolar problem, whose existence in the space periodic case when $n = 2$ was established in Sect. 5.4, converges in the sense of semidistance to the compact attractor for the corresponding non-Newtonian problem as the higher-order viscosity converges to zero.

Finally, Chap. 6 considers (1) the problem of the existence of an inertial manifold for bipolar, incompressible, viscous flows, and the associated phenomena of orbit squeezing, and (2) the question of whether a maximal compact global attractor exists for the flow of an incompressible bipolar viscous fluid in an unbounded, parallel walled channel; existence of solutions for this latter problem was proven in Sect. 4.4. Following the analysis in [BH3] it is proven, in Sect. 6.2, that an inertial manifold exists for the incompressible, nonlinear, bipolar viscous flow problem, subject to space periodic conditions, in both dimensions $n = 2$ and 3. The work in Sect. 6.2 also establishes a squeezing property for the orbits of the associated solution operator; a more fundamental (L^2) squeezing property is shown to hold in Sect. 6.3 by using results in [BH1]. In Sect. 6.4 we employ the analysis which appeared in [BH5] to prove the existence of a global compact attractor for the equations governing nonlinear bipolar fluid flows in unbounded two-dimensional channels. Finally, in Sect. 6.5, we survey some related recent work on the asymptotic behavior of solutions to problems for incompressible bipolar and non-Newtonian flows by other authors, highlighting developments connected with proving the existence of a global attractor.

Three appendices may be found at the end of this book, the first of which, Appendix A, sets the notation we have tried to use, consistently, throughout this volume. In the few instances when the same notation has been used with different meanings in different chapters (or sections), this circumstance has been carefully pointed out. Appendix A also reviews some important basic analysis results and definitions, including embedding and interpolation theorems for Sobolev spaces and some fundamental Sobolev space estimates. Appendix B establishes several key lemmas involving the rate of deformation tensor, including two inequalities of Korn type which are used repeatedly in Chaps. 4–6. The spectral gap condition, an essential ingredient in the proof of the existence of an inertial manifold in Chap. 6 is established in Appendix C. A reasonably comprehensive bibliography accompanies this volume; however, a substantial portion of the literature on non-Newtonian flows which is unrelated to the nonlinear bipolar model has not been detailed. In addition, by the time this volume appears, it is somewhat likely that new work within the realm of incompressible, nonlinear, bipolar viscous flow will have been published; for all such unintended omissions, the authors offer an a priori apology.

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