Pre-main Sequence Evolution and the Hydrogen-Burning Minimum Mass

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Abstract There is a lower limit to the mass of the main-sequence stars (the hydrogen-burning minimum mass) below which the stars cannot replenish the energy lost from their surfaces with the energy released by the hydrogen burning in their cores. This is caused by the electron degeneracy in the stars which suppresses the increase of the central temperature with contraction. To find out the lower limit we need the accurate knowledge of the pre-main sequence evolution of very low-mass stars in which the effect of electron degeneracy is important. We review how Hayashi and Nakano (1963) carried out the first determination of this limit.

1 Introduction

The stars on the main sequence replenish the energy lost outward from their surfaces with the energy released by the hydrogen burning in their cores. Because the nuclear energy is huge, the stars can maintain this steady state for a long time. There is a lower limit to the mass of such main-sequence stars, which we call *the hydrogen-burning minimum mass* hereafter.

The self-consistent determination of the hydrogen-burning minimum mass was first carried out by Hayashi and Nakano (1963). In this article I will review how they performed this determination.

The failure of attaining the steady state due to hydrogen burning is caused by the electron degeneracy in the star. The electron degeneracy is a quantum mechanical phenomenon. Electrons are fermions and obey the Pauli exclusion principle which allows only one particle per quantum state. For the gas of free electrons only two electrons (because of the two spin states) can take the state of the momentum

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$p = 0$, and the other electrons are forced to have non-zero momentum even at the temperature of $T = 0$ K. Therefore, the electron gas has non-zero pressure even at $T = 0$ K. The deviation from the ideal gas induced by the electron degeneracy is large at very high densities or very low temperatures. Protons (or neutrons) may also degenerate but only at much higher densities than electrons, e.g., in neutron stars, because of their large masses.

The pressure of the gas with degenerate electrons is much higher than the pressure of the gas with the electrons assumed to be in the Boltzmann distribution when compared at the same temperature and density. Therefore, the star with degenerate electrons can be in hydrostatic equilibrium even at the temperature of 0 K as long as the stellar mass is smaller than the Chandrasekhar limit given by Eq. (8) below. Therefore, the nuclear burning may not occur in stars with degenerate electrons.

To determine the stellar mass below which the steady state with the hydrogen burning cannot be attained, we need the accurate knowledge of the evolution of the stars in the pre-main sequence stage.

2 Stars with Degenerate Electrons

Stellar structure with degenerate electrons was investigated in the early twentieth century as the model of the white dwarfs (Chandrasekhar 1939). We give in the following a brief summary on the stellar structure with degenerate electrons and the start of electron degeneracy in the stars.

2.1 Stellar Structure

In hydrostatic equilibrium the central pressure P_c and the central density ρ_c of the star are given by (see e.g., Hayashi et al.1962)

$$P_c = aGM^2/R^4, \tag{1}$$

$$\rho_c = bM/R^3, \tag{2}$$

where G is the gravitational constant, M and R are the stellar mass and radius, respectively, and a and b are constants of order 1 whose values are determined by the structure, e.g. by the polytropic index for the polytropic stars. See Eq. (5) below for the polytropic index. We are not concerned about the stars with some special structure such as red giants which have very dense cores and extended envelopes with nuclear burning shells in between. By eliminating R from Eqs. (1) and (2) we obtain

$$P_c \propto M^{2/3}\rho_c^{4/3}. \tag{3}$$

This relation holds for the stars in hydrostatic equilibrium irrespective of the equation of state. The equation of state is an equation which gives the relation among the density ρ, the temperature T, and the pressure P of the gas, and is indispensable in solving the stellar structure.

Let $p_{\rm F}$ be the Fermi momentum of the electron, i.e., the momentum corresponding to the maximum energy of the free electron when all the lower energy levels are occupied. The number of the quantum states per unit volume for the free electrons with the momentum between p and $p + dp$ is equal to $8\pi p^2 dp/h^3$, where h is the Planck constant. Therefore, we find that $p_{\rm F}$ is proportional to $(\rho/\mu_{\rm e})^{1/3}$, where ρ is the density of the gas by mass and $\mu_{\rm e}$ is the mean molecular weight for electrons, or the number of nucleons per electron. Let X, Y, and Z be the fractions by mass of hydrogen, helium, and the heavier elements, respectively. Then we have $\mu_{\rm e}^{-1} = X + (Y + Z)/2 = (1 + X)/2$ because the heavier elements are dominated by C, N, and O whose atomic numbers are half their mass numbers.

The pressure P of the gas is the momentum flux of the gas particles, and is on the order of $n\bar{v}\bar{p}$, where n is the number density of the gas particles and $\bar{v}$ and $\bar{p}$ are the mean values of the velocity and the momentum of the particles, respectively.

When electrons are completely degenerate and non-relativistic, $\bar{p}$ is some fixed fraction of $p_{\rm F}$, and then the pressure P is proportional to $p_{\rm F}^2(\rho/\mu_{\rm e})$, and we obtain the equation of state omitting the coefficient

$$P \propto (\rho/\mu_{\rm e})^{5/3}. \tag{4}$$

Thus, the structure of the star in this situation is represented by a polytrope of index $N = 1.5$.

The polytropic index N characterizes the relation between the distribution of P and the distribution of ρ in the star, and is defined by

$$1 + \frac{1}{N} = \frac{d \log P/dr}{d \log \rho/dr}, \tag{5}$$

where r is the distance from the center. Thus N is in general a function of the position r in the star. But with the equation of state (4), N takes a constant value 1.5 throughout the star.

By eliminating $P_{\rm c}$ and $\rho_{\rm c}$ from Eqs. (2) and (3) using the equation of state (4) and bringing back the coefficient we find

$$R = 0.0400\mu_{\rm e}^{-5/3}(M/1M_\odot)^{-1/3}R_\odot. \tag{6}$$

For the stars supported by the gas of non-relativistic completely degenerate electrons, the radius R is a decreasing function of the mass M.

Even when electrons are only partially degenerate and non-relativistic, the stellar structure is also represented by a polytrope of $N = 1.5$ as long as the distribution of the specific entropy s in the star is uniform, or the star is fully convective (Hayashi et al. 1962). See Sect. 3.2 below for the relation between the distribution of s and the convection.

Eliminating P_c from Eqs. (3) and (4) we find $\rho_c \propto M^2$ for the stars with non-relativistic degenerate electrons. As M increases, ρ_c, and then p_F, increase, and at some stellar mass electrons become relativistic.

When electrons are extremely relativistic and completely degenerate, the pressure P is proportional to $p_F(\rho/\mu_e)$ because most of the electrons have a velocity, or $\bar{v}$, close to the light velocity, and we obtain the equation of state

$$P \propto (\rho/\mu_e)^{4/3}. \tag{7}$$

Thus, the star in this situation has a polytropic structure with $N = 3$. However, because this pressure in the equation of state and the pressure for hydrostatic equilibrium given by Eq. (3) have the same dependence on the density, the equilibrium with this equation of state is realized only at a special value of M, which is called the Chandrasekhar mass and is given numerically by (e.g., Hayashi et al. 1962)

$$M_{\rm Ch} = 5.75\mu_e^{-2} M_\odot. \tag{8}$$

For the white dwarfs composed of pure helium or pure carbon, we find $\mu_e = 2$, and then we obtain the well-known value $M_{\rm Ch} = 1.44\, M_\odot$. For M slightly smaller than $M_{\rm Ch}$, electrons are not extremely relativistic, and the dependence of the pressure on the density is slightly steeper than in Eq. (7). Therefore, hydrostatic equilibrium can be realized.

For the population I chemical composition, e.g., with 72 and 26 % by mass of hydrogen and helium, respectively, we have $\mu_e = 1.16$ and then $M_{Ch} = 4.27 M_\odot$. This does not mean that a star of mass close to this value can attain the equilibrium state with degenerate electrons without burning hydrogen. In order to find out the mass of the stars which can attain this state, we have to follow the stellar evolution prior to the hydrogen burning.

2.2 Start of Electron Degeneracy

When the temperature is so high and/or the density is so low that $kT \gg p_F^2/(2m_e)$ holds, where k is the Boltzmann constant and m_e is the mass of the electron, only a very small fraction of the quantum levels with the energy less than kT are occupied by electrons, hence the electrons are not degenerate, and the gas follows the ideal gas law.

For the stars with negligible electron degeneracy and negligible radiation pressure the central temperature is given by (see e.g., Hayashi et al. 1962)

$$T_c = f \frac{\mu m_H}{k} \frac{GM}{R}, \tag{9}$$

where μ is the mean molecular weight of the gas, $m_{\rm H}$ is the mass of the hydrogen atom, and f is a constant of order 1 whose value is determined by the structure, e.g. by the polytropic index N for the polytropic stars. Eliminating R from Eqs. (2) and (9) we obtain

$$\rho_{\rm c} \propto \mu^{-3} M^{-2} T_{\rm c}^{3}. \tag{10}$$

The Fermi energy of the non-relativistic electron is given by $p_{\rm F}^2/(2m_{\rm e})$. The Fermi energy at the center is proportional to $(\rho_{\rm c}/\mu_{\rm e})^{2/3}$, which is proportional to R^{-2} for a fixed M as seen from Eq. (2), while the thermal energy at the center $kT_{\rm c}$ is proportional to R^{-1} as Eq. (9) shows. As R decreases by contraction, the Fermi energy increases faster than the thermal energy. Therefore, the degree of electron degeneracy increases as the star contracts. Electron degeneracy becomes nonnegligible when $p_{\rm F}^2/(2m_{\rm e}) \approx kT$ holds because a significant fraction of the quantum levels with the energy less than kT are occupied by electrons. At the center this relation can be written as

$$(\rho_{\rm c}/\mu_{\rm e})^{2/3} \propto T_{\rm c}. \tag{11}$$

Eliminating $\rho_{\rm c}$ from Eqs. (10) and (11) we obtain

$$T_{\rm c} \propto \mu_{\rm e}^{2/3} \mu^2 M^{4/3}. \tag{12}$$

This means that in a star of smaller mass M the electron degeneracy becomes efficient at lower central temperature.

As the degree of degeneracy rises with contraction, the rise in the central temperature $T_{\rm c}$ slows down. Some time later $T_{\rm c}$ takes a maximum value at some radius R, and then decreases. Because the electron degeneracy becomes efficient at lower $T_{\rm c}$ for smaller M, the maximum attainable value of $T_{\rm c}$ is lower for the star of smaller mass M, suggesting the existence of the hydrogen-burning minimum mass.

3 Discovery of the Hayashi Phase

Determination of the hydrogen-burning minimum mass is crucially related to the pre-main sequence evolution. First we review how the Hayashi phase was discovered and what the Hayashi phase is.

3.1 Prior to Hayashi's Theory

The Hertzsprung-Russell diagram (HR diagram) is a useful tool in the study of the stars, in which the stellar luminosity is plotted against the effective temperature.

In the HR diagram of star clusters there is a region in which no stars exist with the effective temperature $T_{\rm eff}$ less than several thousand Kelvin (e.g., Sandage 1957). The reason for this was not known until 1961. For instance, Sandage and Schwarzschild (1952) investigated the stellar evolution after the exhaustion of hydrogen in the core, and were confronted with the unlimited decrease of the effective temperature. They presumed that the set-in of the shell hydrogen burning would increase the luminosity and stop the decrease of the effective temperature. It seems to have been considered in those days that the non-existence of stars in this low effective temperature region was peculiar to the advanced stage of stellar evolution and did not apply to the pre-main sequence stage. Stars in the pre-main sequence stage were considered to be in radiative equilibrium and the evolution calculation was started at fairly low effective temperature (e.g., Henyey et al. 1955).

3.2 Boundary Conditions at the Stellar Surface

The problem in these previous evolution calculations was in the boundary conditions at the stellar surface. In the late-type stars the hydrogen ionization zone lies inside the photosphere and has an effect of making the stellar envelope convectively unstable and suppresses the decrease of the effective temperature. The previous papers cited above neglected the convection induced by the H-ionization zone. Hayashi and Hoshi (1961) solved the structure of the stellar atmosphere taking into account the effect of the H-ionization zone and used this as the boundary condition for the internal structure. We describe this in more detail in the following.

In the stars the energy is transported by radiation and convection. Here we do not consider the thermal conduction by gas particles because it is efficient only in some restricted situations. Strictly speaking, convection occurs in the regions where the specific entropy s decreases outward, $ds/dr < 0$. However, s can effectively be regarded constant in the convection zone except at a very thin layer near the stellar surface because the energy transport by convection is so efficient that the negligibly small outward decrease of s is enough to maintain the necessary energy flux. In the radiative zone the local luminosity L_r, the amount of the energy crossing a sphere of radius r in a unit time, is proportional to the temperature gradient defined by $\nabla_{\rm rad} \equiv (d \log T/d \log P)_{\rm rad}$, the exact expression of which is given by Eq. (13) below. In the convection zone the temperature gradient is given by $\nabla_s \equiv (d \log T/d \log P)_s$, the derivative under the constant s. Convection occurs only where $\nabla_s < \nabla_{\rm rad}$. Usually $\nabla_{\rm rad}$ takes a value ~ 1, while $\nabla_s = 0.4$ for the fully ionized ideal gas with negligible radiation pressure. Therefore, in order to find out where the convection occurs, we have to solve the stellar structure accurately.

In the H-ionization zone in the stellar atmosphere, ∇_s is much smaller than 1 because the ionization potential of the hydrogen atom is much larger than the thermal energy kT. Therefore, convection easily occurs. Because $\nabla_s \ll 1$, the decrease of $\log T$ outward in the H-ionization zone is much smaller than the decrease of $\log P$ and $\log \rho$. Because $\log P$ and $\log \rho$ decrease by orders of magnitude in the H-ionization zone, the outer boundary of the H-ionization zone has very low density, and hence is usually not very far from the photosphere. Thus, the effective temperature $T_{\rm eff}$ is not very low compared with the temperature at the outer boundary of the H-ionization zone.

Hayashi and Hoshi (1961) determined by numerical calculation the critical effective temperature $T_{\rm eff}^{(\rm cr)}$ as a function of the stellar mass and radius, which is several times 10^3 K and has the following characteristics. For $T_{\rm eff} = T_{\rm eff}^{(\rm cr)}$ the stellar structure is represented by the Emden solution for a polytrope of index $N = 1.5$ throughout the star, and the star is fully convective. For $T_{\rm eff} > T_{\rm eff}^{(\rm cr)}$ the solution of the Lane-Emden equation for $N = 1.5$ is of the centrally condensed type singular at the center $r = 0$ with $\rho = \infty$ and $M_r > 0$, where M_r is the mass included inside a sphere of radius r, indicating that there is a point mass at the center. This solution can be fitted at a finite radius $r > 0$ with a regular core solution with the effective polytropic index, the mean value of N defined by Eq. (5) in the core, $\bar{N} > 1.5$. Thus, the star with $T_{\rm eff} > T_{\rm eff}^{(\rm cr)}$ is composed of a convective envelope and a radiative core. For $T_{\rm eff} < T_{\rm eff}^{(\rm cr)}$ the solution of the Lane-Emden equation for $N = 1.5$ is of the collapsed type in which M_r decreases to 0 before reaching the center $r = 0$. This solution can be fitted at a position with $M_r > 0$ only with a regular core solution with the effective polytropic index $\bar{N} < 1.5$. However, in such a core solution the entropy s decreases outward and violent convection occurs changing the distribution of s in a dynamical time scale. Therefore, stars cannot be in hydrostatic equilibrium with such $T_{\rm eff}$. Hayashi and Hoshi (1961) found that the red giant branches of the star clusters in the HR diagram are close to the lines of $T_{\rm eff} = T_{\rm eff}^{(\rm cr)}$ for appropriate values of the stellar mass.

3.3 Hayashi's Theory

Because there is no equilibrium state at $T_{\rm eff} < T_{\rm eff}^{(\rm cr)}$ in any stage of stellar evolution, Hayashi (1961) considered that at least the final phase of star formation is dynamical and the star appears on the line of $T_{\rm eff} = T_{\rm eff}^{(\rm cr)}$ in the HR diagram. The line of $T_{\rm eff} = T_{\rm eff}^{(\rm cr)}$ is now called the Hayashi line and the region of $T_{\rm eff} < T_{\rm eff}^{(\rm cr)}$ is called Hayashi's forbidden region.

Hayashi (1961) considered that the star, which appeared on the Hayashi line, contracts along the Hayashi line decreasing its luminosity, and changes the path to the higher temperature part of the radiative path (Henyey et al. 1955) near the cross point of these lines. The path along the Hayashi line is called the Hayashi track and the radiative path the Henyey track. The phase on the Hayashi track is called the Hayashi phase. The stellar luminosity on the Hayashi track is higher than that on the lower temperature part of the radiative path in the old theory when compared at the same stellar radius, and the contraction time of the star along the Hayashi track is shorter than along the lower temperature part of the radiative path in the old theory.

A possibility of the high luminosity phase for the primitive sun greatly stimulated the research on the origin of the solar system.

The reason why the star changes from the Hayashi track to the Henyey track may be explained in the following way. The temperature gradient $\nabla_{\rm rad}$ is given by Hayashi et al. (1962)

$$\nabla_{\rm rad} = g \frac{P}{T^4} \frac{\kappa L_r}{M_r}, \tag{13}$$

where κ is the Rosseland mean opacity and g is a constant. This gives the temperature gradient necessary to transport the energy flow L_r by the radiation alone. In the stars of intermediate and small mass the opacity is mainly contributed by the bound-free and free-free transitions and can be approximated by the Kramers' law $\kappa \propto \rho T^{-3.5}$. The star on the Hayashi track is fully convective and its structure is represented by the Emden solution for $N = 1.5$. Introducing $\theta \equiv T/T_{\rm c}$, which is called the Emden function, we have $\rho/\rho_{\rm c} = \theta^{1.5}$ and $P/P_{\rm c} = \theta^{2.5}$. Substituting these relations into Eq. (13) and eliminating $P_{\rm c}$, $\rho_{\rm c}$ and $T_{\rm c}$ by using Eqs. (1), (2) and (9) we obtain

$$\nabla_{\rm rad} \propto \theta^{-3.5} \frac{R^{0.5} L}{M^{5.5}} \left(\frac{L_r/L}{M_r/M} \right), \tag{14}$$

where L is the stellar luminosity. The energy release rate per unit mass, dL_r/dM_r, is proportional to $T ds/dt$ according to the first law of thermodynamics. Therefore, we have $L_r = \int_0^{M_r} (dL_r/dM_r) dM_r \propto (ds/dt) \int_0^{M_r} T dM_r$ because s is uniform in the star. Thus, L_r/M_r is proportional to the mean temperature inside M_r, which we write $T_{\rm c}\bar{\theta}$. Because L is proportional to $T_{\rm c} ds/dt$, Eq. (14) can be rewritten as

$$\nabla_{\rm rad} \propto \theta^{-2.5} \left(\frac{\bar{\theta}}{\theta} \right) \frac{R^{0.5} L}{M^{5.5}}. \tag{15}$$

The Emden function θ is a decreasing function of r/R. The mean interior temperature $\bar{\theta}$ also decreases outward, but more slowly than θ. Therefore, $\nabla_{\rm rad}$ takes a minimum value at the center $r = 0$. Because L is nearly proportional to R^2 on the Hayashi track, $\nabla_{\rm rad}$ decreases as the star contracts, and finally becomes smaller than $\nabla_s = 0.4$ at the center. In this way the convection stops at the center and a radiative core appears. With a radiative core the effective temperature $T_{\rm eff}$ becomes higher than the critical value $T_{\rm eff}^{\rm (cr)}$ which corresponds to the Hayashi line. As the star contracts, the radiative core grows and the star gradually moves away from the Hayashi line.

Hayashi et al. (1962) investigated numerically the evolution from the Hayashi phase to the main sequence via the Henyey track for some values of the stellar mass and showed the evolutionary paths on the HR diagram in their Fig. 10-2.

4 Pre-main Sequence Evolution of Low-mass Stars

In the case of low-mass stars we have to take into account some effects which are not important for the stars of $M \gtrsim 1 M_\odot$. One is the effect of H_2 molecules on the structure of the atmosphere and another is the electron degeneracy.

4.1 The Effect of H_2-Dissociation Zone

As shown by Hayashi and Hoshi (1961), $T_{\mathrm{eff}}^{(\mathrm{cr})}$ decreases as the stellar mass decreases when compared at a fixed stellar radius. Therefore, for stars of sufficiently small mass the H_2-dissociation zone must lie inside the photosphere. The H_2-dissociation zone has the effect of making the stellar envelope convectively unstable and suppressing the decrease of the effective temperature as the H-ionization zone does. Hayashi and Nakano (1963) investigated the pre-main sequence evolution of low-mass stars taking into account the effect of the H_2-dissociation zone.

The H_2-dissociation zone has been found to have a great effect. If the formation of H_2 molecules is neglected, the effective temperature on the Hayashi line decreases rapidly as the stellar radius decreases while with H_2 formation the decrease of the effective temperature is very slow as long as the stellar radius is somewhat larger than the limiting value of the degenerate star given by Eq. (6), as shown in figure 3 of Hayashi and Nakano (1963).

4.2 The Zero-Age Main Sequence and Its Minimum Mass

In the pre-main sequence stage of the stars with mass $M \gtrsim 0.4 M_\odot$ the effect of electron degeneracy is negligible, and the central temperature T_c increases in proportion to R^{-1} as shown in Eq. (9). Finally at $T_c \approx 10^7$ K the hydrogen burning sets in and soon the stars settle down on the main sequence (the zero-age main sequence: ZAMS).

ZAMS is the stage at which $L_H = L$ holds for the first time in the pre-main sequence contraction phase, where L_H is the energy released by hydrogen burning in the star per unit time and L is the stellar luminosity or the energy emitted from the stellar surface per unit time.

L_H is determined by the structure of the central part of the star though it is not completely independent of the structure of the outer part. If the star is in the Hayashi phase, the structure is represented by the polytrope of $N = 1.5$, and L_H can be determined easily for given M and R by using the Emden solution. To obtain L_H for a star on the Henyey track we have to begin by solving the stellar structure.

To determine L we have to solve in general the whole structure of the star from the center to the surface. The structure of the outer part is sometimes very important in determining L. However, for the stars in the Hayashi phase, L for a given R can be determined easily if the Hayashi line is known. The cross point of the Hayashi line and the line of constant R in the HR diagram gives L.

Thus, to determine ZAMS for a given stellar mass M we need both L_{H} and L as functions of R. Schematically speaking, the cross point of the two curves $L_{\mathrm{H}}(R)$ and $L(R)$ on the ($\log R - \log L_{\mathrm{X}}[L_{\mathrm{H}}$ or $L]$) plane gives ZAMS.

For a star of not very small mass in which the effect of electron degeneracy is very small, the central temperature T_{c} increases in proportion to R^{-1} with contraction. The increase of L_{H} therewith is much steeper because the rate of the nuclear burning is very sensitive to the temperature.

As the star contracts along the Hayashi track, L decreases. If the star moves to the Henyey track, L turns to increase though very slowly. Because this increase is much slower than the increase of L_{H}, the two curves $L_{\mathrm{H}}(R)$ and $L(R)$ on the ($\log R - \log L_{\mathrm{X}}$) plane cross each other at some R, and the state of $L_{\mathrm{H}} = L$ is attained. In this way the ZAMS state is realized. When L_{H} has increased to a significant fraction of L on the Henyey track, the structure is affected by the increase of L_r, or by the increase of ∇_{rad}, in the central region. As a result, L decreases just before the star settles down on the main sequence as shown in the figures of Henyey et al. (1955). Thus, to obtain the exact ZAMS state we have to solve the stellar structure accurately even by taking into account the effect of hydrogen burning.

For the stars which settle down to ZAMS on the Hayashi track, the start of hydrogen burning does not affect the stellar structure. By the increase of L_r in the core ∇_{rad} increases. But ∇_{rad} was larger than ∇_s even before the hydrogen burning sets in. The start of the hydrogen burning just makes the star more convectively unstable, and the star keeps the polytropic structure with $N = 1.5$. Determination of ZAMS on the Hayashi track is much easier than that on the Henyey track.

As discussed in Sect. 2.2, the electron degeneracy becomes efficient at lower T_{c} for a star of smaller mass. The electron degeneracy has an effect of slowing down the increase of T_{c} with contraction, and in course of time T_{c} takes a maximum value at some R, and then decreases. Consequently, L_{H} takes a maximum value at some R, and then decreases. Such effect of electron degeneracy is important in determining ZAMS for low-mass stars with $M \lesssim 0.2M_{\odot}$. The maximum value of L_{H} decreases so fast as the stellar mass M decreases that the two curves $L_{\mathrm{H}}(R)$ and $L(R)$ on the ($\log R - \log L_{\mathrm{X}}$) plane do not cross or touch each other, and the state of $L_{\mathrm{H}} = L$, or ZAMS, cannot be realized at the stellar mass M smaller than some critical value. This critical value is the lower-mass limit to the main sequence, or the hydrogen-burning minimum mass.

There is a paper which claims to have determined the hydrogen-burning minimum mass by assuming the value of L (Kumar 1963). However, the hydrogen-burning minimum mass is not known beforehand, nor is the luminosity of the star of this mass when it settles down on the main sequence as discussed by Nakano (2012).

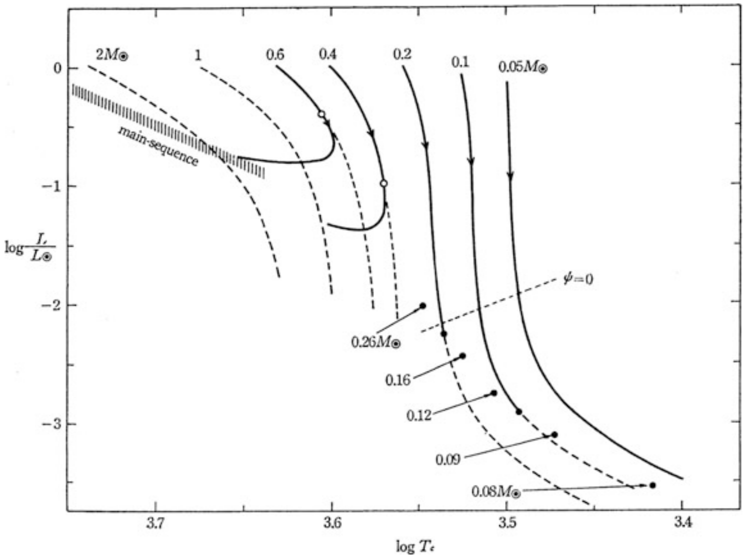

Fig. 1 The evolutionary paths (*solid lines*) and the positions of ZAMS (*filled dots*) of low-mass stars on the HR diagram. The *open circles* on the evolutionary paths indicate the stage at which the fully convective phase ends. The evolutionary path of a star of mass $M = 0.05\,M_\odot$ is also shown. This star cannot settle down on ZAMS, and contracts to the limiting radius given by Eq. (6) cooling down indefinitely. The *solid lines* except below the *open circles* and the *thick dashed lines* are the Hayashi lines. The *thin dashed line* represents the state of $\psi = 0$ where the effect of the electron degeneracy becomes nonnegligible for the fully convective stars. This is a copy of figure 2 of Hayashi and Nakano (1963)

4.3 Numerical Results

Hayashi and Nakano (1963) investigated the pre-main sequence evolution for the stars of mass $M \leq 0.6 M_\odot$. Their results are shown in Fig. 1, which is a copy of their figure 2.

The open circles on the evolutionary paths in Fig. 1 (solid lines) indicate the stage at which the fully convective phase ends. Afterwards the radiative core grows and the star moves to the Henyey track. The length of the Henyey track until the star arrives at the main sequence is shorter for stars of smaller mass. The stars with $M \leq 0.26\,M_\odot$ do not experience the phase on the Henyey track. When these stars settle down on the main sequence, they have the fully convective structure. The filled dots in Fig. 1 show the positions of ZAMS for some values of the stellar mass down to $0.08\,M_\odot$.

Hayashi and Nakano (1963) also investigated the evolution of a star of mass $M = 0.07\,M_\odot$. The evolutionary sequence of this star is shown in Table 1, which is a reproduction of their table III. In this table, ψ in the first column represents the degree of electron degeneracy, or the chemical potential of the

Table 1 The evolutionary sequence of a star of $M = 0.07\,M_\odot$ with the population I chemical composition. This is a reproduction of table III of Hayashi and Nakano (1963)

ψ[a]	$\log L/L_\odot$	$\log T_{\rm eff}$	$\log R/R_\odot$	$\log T_{\rm c}$	$\log \rho_{\rm c}(\rm g\,cm^{-3})$	$L_{\rm H}/L$	Age (10^8 year)
3.0	−2.90	3.48	−0.89	6.52	2.43	0.015	0.8
5.0	−3.23	3.45	−0.99	6.54	2.74	0.090	1.5
7.0	−3.45	3.42	−1.05	6.52	2.82	0.17	3.1
9.0	−3.63	3.40	−1.09	6.49	3.05	0.23	4.6
11.0	−3.74	3.38	−1.12	6.45	3.26	0.19	6.2

[a] ψ is the degree of electron degeneracy, or the chemical potential of the electron divided by kT

electron divided by kT. The effect of electron degeneracy becomes nonnegligible around the state of $\psi = 0$, which is shown by the dashed line in Fig. 1 for the fully convective stars. In the stars with uniform s, ψ is also uniform. Around the stage $\psi = 5.0$ the central temperature takes the maximum value $\log T_{\rm c} \approx 6.54$, and thereafter the core gradually cools down. At $\psi \approx 9.0$, $L_{\rm H}/L$ takes the maximum value ≈ 0.23 and then decreases. This maximum value is far below the value for the steady state hydrogen burning, $L_{\rm H}/L = 1$. Therefore, this star cannot attain the ZAMS state. In this way Hayashi and Nakano (1963) concluded that the hydrogen burning minimum mass is between 0.08 and 0.07 $M_\odot$.

This is for the population I chemical composition. The abundance of heavy elements might have some effect on the Hayashi line, and then on the hydrogen-burning minimum mass, through the opacity. Hayashi and Nakano (1963) confirmed that the minimum mass is hardly affected by the change of the heavy element abundance even by a factor of 5, although it is significantly affected by the change of the helium content through the change of the mean molecular weight of electrons $\mu_{\rm e}$. Because it is known now that the helium content is almost the same for population I and II, the hydrogen-burning minimum mass for the population II stars is almost the same as the one for the population I stars.

The stars of mass smaller than this critical value cool down indefinitely, and are now called brown dwarfs. Hayashi and Nakano (1963) investigated the evolution of a star of mass $M = 0.05 M_\odot$ as an example of such stars although the term "brown dwarf" was not yet used in those days (see the chapter by J. Tarter in this volume). The evolutionary path of this star on the HR diagram is shown in Fig. 1. This must be the first evolutionary path of brown dwarfs drawn on the HR diagram.

The lithium burning becomes efficient around $T \approx 3 \times 10^6$ K, significantly lower than the temperature of hydrogen burning. Lithium in the stellar envelope is depleted by the burning near the bottom of the convective envelope in the pre-main sequence stage. For the star of larger mass the radiative core appears at a lower central temperature. Therefore, the amount of lithium depletion in the atmosphere depends on the stellar mass. Hayashi and Nakano (1963) investigated this dependence at $M \geq 0.4\,M_\odot$. At the smaller stellar masses where the electron degeneracy can be efficient, the maximum central temperature attained during the contraction is lower for the smaller stellar mass. As seen from Table 1, the maximum $T_{\rm c}$ for the star of $M = 0.07\,M_\odot$ is 3.5×10^6 K. At slightly smaller M, lithium hardly burns. Later on it was pointed out that the abundance of lithium in the stellar atmosphere can be used to test whether the candidate stars for brown dwarfs are really brown dwarfs or not (e.g., Rebolo et al. (1992), see also the chapters by R. Rebolo and G. Basri in this volume).

Finally, I would like to point out that the newest results I know give the hydrogen-burning minimum mass between 0.08 and 0.075 $M_{\odot}$ (Burrows et al. 1997), which is almost the same as the results of Hayashi and Nakano (1963).

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