

Chapter 2

Piezoceramic Elements for PEAT

Abstract Mathematical models of fluctuations, which arise in piezoceramic elements of various forms and designs are resulted in this chapter. Work principle and requirements to piezoelectric resonators and transformers, and also designs of bimorph and trimorph piezoelements are described in this chapter.

The industry makes a significant number of standard sized piezoceramic elements from various materials [1–7]. Some are shown in Fig. 2.1.

In the given work transducers with piezoelements in the form of a disk polarised on a thickness, and in the form of the hollow cylinder polarised on radius are studied.

At studying of piezoelements we can consider them from the different points of view. For example, we can consider a piezoelement as the electromechanical system fluctuating on resonant frequency under the influence of electric voltage (radiator) or in pre-resonance of area and on resonant frequency under the influence of mechanical pressure (receiver).

We can consider also a piezoelement with two electrodes as the resonator, and with three and more electrodes—as the piezoelectric transformer.

If to a piezoelement or two piezoelements to paste a metal plate (bimorph or trimorph element), characteristics of this oscillatory system will essentially differ from characteristics of monomorph piezoelement. Therefore such devices are considered by us separately.

2.1 Fluctuations of Piezoceramic Disk Polarised on a Thickness

The analysis of fluctuations of thin piezoceramic disk is executed by N.A. Shulga and A.M. Bolkisev [8].

Let's consider a piezoelectric disk, radius R which considerably surpasses its thickness (Fig. 2.2). Fluctuations are raised by potential difference ΔV enclosed to electrodes, located on face surfaces. Let's admit, that the thickness of a disk is so small, that change of pressure along axis Z can be neglected. We come to a problem

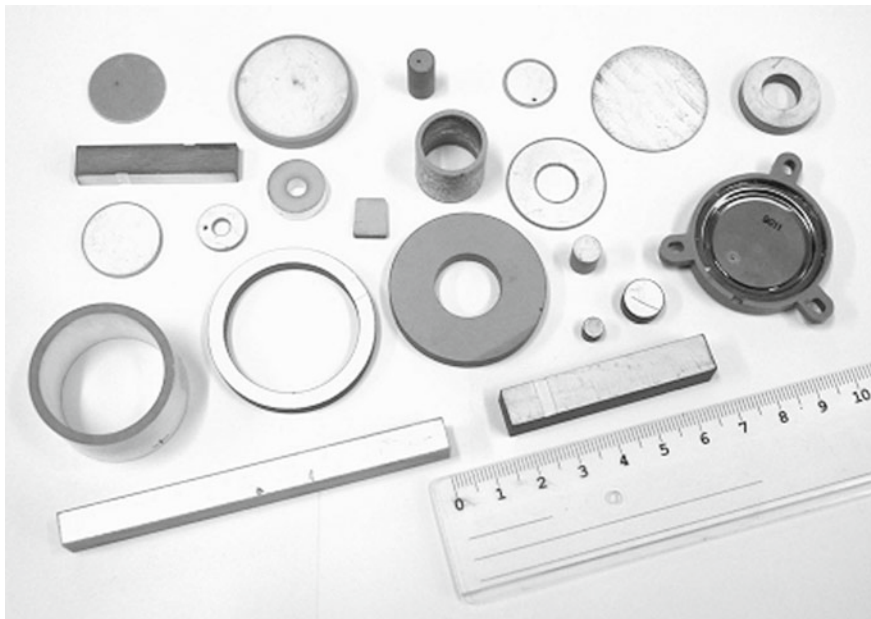
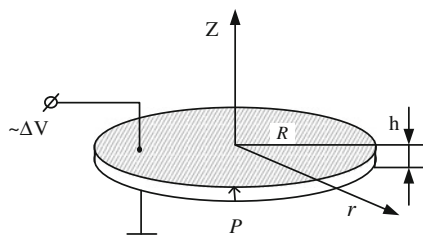


Fig. 2.1 Piezoceramic elements for PEAT

Fig. 2.2 Piezoelement in the form of disk



about a flat intense condition, in which

$$\sigma_{zz} = \sigma_{rz} = \sigma_{\theta z} = 0.$$

Other components of pressure and movings to disk planes do not depend from r . Besides, from axial symmetry follows $u_\theta = 0$, and then geometrical parities can be written down in a kind

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{1}{r}u_r, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \varepsilon_{r\theta} = \varepsilon_{\theta z} = 0 \quad (2.1)$$

As the thin plate is concluded between two equipotential surfaces electric field along radius is considered by the homogeneous. Thus, the condition equation is chosen in shape

$$\begin{aligned}\varepsilon_{rr} &= s_{11}^E \sigma_{rr} + s_{12}^E \sigma_{\theta\theta} + d_{31} E_z, \\ \varepsilon_{\theta\theta} &= s_{12}^E \sigma_{rr} + s_{11}^E \sigma_{\theta\theta} + d_{01} E_z, \\ \varepsilon_{zz} &= s_{13}^E (\sigma_{rr} + \sigma_{\theta\theta}) + d_{33} E_z, \\ D_z &= d_{31} (\sigma_{rr} + \sigma_{\theta\theta}) + \varepsilon_{33}^T E_z,\end{aligned}\tag{2.2}$$

The third equation of system (2.2) appears superfluous and can be used for definition of displacement on a thickness.

The planar factor of electromechanical communication k_p is defined by a share of electric energy from the full entrance energy created by system of pressure $\sigma_{rr} = \sigma_{\theta\theta} = p_0$, $\sigma_{zz} = 0$. Under the influence of such system of pressure in a disk there are deformations and electric field:

$$\begin{aligned}\varepsilon_{rr} = \varepsilon_{\theta\theta} &= s_{11}^E (1 - \nu) \left(1 - \frac{2d_{31}^2}{s_{11}^E (1 - \nu) \varepsilon_{33}^T} \right) p_0 \\ E_z &= -\frac{2d_{31}}{\varepsilon_{33}^T} p_0\end{aligned}$$

where $\nu = -s_{12}^E/s_{11}^E$ —Pousson's coefficient. The density of reserved electric energy is calculated under the formula

$$W_{el} = \frac{1}{2} \varepsilon_{33}^T \left(1 - \frac{2d_{31}^2}{s_{11}^E (1 - \nu) \varepsilon_{33}^T} \right) E_z^2\tag{2.3}$$

The density of brought mechanical energy is defined as

$$W_{mech} = \frac{1}{s_{11}^E (1 - \nu)} \varepsilon_{rr}^2 = s_{11}^E (1 - \nu) \left(1 - \frac{2d_{31}^2}{s_{11}^E (1 - \nu) \varepsilon_{33}^T} \right) p_0^2$$

Considering (2.3), for planar coefficient of communication is received

$$k_p^2 = \frac{W_{el}}{W_{mech} + W_{el}} = \frac{2}{1 - \nu} \cdot \frac{d_{31}^2}{s_{11}^E \varepsilon_{33}^T}$$

Radial fluctuations of a disk are described by the movement equation

$$\frac{\partial \sigma_{kk}}{\partial r} - \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) + \rho \omega^2 u_r = 0.\tag{2.4}$$

To substitute parities (2.2) in the movement equation, it is necessary to express components of mechanical pressure through components of deformations. Taking into consideration (2.1), we receive

$$\begin{aligned}\sigma_{rr} &= \frac{1}{s_{11}^E(1-\nu^2)} \left(\frac{du_r}{dr} + \nu \frac{1}{r} u_r \right) - \frac{d_{31}}{s_{11}^E(1-\nu)} E_z, \\ \sigma_{\theta\theta} &= \frac{1}{s_{11}^E(1-\nu^2)} \left(\nu \frac{du_r}{dr} + \frac{1}{r} u_r \right) - \frac{d_{31}}{s_{11}^E(1-\nu)} E_z\end{aligned}\quad (2.5)$$

The Eqs. (2.4) and (2.5) can be reduced to an initial form of Bessel equation, entering a variable $z_1 = \sqrt{\rho s_{11}^E(1-\nu^2)} \omega r$:

$$\frac{d^2 u_r}{dz_1^2} + \frac{1}{z_1} \frac{du_r}{dz_1} + \left(1 - \frac{1}{z_1^2} \right) u_r = 0. \quad (2.6)$$

The common decision (2.6) is a linear combination of Bessel functions of the first and second sort

$$u_r = A J_1(kr) + B Y_1(kr), \quad (2.7)$$

where $k^2 = \rho \omega^2 s_{11}^E(1-\nu^2)$. Any constants A, B are defined from boundary conditions

$$u_r = 0, \quad r = 0, \quad \sigma_{rr} = 0, \quad r = R. \quad (2.8)$$

As at $r = 0$ function $Y_1 \rightarrow -\infty$, that $B = 0$. From the second condition (2.8) it is received

$$\begin{aligned}A &= \frac{(1+\nu)d_{31}E_z R}{\chi J_0(\chi) - (1-\nu)J_1(\chi)}, \\ \sigma_{rr} &= \frac{d_{31}E_z}{s_{11}^E(1-\nu)} \left[\frac{\chi J_0(kr) - (1-\nu)\frac{R}{r}J_1(kr)}{\chi J_0(\chi) - (1-\nu)J_1(\chi)} - 1 \right], \\ \sigma_{\theta\theta} &= \frac{d_{31}E_z}{s_{11}^E(1-\nu)} \left[\frac{\chi \nu J_0(kr) - (1-\nu)\frac{R}{r}J_1(kr)}{\chi J_0(\chi) - (1-\nu)J_1(\chi)} - 1 \right],\end{aligned}\quad (2.9)$$

where $\chi = \omega R \sqrt{\rho s_{11}^E(1-\nu^2)}$ —dimensionless frequency. Following step of the decision is definition of full conductivity of a disk. For this purpose we will integrate a physical parity (2.2) for D_z on the electrode area. From (2.9), considering, that $\Delta V = E_z h$, we find

$$I = i\omega \frac{2\pi R d_{31} A}{s_{11}^E(1-\nu)} J_1(\chi) + i\omega \frac{\pi R^2}{h} \left(\varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E(1-\nu)} \right) \Delta V.$$

Thus, full conductivity of a disk on a radial mode is defined under formula

$$Y_6 = i\omega \frac{\pi R^2 \varepsilon_{33}^T}{h} \left(1 - k_p^2 + k_p^2 \frac{1 + \nu}{\Delta} J_1(\chi) \right). \quad (2.10)$$

Here $\Delta = \chi J_0(\chi) - (1 - \nu)J_1(\chi)$ —the frequency equation, which roots define values of resonant frequencies. As we see, resonant frequencies depend on pliability s_{11}^E and Puasson's coefficient.

These two parameters of piezoceramic material can be defined on measurements of two resonant frequencies.

First four dimensionless resonant frequencies χ enough a thin disk ($h/R < 0, 1$) have the values concerning approximately as 2,05:5,40:8,58:11,73, Whereas the basic fashion thickness fluctuations $\chi \approx 20$. Hence, on the several first resonances the assumptions made at the decision of a problem about radial fluctuations, will hold good.

The relation of the basic frequency to frequency overtone $\chi_{r0}/\chi_{r1} = f_{r0}/f_{r1}$ does not depend on a pliability s_{11}^E . Having constructed on the frequency equation dependence χ_{r0}/χ_{r1} from Puasson's coefficient, we will define it on measurement f_{r0}/f_{r1} . Further the pliability coefficient is calculated under the formula

$$s_{11}^E = \chi_{r0}^2 / 4\pi^2 \rho (1 - \nu^2) R^2 f_{r0}^2. \quad (2.11)$$

The decision of a problem on disk fluctuations can be generalised on a problem about radial fluctuations of a ring with thickness polarisation for which the general decision also looks like (2.7), and constants A, B are defined from boundary conditions

$$\sigma_{rr} = 0, \quad r = r_1, r_2, \quad (2.12)$$

where r_1 and r_2 —internal and external radiuses of a ring. The particular decision satisfying (2.12), looks like

$$u_r(r) = \frac{(1 + \nu)d_{31}E_z r_2}{\Delta k} \{ [\chi_1 \Delta_2(\chi_2) - \chi_2 \Delta_2(\chi_1)] J_1(kr) + [\chi_2 \Delta_1(\chi_1) - \chi_1 \Delta_1(\chi_2)] Y_1(kr) \}, \quad (2.13)$$

where

$$\begin{aligned} \chi_1 &= kr_1, \chi_2 = kr_2, \quad \Delta_1(\chi) = \chi J_0(\chi) - (1 - \nu)J_1(\chi), \\ \Delta_2(\chi) &= \chi Y_0(\chi) - (1 - \nu)Y_1(\chi); \end{aligned}$$

$\Delta = \Delta_1(\chi_1)\Delta_2(\chi_2) - \Delta_1(\chi_2)\Delta_2(\chi_1)$ —frequency equation. The analysis of last equation shows, that frequency of the basic fashion at increase r_1/r_2 decreases, frequencies of the first and second overtones at first decrease, and then increase, i.e. at $0.05 < r_1/r_2 < 0.3$ there is a minimum. Influence of Puasson's coefficient on roots of the frequency equation for a disk with a small aperture ($r_1/r_2 \leq 0.4$) It appears essential (an order 2%). At the expense of a corresponding choice of the

geometrical sizes efficiency of transformation of energy on an overtone of the basic fashion can be increased. Let's define conductivity of a ring on radial fashions.

Having integrated a physical parity (2.2) for D_z on the electrode area, we will receive a displacement current

$$I = i\omega \frac{2\pi d_{31}}{s_{11}^E(1-\nu)} \{A[r_2 J_1(\chi_2) - r_1 J_1(\chi_1)] + B[r_2 Y_1(\chi_2) - r_1 Y_1(\chi_1)]\} \\ + i\omega\pi \left(\varepsilon_{33}^T - \frac{2d_{31}^2}{s_{11}^E(1-\nu)} \right) (r_2^2 - r_1^2).$$

The relation of a current to brought potential difference $E_2 h$ gives full conductivity of a ring

$$Y_7 = i\omega \frac{S\varepsilon_{33}^T}{h} \left\{ 1 - k_p^2 + \frac{(1+\nu)k_p^2}{\Delta k^2(r_2^2 r_1^2)} \right. \\ \left. \times [(\chi_1 \Delta_2(\chi) - \chi_2 \Delta_2(\chi_1)) \cdot (\chi_2 J_1(\chi_2) - \chi_1 J_1(\chi)) \right. \\ \left. + (\chi_2 \Delta_1(\chi_1) - \chi_1 \Delta_1(\chi_2)) \cdot (\chi_2 Y_1(\chi_2) - \chi_1 Y_1(\chi_1))] \right\} \quad (2.14)$$

where S —electrode area.

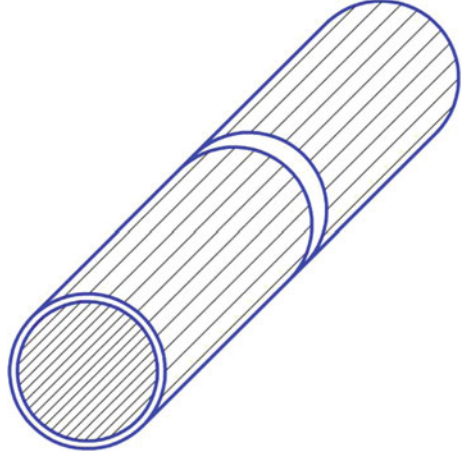
2.2 The Fluctuations of Radially Polarised Cylinder

Let's consider the fluctuations of the cylinder raised by external harmonious loadings. Electrical field or mechanical efforts, for example pressure from an environment on cylindrical surfaces can be such (Fig. 2.3).

According to character loading and consequently, kind of regional conditions conveniently resolving system of the equations is present in the mixed kind. Let's choose as independent functions variables u_z , σ_{rr} , σ_{rz} , u_r , φ , D_r . After simple transformations we will lead resolving system to the form [9]:

$$\frac{\partial u_z}{\partial r} = -\frac{\partial u_r}{\partial z} + \frac{1}{c_{55}^E} \left(\sigma_{rz} - e_{15} \frac{\partial \varphi}{\partial z} \right), \\ \frac{\partial \sigma_{rr}}{\partial r} = \frac{1}{r} \left(c_{12}^E + \frac{\Delta_2}{\Delta_1} \right) \frac{\partial u_z}{\partial z} - \frac{1}{r} \left(1 - \frac{\Delta_4}{\Delta_1} \right) \sigma_{rr} \\ - \frac{\partial \sigma_{rz}}{\partial z} - \left[\rho\omega^2 - \frac{1}{r^2} \left(c_{11}^E + \frac{\Delta_2}{\Delta_1} \right) \right] u_r + \frac{1}{r} \frac{\Delta_3}{\Delta_1} D_r, \\ \frac{\partial \sigma_{rz}}{\partial r} = -\rho\omega^2 u^2 - \left(c_{11}^E + \frac{\Delta_2}{\Delta_1} \right) \frac{\partial^2 u_r}{\partial z^2} - \frac{\Delta_4}{\Delta_1} \frac{\partial \sigma_{rr}}{\partial z} - \frac{1}{r} \sigma_{rz} \quad (2.15)$$

Fig. 2.3 Radially polarised cylinder



$$\begin{aligned}
 & -\frac{1}{r} \left(c_{12}^E + \frac{\Delta_2}{\Delta_1} \right) \frac{\partial u_r}{\partial z} - \frac{\Delta_3}{\Delta_1} \frac{\partial D_r}{\partial z}, \\
 \frac{\partial \varphi}{\partial r} &= \frac{1}{\Delta_1} \left(-\Delta_3 \frac{\partial u_z}{\partial z} + \varepsilon_{33}^S \sigma_{rr} - \frac{1}{r} \Delta_4 u_r + e_{33} D_r \right), \\
 \frac{\partial \varphi}{\partial r} &= \frac{1}{\Delta_1} \left(-\Delta_3 \frac{\partial u_z}{\partial z} + e_{33} \sigma_{rr} - \frac{1}{r} \Delta_3 u_r - c_{33}^E D_r \right), \\
 \frac{\partial D_r}{\partial r} &= \frac{1}{c_{55}^E} \left(-e_{31} \frac{\partial \sigma_{rz}}{\partial z} + \Delta_5 \frac{\partial^2 \varphi}{\partial z^2} \right) - \frac{1}{r} D_r.
 \end{aligned}$$

Here designations are used

$$\begin{aligned}
 \Delta_1 &= e_{33}^2 + c_{33}^E \varepsilon_{33}^S; \quad \Delta_2 = c_{33}^E e_{31}^2 + 2c_{13}^E e_{31} e_{33} - c_{13}^2 \varepsilon_{33}^S; \quad \Delta_3 = c_{13}^E e_{33} - c_{33}^E e_{31}; \\
 \Delta_4 &= c_{13}^E \varepsilon_{33}^S + e_{31} e_{33}; \quad \Delta_5 = e_{15}^2 + c_{55}^E \varepsilon_{11}^S;
 \end{aligned}$$

Let's enter systems of basic functions $1, \cos \alpha, \cos 2 \alpha, \dots, \cos n \alpha, \dots$ and $\sin \alpha, \sin 2 \alpha, \dots, \sin n \alpha, \dots$ If to accept

$$\{u_z(r, z); \sigma_{rz}(r, z)\} = \sum_{n=0}^{\infty} \left\{ u_z^{(n)}(r), \sigma_{rz}^{(n)}(r) \right\} \cos x_n z,$$

$$\begin{aligned}
 & \{\sigma_{rr}(z, z); u_r(r, z); \varphi(r, z); D_r(r, z)\} \\
 &= \sum_{n=0}^{\infty} \left\{ \sigma_{rr}^{(n)}(r); u_r^{(n)}(r); D_r^{(n)}(r) \right\} \sin x_n z,
 \end{aligned}$$

$$x_n = n\pi/l,$$

boundary conditions

$$\begin{aligned}\sigma_{zz}(r) \big|_{z=0,l} &= u_r(r) \big|_{z=0,l} = 0, \\ \varphi(r) \big|_{z=0,l} &= 0, \\ D_z(r) \big|_{z=0,l} &= 0.\end{aligned}$$

Will be satisfied precisely, and in system (2.15) Division of variables is possible. To formulate regional conditions on cylindrical surfaces concerning functions $u_z^{(n)}$, $\sigma_{rr}^{(n)}(r)$, $\sigma_{rz}^{(n)}(r)$, $\phi^{(n)}(r)$, $D_r^{(n)}(r)$, subject to search, external power factors also are necessary for spreading out on basic functions. So, the electric potential set on lateral surfaces $\phi(z)|_{r=r_0 \pm h} = \pm \frac{1}{2}V_0$ will become

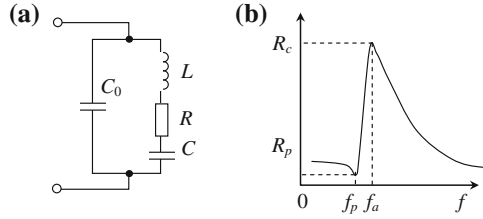
$$\varphi(z) \big|_{r_0 \pm h} = \pm 2V_0 \sum_n \frac{1}{n\pi} \sin \frac{n\pi}{l} z, n = 1, 3, 5, \dots \quad (2.16)$$

Thus, the decision problems about the compelled fluctuations of radially polarised cylinder at electric loading and homogeneous conditions at end faces is reduced to the decision of infinite sequence of systems of the ordinary differential equations

$$\begin{aligned}\frac{du_z^{(n)}}{dr} &= -x_n u_r^{(n)} + \frac{1}{c_{55}^E} \left(\sigma_{rz}^{(n)} - x_n e_{15} \varphi^{(n)} \right), \\ \frac{d\sigma_{rr}^{(n)}}{dr} &= -\frac{1}{r} x_n \left(c_{12}^E + \frac{\Delta_2}{\Delta_1} \right) u_z^{(n)} - \frac{1}{r} \left(1 - \frac{\Delta_4}{\Delta_1} \right) \sigma_{rr}^{(n)} \\ &\quad + x_n \sigma_{rz}^{(n)} - \left[\rho \omega^2 - \frac{1}{r^2} \left(c_{11}^E + \frac{\Delta_2}{\Delta_1} \right) \right] u_r^{(n)} + \frac{1}{r} \frac{\Delta_3}{\Delta_1} D_r^{(n)}, \\ \frac{d\sigma_{rz}^{(n)}}{dr} &= \left[-\rho \omega^2 + x_n^2 \left(c_{11}^E + \frac{\Delta_2}{\Delta_1} \right) \right] u_z^{(n)} - x_n \frac{\Delta_4}{\Delta_1} \sigma_{rr}^{(n)} \\ &\quad - \frac{1}{r} \sigma_{rz}^{(n)} - \frac{x_r}{r} \left(c_{12}^E + \frac{\Delta_2}{\Delta_1} \right) u_r^{(n)} - x_n \frac{\Delta_3}{\Delta_1} D_r^{(n)}, \\ \frac{du_r^{(n)}}{dr} &= \frac{1}{\Delta_1} \left(x_n \Delta_4 u_z^{(n)} + \varepsilon_{33}^S \sigma_{rr}^{(n)} - \frac{\Delta_4}{r} u_r^{(n)} + e_{33} D_r^{(n)} \right), \\ \frac{d\varphi^{(n)}}{dr} &= \frac{1}{\Delta_1} \left(x_n \Delta_3 u_z^{(n)} + e_{33} \sigma_{rr}^{(n)} - \frac{\Delta_3}{r} u_r^{(n)} - c_{33}^E D_r^{(n)} \right), \\ \frac{dD_r^{(n)}}{dr} &= \frac{x_r}{c_{55}^E} \left(e_{15} \sigma_{rz}^{(n)} - x_n^2 \Delta_5 \varphi^{(n)} \right) - \frac{1}{r} D_r^{(n)}\end{aligned} \quad (2.17)$$

with boundary conditions

Fig. 2.4 **a** Equivalent electric scheme of the resonator. **b** Typical frequency characteristic



$$\varphi^{(n)}|_{r_0 \pm 0h} = \pm \frac{2V_0}{n\pi}, \quad \left[\sigma_{rr}^{(n)}(r) = \sigma_{rz}^{(n)}(r) \right] |_{r_0 \pm h} = 0.$$

In decomposition (2.16) composed with the even n are equal to zero, therefore even harmonics are not raised and systems (2.17) is necessary to solve at $n = 1, 3, 5, \dots$

Conductivity of piezocylinder will be defined as

$$Y = i\omega \frac{2\pi(r_0 + h)}{V_0} \int_0^i D_r^{(n)}(r)|_{r=r_0+h} dz.$$

Using expression for a radial component of a vector of an induction through basic functions (2.15), we receive a parity for conductivity definition

$$Y = 4i\omega \frac{(r_0 + h)}{V_0} \sum_n \frac{1}{n} D_r^{(n)}, \quad n = 1, 3, 5, \dots \quad (2.18)$$

Series (2.18) is not harmonious, and sign-alternating, therefore it is convergent. Though the potential difference also enters into this parity V_0 , conductivity will not depend on it, as the private decision is proportional V_0 , that follows from a boundary condition (2.16). Conductivity has purely jet character and has a pole and a zero on frequencies of a resonance and an antiresonance without taking into account dissipation.

2.3 Piezoelectric Resonators

Piezoelements with two electrodes are named ‘resonators’.

When the frequency voltage (or pressure) is coincided to the piezoelement fluctuation own (natural) frequency, there is a electromechanical resonance phenomenon.

For systems with the distributed parametres, what piezoelements are, resonance is characterised by that in a direction of distribution of waves the integer number of half waves must indine.

For the elementary equivalent electric circuit of the resonator—series-parallel contour (Fig. 2.4)—it means equality of inductive and capacitor resistance.

Various fluctuation types can be raised in a piezoelement: longitudinal, fluctuations on thickness, radial, torsion, curving, and shift. Physically the resonance is characterized by a directional distribution of waves. At the resonator simultaneously, there are some kinds of interconnected fluctuations that create hindrances to the fluctuations in the basic direction; therefore it is necessary to create conditions so one kind of fluctuation prevails.

Most often met ‘resonance frequencies’ are fluctuations under formulas: longitudinal fluctuations:

$$f_p = \frac{n}{2l} \sqrt{\frac{Y}{\rho}}; \quad (2.19)$$

shift fluctuations on a thickness:

$$f_p = \frac{n}{2a} \sqrt{\frac{Y}{\rho} \frac{1 - \delta}{(1 + \delta)(1 - 2\delta)}}; \quad (2.20)$$

radial fluctuations:

$$f_p = \frac{z_n}{2\pi r} \sqrt{\frac{Y}{\rho(1 - \delta)^2}}, \quad (2.21)$$

where

Y—Young’s module

ρ —density of piezoelement material

δ —Pouasson’s coefficient

n—harmonic number

l, a, r—length, thickness, piezoelement radius

zn—parameter defined through Bessel functions

For longitudinal fluctuation resonant frequency definition use approached formula::

$$f_p \approx \frac{c}{2l}, \quad (2.22)$$

where c—speed of a sound in the piezoelement material

l—resonant size;

$$f_p \approx \frac{200}{l}, \text{ kHz}$$

or

Resonance frequency of a disk’s radial fluctuations [10]

$$f_p \approx \frac{1,35c}{4r}, \quad (2.23)$$

For piezoelectric resonator with continuous resonant electrodes and anti-resonant frequencies, it is possible to express parameters of the equivalent scheme (Fig. 2.4).

$$\left. \begin{aligned} f_p &= \frac{1}{2\pi\sqrt{LC}}; \\ f_a &= \frac{1}{2\pi\sqrt{\frac{LCc_0}{c+c_0}}} \end{aligned} \right\}; \quad (2.24)$$

A good quality piezorezonator is defined by expression:

$$Q = \frac{2\pi f_p L}{R}, \quad (2.25)$$

Piezorezonator's equivalent schemes, considering electrophysical and mechanical parameters, and also methods of calculation of resonators, are resulted in [5].

2.4 Piezoceramic Transformers

The piezoelectric transformer, conditionally name a piezoelectric element with three and more electrodes connected to one or several sources of an electric signal and loadings [2, 5]. In the elementary case the piezoelectric transformer represents a piezoelement with three electrodes, forming two system of electrodes. The part of the piezoelectric transformer connected to a source of an electric signal, name the activator, and a part connected to loading,—the generator.

In the activator the variable electric signal at the expense of the return piezoeffect will be transformed to energy of acoustic waves which, arising on borders of electrodes, extend on all volume of the transformer. On the frequency equal to one of resonant mechanical frequencies of the transformer, the standing wave with the maximum amplitude of fluctuations is formed. In the generator of the piezoelectric transformer mechanical pressure at the expense of direct piezoeffect will be transformed to an electric signal. On resonant frequencies the transformation factor has the maximum value [11–14].

On a way of transformation of energy in the activator and the generator piezoelectric transformers can be classified as transverse-transverse, longitudinal-longitudinal, transverse-longitudinal, longitudinal-transverse.

As fluctuations piezoelectric transformers subdivide into transformers with excitation of fluctuations longitudinal, radial, shift and a bend.

The basic designs of piezoelectric transformers are shown in Fig. 2.5 [2]. The transformer with longitudinal polarization of the activator and the generator (Fig. 2.5a) is called as the transformer of ring type, and with transverse-longitudinal and transverse polarization (Fig. 2.5b, c)—the transformer of transverse type. The disk transformer (Fig. 2.5d), also is the transformer of transverse type, but has some features of work, therefore disk transformers allocate in separate group.

Longitudinal-longitudinal and transverse-transverse transformers are symmetric. Their factor of transformation does not depend on the geometrical sizes and reaches several tens and even hundreds units.

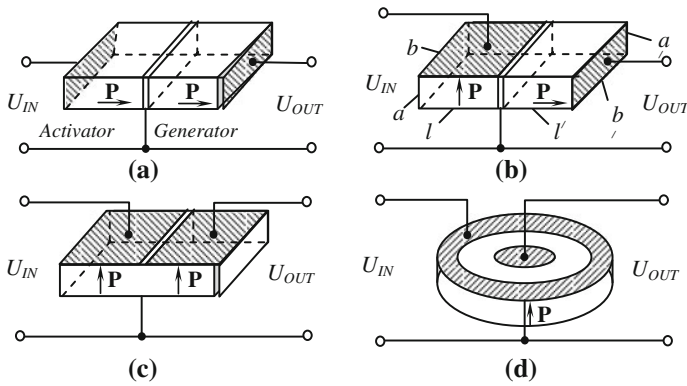


Fig. 2.5 Designs of piezoelectric transformers

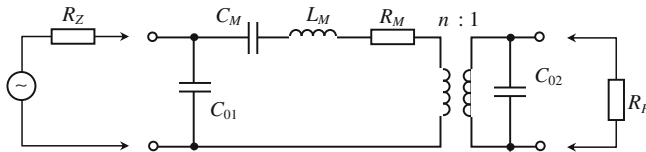


Fig. 2.6 Equivalent scheme *disk* piezoelectric transformer

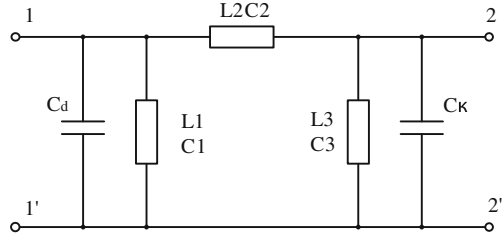
The transverse-longitudinal transformer is asymmetrical, and its coefficient of transformation depends on a parity of the geometrical sizes. This design of the transformer represents the greatest practical interest. The coefficient of transformation of the transverse-longitudinal transformer can reach several thousand [2].

Coefficient of transformation of the disk transformer above, than at the transverse. At excitation on a radial fashion fluctuation the energy stream through a cylindrical surface remains invariable for any radius, and, hence, in the disk centre there is a concentration of energy. If generating section of the transformer to arrange in the disk centre, there is an additional increase of coefficient of transformation at the expense of concentration of energy [2].

Reduction of a thickness of the transformer leads to increase in factor of transformation. Input and output impedances are defined mainly by capacities of systems of electrodes.

The analysis of work of piezoelectric transformers, as well as piezoelectric resonators, can be spent by means of equivalent schemes. The equivalent scheme of the piezoelectric transformer turns out from equivalent schemes of two piezoelectric resonators, one of which is the activator, another—the generator and for disk piezotransformer looks like, shown in Fig. 2.6 [2].

Expression for transformation coefficient on the voltage, the transformer expressed through parametres:
for transverse-longitudinal

Fig. 2.7 Equivalent scheme piezoelectric transformer

$$K_{u0} = \frac{4Q_M}{\pi^2} \frac{Y_{33}^E g_{33} d_{31}}{1 - k_{33}^2} \frac{l}{a}; \quad (2.26)$$

for longitudinal-longitudinal

$$K_{u0} = \frac{4Q_M k_{33}^2}{\pi^2} = \frac{4Q_M g_{33} d_{33} Y_3}{\pi^2}; \quad (2.27)$$

for transverse-transverse

$$K_{u0} = \frac{4Q_M}{\pi^2} \frac{k_{31}^2}{1 - k_{31}^2} = \frac{4Q_M}{\pi^2} \frac{g_{31} d_{31} Y_3}{\pi^2}. \quad (2.28)$$

As it was specified above, the maximum coefficient of transformation has the transverse-longitudinal piezoelectric transformer. If to assume, that the sizes $l \rightarrow \infty$, $a \rightarrow \infty$, $K_{u0} \rightarrow \infty$. However the sizes are defined by admissible overall dimensions, complexity of technology of their performance and losses. If to consider, that the length l should not exceed 100 mm, and the thickness should not be less than 0,25 mm, geometry coefficient $N_G = \frac{l}{a} = 200$. To this limit for the transformer from piezoceramic L[TC-23 (PZT) there corresponds the coefficient of transformation equal 7,000 [15]. In practice this size is much less [2].

The model of a disk piezoelement with electrodes in the form of a ring and a disk is investigated in [6, 15–17], on the basis of what the equivalent scheme of piezoelectric transformer is constructed (Fig. 2.7).

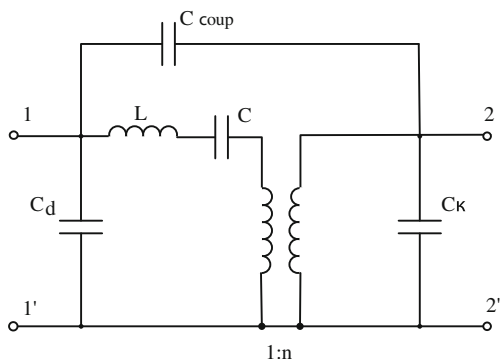
Reception of such scheme is represented quite obvious of consideration of physics of processes, occurring in piezoelectric transformer. Really, at excitation of fluctuations in piezoelectric transformer the input section can be considered as a part resonance a fluctuating disk with parameters C_d , L_1 , C_1 , output section—also as a part resonance a fluctuating disk with parameters C_k , L_3 , C_3 .

Communication between sections is carried out by one more part of a disk with parameters L_2 , C_2 .

The received scheme is fair as for the frequencies close to resonant, and for pre-resonance area.

One more variant of the electric equivalent scheme of disk piezoelectric transformer is shown in Fig. 2.8.

Fig. 2.8 Equivalent scheme piezoelectric transformer with the coupling capacity C_{coup}



In this case the known scheme of voltage transfer of on an exit of piezoelectric transformer is used by means of the ideal transformer, however the coupling capacity is in addition entered C_{coup} between an input and output.

This capacity is real capacity, representing consecutive connection C_d and C_k . Such communication (elastic) is carried out on a material piezoelectric transformer (L_2 , C_2) as follows from Fig. 2.8.

The understanding of physics of processes allows to operate characteristics piezoelectric transformer with means of additional condensers and inductance, included between an input and output [6].

2.5 Bimorph and Trimorph Piezoelements

Bimorph piezoelements (BPE) consist of two parts—two interconnected piezoelements or a piezoelement and a metal plate, soldered or glued to each other by epoxy compound.

Bimorph elements, consisting of two piezoelements, are named symmetric. Bimorph elements, consisting of a piezoelement and a metal plate, are named asymmetric [5, 6].

Elements, consisting of two piezoelements and a metal plate, are called trimorph [9].

It is necessary to note, that joining of two piezoelements or a piezoelement and a metal plate in one design leads to essential change of sensor characteristics. For example, minimal resonant frequency of PZTTC-19 piezoceramic piezoelement (30 mm in diameter and 0.3 mm thick) is ~ 70 kHz (radial vibrations). Its sensitivity to the sound field at the frequency of 100 Hz is ~ 1 mV/Pa, for example. Joining of these two piezoelements in a symmetric bimorph leads to occurrence of resonant frequencies ~ 2.5 and 3.5 kHz (flexural vibrations). Its sensitivity increases to 20–30 mV/Pa under the same conditions. Thus, it increases not in two, but in 20–30 times.

Fig. 2.9 Fluctuations of bimorph elements: **a** rigid fastening on the forming, **b** free arrangement

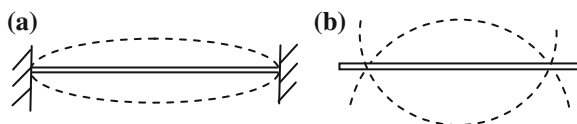
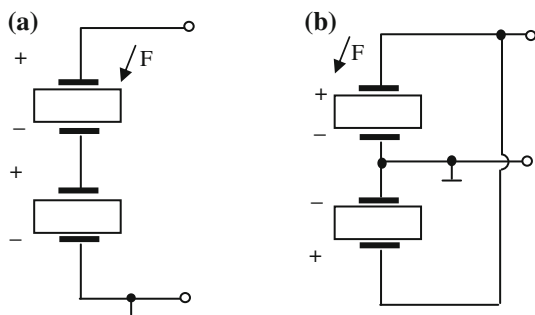


Fig. 2.10 Communication diagrams of piezoelements in symmetric bimorph transducers: **a** series, **b** parallel



At the same time, joining of piezoceramic and metal (amorphous) plates in an asymmetric bimorph increases sensitivity in 10–20 times. [5, 6].

Piezoelements in symmetric BPE and piezoelement and a metal plate in the asymmetric BPE Incorporate among themselves usually to the help epoxy compound or a fusible alloy or solder. The technology glueing together is described in [5, 6].

Occurrence of curving fluctuations in BPE is caused by anisotropy of mechanical properties of a piezoelement, a metal plate and glutinous connection.

Two basic resonances fluctuations of curving for BPE in the form of disk exists, at least (Fig. 2.9).

BPE is fixed on the forming in first case (Fig. 2.9a). It is the lowest resonant frequency for BPE the given size. BPE is located freely in the second case (Fig. 2.9b). In this case the basic resonant frequency approximately in 1.4 times above, than for the first case.

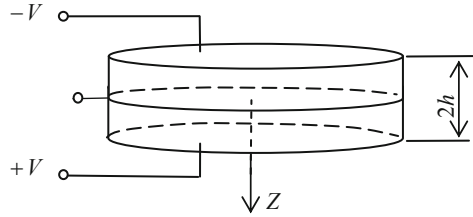
Symmetric Bimorph Piezoelements

Two schemes of piezoelements communication are known: series and parallel (Fig. 2.10a, b, accordingly) [5].

Traditionally these transducers are made of identical-dimensioned plates. Their thickness is also identical. This is very important (Fig. 2.10). This assures maximal sensitivity. However, it is necessary to remark if dimensions of piezoelements are identical sensitivity of the voltage sensor in parallel circuit and its own resistance is four times smaller than the corresponding characteristics of the sensor in series communication circuit.

Static electromechanical coupling coefficients (EMCC) are quantitative effectiveness measure of the homogeneous deformations coordination discussed. Homogeneous planar deformation occurs in electric field of the single-layered plate, polarized along its thickness. Static EMCC C_P for this deformation type is a table value. It has different values for various compositions of piezoceramics [18].

Fig. 2.11 Symmetric bimorph transducer with identically thick piezoelements



Dynamic (effective) EMCC is introduced to consider the efficiency of electro-mechanical energy transformation under oscillatory deformations. It is calculated by simple formulas [17] for homogeneous deformation. For inhomogeneous deformation the effective EMCC is found by solution of electroelasticity boundary-value problem and with the help of “energetic theory” (Fig. 2.11).

The bimorph plate deformation is generally flexural. However, it is synthesized from two opposed planar (homogeneous) rigidly connected single-layer (monomorph) plates. It is offered in [18] to evaluate the effective EMCC value of round bimorph flexural vibrations K_D . The bimorph consists of identically thick plates in comparison with the static EMCC value of each plate planar deformation C_P . It should also be compared with the effective EMCC theoretical value of the bimorph element. This value is found in accordance with “energetic theory” [18].

The analysis of vibrations and effective EMCC expression for a symmetric bimorph piezoelement are received in [18].

The flexural vibrations problem of a round bimorph plate with radius $r = a$ under the action of mechanical force, changing according to harmonious law, is considered. Voltages U_1 and U_2 of the same frequency occurred on the piezoelements electrodes.

The flexural vibrations equation of the piezoceramic plate, relative to inflection function of middle surface, looks like this:

$$\nabla^2 \nabla^2 W + \frac{\rho h}{\tilde{D}} \frac{\partial^2 W}{\partial t^2} = \frac{q}{\tilde{D}} \quad (2.29)$$

where $\tilde{D} = \frac{h^3}{12S_{11}^E(1-\tilde{\nu}^2)} \left(1 + \frac{1+\nu}{8} \frac{K_p^2}{1-K_p^2} \right)$ —rigidity under cylindrical bending of piezoceramic plate;

$$\tilde{\nu} = \frac{\nu + \frac{1+\nu}{8} \frac{K_p^2}{1-K_p^2}}{1 + \frac{1+\nu}{8} \frac{K_p^2}{1-K_p^2}} \text{—reduced Poisson coefficient;}$$

$$K_p^2 = \frac{2}{1-\nu} \frac{d_{31}^2}{S_{11}^E \varepsilon_{33}^T} \text{—static planar EMCC;}$$

q —lateral distributed load;

$W(x, y, z)$ —inflection function;

h —thickness of the plates;

ρ —density of plates material.

The solution of this equation for a bimorph round plate looks like this [26]:
with open electrodes:

$$A_{OPEN} = \frac{1}{2} \pi \beta^2 \tilde{D} \left[(\beta a)^2 \left\{ (A J_1(\beta a) - B I_1(\beta a))^2 + (A J_1(\beta a) - B I_1(\beta a))^2 \right\} \right], \quad (2.30)$$

with short-circuited electrodes:

$$A_{SH-CIR} = \frac{1}{2} \pi \beta^2 \tilde{D} \times \left\{ (\beta a)^2 \left[(A J_0(\beta a) - B I_0(\beta a))^2 + (A J_1(\beta a) - B I_1(\beta a))^2 \right] \right\} \quad (2.31)$$

Then the effective EMCC expression C_D will look like this:

where U_{OPEN} —the energy reserved in a plate at bends in case of opened electrodes;

$\beta = \sqrt[4]{\frac{\rho \omega^2 h}{\tilde{D}}}$ —coefficient, introduced to simplify the calculations;

$a = r$ —radius of the bimorph plates;

$A = -\frac{h^2}{4} \frac{d_{31} a}{S_{11}^E (1-\nu)} \frac{1}{\beta \tilde{D}} \left(\frac{V_0}{h} \right) \frac{I_1(\beta a)}{\Delta}$ and $B = -A \frac{J_1(\beta a)}{I_1(\beta a)}$ —constants, with $J_1(z) = -\frac{dJ_0}{dz}$; $J_1(z) = -\frac{dJ_0}{dz}$; $I_1(z) = \frac{dI_0}{dz}$; $J_0(\beta r)$ constants, with $I_0(\beta r)$ —Bessel functions.

This is the value for piezoceramics II TC-19 composition: $\nu = 0,33$; $K_p^2 = (0,58)^2 = 0,34$.

To compare the effective EMCC value for flexural (inhomogeneous) deformations with the value of static planar EMCC C_p^2 one should proceed from vibrations to static deformations, i.e. to consider C_D^2 when vibrations frequency tends to zero. Then

$$\lim_{\beta a \rightarrow 0} K_D^2 = 0,252 \approx \frac{3}{4} K_p^2. \quad (2.32)$$

This result conforms to EMCC theoretical value for inhomogeneous deformation of the bimorph element, received in [17]:

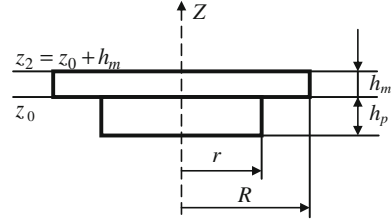
$$K^2 = \frac{A_{OPEN} - A_{SH-CIR}}{A_{OPEN}} = \frac{3}{4} K_p^2 = 0,255. \quad (2.33)$$

The constants values for II TC-19 piezoceramics are:

$S_{11}^E = 12,3 \cdot 10^{-12} \frac{\text{m}^2}{\text{N}}$, $\varepsilon_{33}^T = 1300 \cdot 8,85 \cdot 10^{-12} \frac{\text{F}}{\text{m}^2}$, $d_{31} = -5,2 \frac{\text{Kl}}{\text{m}^2}$, accordingly.

Then after numerical search [19] of the first three ω_a and ω_r values, corresponding to roots of antiresonance and resonance equations (first three main vibration modes), when changes βa with 0,1 step in 0...10 interval, the corresponding values are found by Mason formula K_D^2 : $K_{D1}^2 = 0,23$, $K_{D2}^2 = 0,12$, $K_{D3}^2 = 0,06$.

Fig. 2.12 Asymmetric bimorph piezosensor



Asymmetric Bimorph Piezoelements

Asymmetric bimorph piezoelements are more mechanically durable. They consist of a metal plate and a flat piezoelement, polarized along its thickness.

Idle sensitivity is an important characteristic of sensors, working in reception/radiation mode. A mathematical model of this transducer type should be constructed to estimate the sensitivity and other dynamic characteristics of bimorph pressure sensors.

A sensor, consisting of glued to each other round metal and thickness-polarized piezoceramic plates with radius R and r , accordingly, is considered (Fig. 2.12). Their thicknesses are denoted h_m and h_p , accordingly. A cylindrical system of coordinates r, θ and z is used. Its axis OZ coincides with two-layer disk axis. $z = 0$ relative to the reduction surface. Its position is determined lower. Coordinates of interface (separation surface), piezoelement lower surface and metal plate top surface are denoted z_0 ; $z_1 = z_0 - h_p$; $z_2 = z_0 + h_m$, accordingly.

Potential difference $U_x e^{j\omega t}$ is generated on open piezoelement electrodes under the harmonious load (sound pressure) influence $p e^{j\omega t}$ ($p = \text{const}$), normally applied to the flat surface of the metal plate. The thickness of electrodes, covering flat surfaces of the piezoceramic disk and glued joint between the plates, are neglected. The mathematical model of this transducer is constructed by A.N. Shulga and other scientists [8]. Kinematic hypotheses are used to construct a mathematical axially symmetric model of asymmetric bimorph vibrations.

$$u_r(r, z) = u(r) + z\psi(r); \quad u_z(r, z) = \varpi(r) \quad (2.34)$$

Here u and ϖ —tangential and normal displacement of initial surface, ψ —rotation angle of the normal; time factor (multiplier) $e^{j\omega t}$ is always omitted. These are expressions for deformation tensor components:

$$\begin{aligned} e_{rr} &= \varepsilon_r + z\chi_r; & e_{\theta\theta} &= \varepsilon_\theta + z\chi_\theta; & e_{rz} &= \varepsilon_{rz}; \\ e_{zz} &= 0, \end{aligned} \quad (2.35)$$

In which:

$$\varepsilon_r = \frac{du}{dr}; \quad \varepsilon_\theta = \frac{u}{r}; \quad \chi_r = \frac{d\psi}{dr}; \quad \chi_\theta = \frac{\psi}{r}; \quad (2.36)$$

$$2\varepsilon_{rz} = \frac{d\varpi}{dr} + \psi.$$

Hypotheses (2.34), mastering the law of movement variation along the two-layer plate thickness, are expanded by the hypothesis of electric potential change along the piezoceramic disk thickness. Quadratic approximation is used as this hypothesis. It was used in the theory of homogeneous covers piezoceramic [8] with electrodes on lateral surfaces

$$\varphi(r, \bar{z}) = \frac{U_x}{h_p} \bar{z} + \frac{3}{2} \left(1 - \frac{4\bar{z}^2}{h_p^2} \right) \Phi(r) \quad (2.37)$$

Here U_x —amplitude of an unknown potential difference on the open electrodes. In expression (2.37) $\bar{z}$ —the co-ordinate, calculated from the middle surface of the piezoceramic disk. The distance between this surface and the initial bimorph surface is denoted $b = z_0 - h_p/2$. Formula (2.37) is rewritten in the system of coordinates. The latter is normally connected with the initial surface:

$$\varphi(r, \bar{z}) = \frac{U_x}{h_p} (z - b) + \frac{3}{2} \left(1 - \frac{4(z - b)^2}{h_p^2} \right) \Phi(r). \quad (2.38)$$

Then these are the expressions for components of the electric field intensity vector in the piezoceramic disk:

$$\begin{aligned} E_r(r, z) &= f(z) E_r^{(0)}(r); \quad E_r(r, z) = E_r^{(0)}(r) + (z - b) E_z^{(1)}(r); \\ f(z) &= \frac{3}{2} \left[1 - \frac{4(z - b)^2}{h_p^2} \right], \end{aligned} \quad (2.39)$$

In which:

$$E_r^{(0)} = -\frac{d\Phi}{dr}; \quad E_z^{(0)} = -\frac{V_x}{h_p}; \quad E_z^{(1)} = -\frac{12}{h_p^2} \Phi. \quad (2.40)$$

Generalized Hamilton principle is applied for derivation of vibrations, electrostatics and natural boundary conditions equations [8]. Considering axially symmetric bimorph stationary vibrations, we will start with the functional stationarity condition

$$\begin{aligned} \delta u_p u_z \varphi \left\{ \int_0^R \int_{z_1}^{z_2} (\sigma_{rr} e_{rr} + \sigma_{\theta\theta} e_{\theta\theta} + \sigma_{zz} e_{zz} + 2\sigma_{rz} e_{rz}) r dr dz \right. \\ \left. - \int_0^R \int_{z_1}^{z_0} (D_r E_r + D_z E_z) r dr dz - \frac{\omega^2}{2} \int_0^R \int_{z_1}^{z_2} \rho(z) \{u_r^2 + u_z^2\} r dr dz - \int_0^R p u_z r dr \right\} = 0 \end{aligned} \quad (2.41)$$

Here $\rho(z)$ —piecewise constant function of density. The densities of ceramic and metal plates are denoted ρ_p and ρ_m , accordingly.

After consideration of intense condition integrated characteristics of the two-layer plate:

$$N_r(\theta) = \int_{Z_1}^{Z_2} \sigma_{rr}(\theta\theta) dz; \quad M_r(\theta) = \int_{Z_1}^{Z_2} \sigma_{rr}(\theta\theta) z dz; \quad Q_r = \int_{Z_1}^{Z_2} \sigma_{rz} dz, \quad (2.42)$$

and electric condition of the piezoceramic plate:

$$\hat{D}_r = \int_{Z_1}^{Z_0} D_r f(z) dz; \quad \hat{D}_z^{(0)} = \int_{Z_1}^{Z_0} D_z dz, \quad \hat{D}_z^{(1)} = \frac{12}{h_p^2} \int_{Z_1}^{Z_0} D_r (z - b) dz, \quad (2.43)$$

considering dependences (2.35) and (2.36), variation equation (2.41) can be re-arranged like this:

$$\begin{aligned} & \int_0^R \{ N_r \delta \varepsilon_r + N_\theta \delta \varepsilon_\theta + M_r \delta \chi_r + M_\theta \delta \chi_\theta + 2 Q_r \delta \varepsilon_{rz} \\ & - \hat{D}_r \delta E_r^{(0)} - \hat{D}_z^{(0)} \delta E_z^{(0)} - \frac{h_p^2}{12} \hat{D}_z^{(1)} \delta E_z^{(1)} \\ & - \omega^2 [\rho_1 (u \delta u + \varpi \delta \varpi) + \bar{\rho} (u \delta \psi + \psi \delta u) + \rho_2 \psi \delta \psi] - p \delta \varpi \} r dr = 0, \end{aligned} \quad (2.44)$$

where

$$\begin{aligned} \rho_1 &= \rho_p h_p + \rho_m h_m; \quad \rho_2 = \rho_p \frac{z_0^3 - z_1^3}{2} + \rho_m \frac{z_2^3 - z_0^3}{2}; \\ \bar{\rho} &= \rho_p \frac{z_0^2 - z_1^2}{2} + \rho_m \frac{z_2^2 - z_0^2}{2}. \end{aligned}$$

Vibration equations follow from variation equation (2.44) where independent variations δu , $\delta \varpi$, $\delta \psi$, $\delta \Phi$ and δU_x are considered:

$$\begin{aligned} & \frac{dN_r}{dr} + \frac{1}{r} (N_r - N_\theta) + \rho_1 \omega^2 u + \bar{\rho} \omega^2 \psi = 0; \\ & \frac{dQ_r}{dr} + \frac{1}{r} Q_r + \rho_1 \omega^2 \varpi + p = 0; \\ & \frac{dM_r}{dr} + \frac{1}{r} (M_r - M_\theta) - Q_r + \rho_2 \omega^2 \psi + \bar{\rho} \omega^2 u = 0, \end{aligned} \quad (2.45)$$

electrostatics equation:

$$\frac{d}{dr} \left(r \hat{D}_r \right) - r \hat{D}_z^{(1)} = 0, \quad (2.46)$$

integral relation:

$$\int_0^R \hat{D}_z^{(0)} r dr = 0. \quad (2.47)$$

The fact that displacement current equals zero is the physical sense of condition (2.47). It is accurate within multiplier $2\pi i\omega$ through the middle piezoelement surface. This integral relation is additional for unambiguous determination of potential difference on equipotential surfaces of the piezoceramic disk.

Natural boundary conditions follow from the equality:

$$\left[N_r \delta u + M_r \delta \psi + Q_r \delta \varpi + \hat{D} \delta \Phi \right]_{r=0}^{r=R} = 0,$$

which follows from (2.44).

Electroelasticity ratios of asymmetric bimorph are received by integration, using formulas (2.42) and (2.43) of three-dimensional state equations, simplified according to assumptions of the thin plates theory. Forces and the moments are written in the following way:

$$\begin{aligned} N_r &= C_1 \varepsilon_r + C_2 \varepsilon_\theta + B_1 \chi_r + B_2 \chi_\theta - N_{EL}; \\ N_\theta &= C_2 \varepsilon_r + C_1 \varepsilon_\theta + B_2 \chi_r + B_1 \chi_\theta - N_{EL}; \\ M_r &= B_1 \varepsilon_r + B_2 \varepsilon_\theta + D_1 \chi_r + D_2 \chi_\theta - M_{EL}; \\ M_\theta &= B_2 \varepsilon_r + B_1 \varepsilon_\theta + D_2 \chi_r + D_1 \chi_\theta - M_{EL}. \end{aligned} \quad (2.48)$$

Rigid characteristics and the electric components N_{EL} and M_{EL} from correlations (2.48) are determined by the equalities:

$$N_{EL} = \frac{d_{31}}{S_{11}^p (1 - \nu_p)} \int_{z_1}^{z_0} E_z dz; \quad M_{EL} = \frac{d_{31}}{S_{11}^p (1 - \nu_p)} \int_{z_1}^{z_0} E_z z dz; \quad (2.49)$$

$$\begin{aligned} C_1 &= h_p c_{11}^p + h_m c_{11}^m; \quad C_2 = h_p \nu_p c_{11}^p + h_m \nu_m c_{11}^m; \\ B_1 &= \frac{z_0^2 - z_1^2}{2} c_{11}^p + \frac{z_2^2 - z_0^2}{2} c_{11}^m; \quad B_2 = \frac{z_0^2 - z_1^2}{2} c_{11}^p \nu_p + \frac{z_2^2 - z_0^2}{2} \nu_m c_{11}^m; \\ D_1 &= \frac{z_0^3 - z_1^3}{3} c_{11}^p + \frac{z_2^3 - z_0^3}{3} c_{11}^m; \\ D_2 &= \frac{z_0^3 - z_1^3}{3} \nu_p c_{11}^p + \frac{z_2^3 - z_0^3}{3} \nu_m c_{11}^m. \end{aligned} \quad (2.50)$$

In equalities (2.49) d_{31} is a piezomodule; s_{11}^p and ν_p -compliance and Poisson coefficient of the piezoceramic disk. These are the notations, introduced to equalities (2.50):

$$c_{11}^p = \frac{1}{s_{11}^p (1 - \nu_p^2)} \quad \text{and} \quad c_{11}^m = \frac{1}{s_{11}^m (1 - \nu_m^2)}.$$

In the theory of homogeneous elastic (electroelastic) plates and covers the simplest connection of forces and moments with middle surface deformations is established by selection of the middle surface, used as the reduction surface. The forces depend only on tangential, and the moments—only on flexural deformations [8]. Correlations (2.48) give more complicated connection between the specified characteristics of the stressed and deformed states. This additional connection (coupling) from homogeneous plates view point is fulfilled by rigid characteristics B_1 and B_2 . The expressions analysis for these characteristics shows that if Poisson coefficients of both plates are considered equal conditions $B_1 = B_2 = 0$ are met by the corresponding choice of the initial surface. Then dependences (2.48) will be as simple as for homogeneous problem. This simplification is reached only if Poisson coefficients of layers material are equal [8].

The position of reduction surface is determined from condition $B_1 = B_2 = 0$ (with $\nu_p = \nu_m$) by the dependences:

$$z_0 = h_p \gamma_0; \quad z_1 = h_p (\gamma_0 - 1); \quad z_2 = h_p (\gamma_0 + \beta);$$

$$\gamma_0 = \frac{1}{2} \frac{\alpha - \beta^2}{\alpha + \beta}, \quad (2.51)$$

in which $\alpha = \frac{s_{11}^m}{s_{11}^p}$ and $\beta = \frac{h_m}{h_p}$.

Special case $\beta = 0$ ($h_p \neq 0$) corresponds to the single-layer piezoceramic plate with middle initial surface. In case $\alpha = 1$ ($\beta \neq 0$) the initial surface coincides with the middle surface of the two-layer package.

Thus, under condition $\nu_m = \nu_p = \nu$ material correlations for bimorph can be written like this:

$$\begin{aligned} N_r &= C_{11} (\varepsilon_r + \nu \varepsilon_\theta) - e_{31} h_p E_z^{(0)}; \quad N_\theta = C_{11} (\nu \varepsilon_r + \varepsilon_\theta) - e_{31} h_p E_z^{(0)} \\ M_r &= D_{11} (\chi_r + \nu \chi_\theta) - e_{31} h_p^2 \left[E_z^{(0)} \left(\gamma_0 - \frac{1}{2} \right) + \frac{h_p}{12} E_z^{(1)} \right]; \\ M_\theta &= D_{11} (\nu \chi_r + \chi_\theta) - e_{31} h_p^2 \left[E_z^{(0)} \left(\gamma_0 - \frac{1}{2} \right) + \frac{h_p}{12} E_z^{(1)} \right]; \end{aligned} \quad (2.52)$$

$$Q_r = C_{44} 2\varepsilon_{rz} - e_{15} h_p E_r^{(0)}; \quad \hat{D}_r = \varepsilon_{11} h_p E_r^{(0)} - e_{15} h_p 2\varepsilon_{rz};$$

$$\begin{aligned}\hat{D}_z^{(0)} &= \varepsilon_{33} h_p E_z^{(0)} + e_{31} h_p (\varepsilon_r + \varepsilon_\theta) + e_{31} h_p^2 (\gamma_0 - 1/2) (\chi_r + \chi_\theta); \\ \hat{D}_z^{(1)} &= \varepsilon_{33} h_p E_z^{(1)} + e_{31} h_p (\chi_r + \chi_\theta).\end{aligned}$$

These notations are offered:

$$\begin{aligned}C_{11} &= \frac{h_p}{s_{11}^p (1 - \nu^2)} \left(1 + \frac{\beta}{\alpha}\right); \quad C_{44} = \frac{h_p}{s_{44}^p} \left(1 + \frac{\beta}{\alpha}\right); \quad e_{31} = \frac{d_{31}}{s_{11}^p (1 - \nu)}; \\ e_{15} &= \frac{d_{15}}{s_{44}^p}; \quad D_{11} = \frac{h_p^3}{s_{11}^p (1 - \nu^2)} \left(\gamma_0^3 - \gamma_1^3 + \frac{\gamma_2^3 - \gamma_1^3}{\alpha}\right); \\ \varepsilon_{33} &= \varepsilon_{33}^T (1 - K_p^2); \quad \varepsilon_{11} = \varepsilon_{11}^T (1 - K_{15}^2),\end{aligned} \quad (2.53)$$

in which s_{11}^p, s_{44}^p —compliances if electric field is constant; d_{31}, d_{15} —piezomodules; $\varepsilon_{11}^T, \varepsilon_{33}^T$ —dielectric permeability under constant voltage; K_p, K_{15} —planar and shear factors of electromechanical coupling.

Vibration (2.45) and electrostatics (2.52) equations, dependences (2.36), (2.39), material correlations (2.52) and integral condition (2.47) are a closed equations system of axially symmetric bimorph transducer vibrations. The transducer type is metal—piezoceramics with decreased shear rigidity of layers.

The simplified variant of the represented above equations should be used to evaluate dynamic characteristics of a thin bimorph with a high shear rigidity of layers.

Kirchhoff—Love model is considered below. It is assumed that lateral shear deformation equals zero ($\varepsilon_{rz} = 0$) and shear rigidity is infinite ($C_{44} = \infty$). Overcut force, reaching its final value in the limit, can be determined from the third equation of the system. Inertial members are not considered. The rotation angle of the normal is dependent. It is connected with the inflection by equality $\psi = -\frac{d\varpi}{dr}$.

The equation is further simplified, assuming that:

$$\hat{D}_z^{(1)} = 0. \quad (2.54)$$

From the obvious equality:

$$D_z(r, z) = \frac{1}{h_p} \hat{D}_z^{(0)}(r) + \frac{z - b}{h_p} \hat{D}_z^{(1)}(r)$$

It is clear that acceptance of additional restriction (2.54) indicates the transition to a more rigid hypothesis [6]. It is about normal component constancy of electric induction vector along the piezoelement thickness. Using equality (2.54) in (2.52), linear correction of electric field $E_z^{(1)}$ intensity can be done by curvature change parameters of the initial surface:

$$E_z^{(1)} = -\frac{1}{2d_{31}} \frac{K_p^2}{1 - K_p^2} (\chi_r + \chi_\theta),$$

and to exclude it from equations in this way (2.52). Electrostatics equation (2.46) considering (2.54) becomes simpler $\hat{D}_r = \frac{\text{const}}{r}$ and, obviously, is satisfied if there are no charges on the cylindrical surface of the piezoceramic disk.

Simplifications, connected with transition to more rigid mechanical and electric hypotheses, lead to a simpler electromechanical model and a smaller number of the unknown. The rotation angle and function F are now functions of the initial surface inflection.

Material correlations (2.52), simplified according to the discussed above ideas and by use of these correlations (2.35) and (2.40), can be written in the form of equalities:

$$\begin{aligned}
 N_r &= C_{11} \left(\frac{du}{dr} + v \frac{u}{r} \right) + e_{31} U_x; \quad N_\theta = C_{11} \left(v \frac{du}{dr} + \frac{u}{r} \right) + e_{31} U_x; \\
 M_r &= -\bar{D}_{11} \left(\frac{d^2 \varpi}{dr^2} + \bar{v} \frac{1}{r} \frac{d\varpi}{dr} \right) + e_{31} h_p \left(\gamma_0 - \frac{1}{2} \right) U_x; \\
 M_\theta &= -\bar{D}_{11} \left(\bar{v} \frac{d^2 \varpi}{dr^2} + \frac{1}{r} \frac{d\varpi}{dr} \right) + e_{31} h_p \left(\gamma_0 - \frac{1}{2} \right) U_x; \\
 \hat{D}_z^{(0)} &= -\varepsilon_{33} U_x + e_{31} h_p \left[\left(\frac{du}{dr} + \frac{u}{r} \right) - h_p \left(\gamma_0 - \frac{1}{2} \right) \left(\frac{d^2 \varpi}{dr^2} + \frac{1}{r} \frac{d\varpi}{dr} \right) \right].
 \end{aligned} \tag{2.55}$$

$$\tag{2.56}$$

There are reduced flexural rigidity and Poisson coefficient in the ratios for the moments

$$\bar{D}_{11} = \frac{h_p^3}{S_{11}^p (1 - v^2)} d; \quad \bar{v} = \frac{8gv + K}{8g + K},$$

where

$$d = \frac{8g + K}{24}; \quad K = \frac{(1 + v)K_p^2}{1 - K_p^2}; \quad g = \gamma_0^3 - \gamma_1^3 + \frac{\gamma_2^3 - \gamma_1^3}{\alpha}.$$

The expression for unknown potential difference can be found from this integral condition (2.55). The result after integration is

$$U_x = \frac{1}{d_{31}} \frac{K_p^2}{1 - K_p^2} \frac{h_p}{R} \left[u(R) - h_p \left(\gamma_0 - \frac{1}{2} \right) \varpi'(R) \right]. \tag{2.57}$$

Thus, the output potential difference is expressed by tangential displacement values of the initial surface and the rotation angle on the plate edge ($r = R$). Evidently, if the edge is built in (rigidly fastened) there is no potential difference.

Substituting correlations (2.57) in system (2.45) the vibration equations in movements are received. Small inertial members of squared / cubed bimorph thickness are not considered.

$$\Delta u + \left(\lambda^2 - \frac{1}{x^2} \right) u = 0, \quad (2.58)$$

$$\Delta \Delta \varpi - \mu^4 \varpi = q. \quad (2.59)$$

The following notations are introduced:

operator $\Delta = \frac{d^2}{dx^2} + \frac{1}{x} \frac{d}{dx}$;

$x = \frac{r}{R}$ —dimensionless coordinate;

u and ϖ —dimensionless movements, divided by radius; and dimensionless values:

$$\begin{aligned} \lambda^2 &= \alpha_1 \Omega^2; \quad \mu^4 = \alpha_2 \Omega^2; \quad q = \frac{pR^3}{\bar{D}_{11}}; \quad \Omega^2 = \omega^2 R^2 \rho_p s_{11}^p (1 - \nu^2) \\ \alpha_1 &= \alpha \frac{1 + \rho\beta}{\alpha + \beta}; \quad \alpha_2 = \frac{1 + \rho\beta}{\varepsilon^2 d}; \quad \rho = \frac{\rho_m}{\rho_p}; \quad \varepsilon = \frac{h_p}{R}. \end{aligned}$$

Planar and flexural equations of bimorph vibrations (2.58) and (2.59) look similar to analogous equations for a homogeneous isotropic plate. These equations can be solved by functions J_n and modified Bessel functions I_n . Radial displacement, equal zero, and finiteness of the inflection in the plate center are considered:

$$u = A_1 J_1(\lambda x); \quad \varpi = A_2 J_0(\mu x) + A_3 I_0(\mu x) - \frac{q}{\mu^4}. \quad (2.60)$$

Using (2.60), equality (2.57) is written like this:

$$U = \frac{K^2}{1 + \nu} \varepsilon \left[A_1 J_1(\lambda) + \varepsilon \left(\gamma_0 - \frac{1}{2} \right) \{ A_2 J_0(\lambda) + A_3 I_0(\lambda) \} \right], \quad (2.61)$$

where $U = U_x \frac{d_{31}}{R}$ —dimensionless potential. To determine unknown U and integration constants A_1 , A_2 and A_3 equality (2.61) is complemented by boundary conditions.

An algebraic system to determine unknown constants is received from boundary conditions:

$$u(1) = 0; \quad \varpi(1) = 0; \quad M_x(1) = 0 \quad (2.62)$$

and equalities (2.61) if the edge is (hinged) merely supported. Obviously, $A_1 = 0$ and the following expression is received for output voltage if bimorph vibrates (flexural vibrations):

$$\begin{aligned}
U_{OUT} = & \frac{qK\varepsilon^2(\gamma_0 - \frac{1}{2})}{(1+\nu)\mu} \\
& \times \left[\frac{I_0(\mu)J_1(\mu) - J_0(\mu)I_1(\mu)}{-2\mu^2J_0(\mu)I_0(\mu) + \mu\left(1 - \bar{\nu} - \frac{K(\gamma_0 - \frac{1}{2})^2}{d}\right)[I_0(\mu)J_1(\mu) - J_0(\mu)I_1(\mu)]} \right]
\end{aligned} \quad (2.63)$$

If case the edge is for freely supported:

$$N_x(1) = 0; \quad \varpi(1) = 0; \quad M_x(1) = 0, \quad (2.64)$$

Constants A_1 and A_2 , A_3 cannot be determined independently, as there are also planar vibrations, and (2.61) is dependent. A system of algebraic equations is received from boundary conditions (2.64), using (2.61). Its determinant equals zero. It is an equation of own (resonant) frequencies.

$$\begin{aligned}
& [\lambda J_0(\lambda) + (\nu_1 - 1)J_1(\lambda)] \\
& \times \left[-2\mu^2 I_0(\mu)J_0(\mu) + \mu(1 - \nu_2)\{I_0(\mu)J_1(\mu) + I_1(\mu)J_0(\mu)\} \right] \\
& - \bar{K}J_1(\lambda)\mu[J_1(\mu)I_0(\mu) + I_1(\mu)J_0(\mu)] = 0,
\end{aligned} \quad (2.65)$$

where $\nu_1 = \nu + \frac{K\alpha}{\beta + \alpha}$; $\nu_2 = \bar{\nu} + \frac{K(\gamma_0 - \frac{1}{2})^2}{d}$; $\bar{K} = \frac{K^2\alpha(\gamma_0 - \frac{1}{2})}{(\beta + \alpha)d}$.

In the specific case $\beta = 0$ ($\gamma_0 = 1/2$) the equation (2.39) is disintegrated into independent frequency equations of piezoceramic disk radial vibrations. Its electrodes are open:

$$\lambda J_0(\lambda) + (\nu - 1 + K)J_1(\lambda) = 0, \quad (2.66)$$

and own frequencies equations of flexural vibrations:

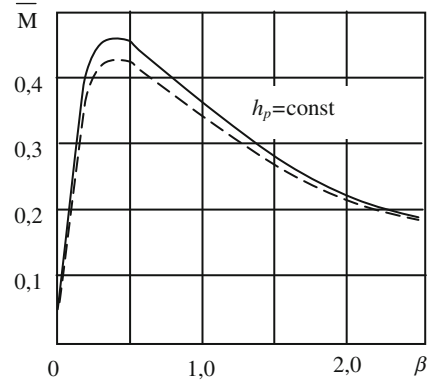
$$-2\mu^2 J_0(\mu)I_0(\mu) + \mu(1 - \bar{\nu})[J_1(\mu)I_0(\mu) + I_1(\mu)J_0(\mu)] = 0. \quad (2.67)$$

Generally $\beta \neq 0$ planar and flexural vibrations are related. Vibrations connectedness is shown as a result of inverse piezoeffect only if the electrodes are open. In the equations this relatedness is realized by dependence (2.61) if boundary conditions are met (2.64).

Correlation (2.61) is not considered in case potential difference on the piezoelement electrodes is given. As a result, planar and flexural vibrations won't be connected.

The dependence of dimensionless sensitivity is shown in Fig. 2.13:

Fig. 2.13 Dependence of dimensionless sensitivity $\bar{M}$ on $\beta = h_m / h_p$ Ratio



$$\bar{M} = \frac{U_x}{p} \frac{d_{31}}{R s_{11}^p (1 - \nu^2)} \quad (2.68)$$

It depends on the ratio of metal and piezoceramic disks thicknesses β for (hinged) merely supported (a continuous line) and freely supported (a dashed line) bimorph.

The calculations are made in below resonance range at dimensionless frequency of $\Omega = 0,0179$ for following physical-mechanical and geometrical parameters of the transducer:

$$\begin{aligned} s_{11}^p &= 15,2 \times 10^{-12} \frac{m^2}{N}; \quad s_{12}^p = -5,8 \times 10^{-12} \frac{m^2}{N}; \quad s_{11}^m = 9,9 \times 10^{-12} \frac{m^2}{N}; \\ \varepsilon_{33}^T &= 1540 \varepsilon_0 \left(\varepsilon_0 = 8,854 \times 10^{-12} \frac{F}{m} \right); \quad d_{31} = -100 \times 10^{-12} \frac{Kl}{N}; \\ \rho_p &= 7,74 \times 10^3 \frac{kg}{m^3}; \quad \rho_m = 8,3 \times 10^3 \frac{kg}{m^3}; \quad R = 9 \times 10^3 m. \end{aligned}$$

The sensitivity dependence on dimensionless parameter β is constructed for the fixed thickness of the piezoceramic element $h_p = 3 \times 10^{-4} m$ (Fig. 2.13). As it is seen from the diagram there are values β^* with which the sensitivity maximum is reached ($\beta^* \approx 0,4$). Consequently, the thickness of the metal plate should be selected from ($h_m \approx 0,4 h_p$) condition to assure the highest sensitivity of the transducer.

Trimorph Piezoelements

Some variants of performance trimorph elements are possible (Fig. 2.14).

Adding one more piezoelement to the asymmetric bimorph element, increases the rigidity of the vibratory system and resonant frequency. Simultaneously sensitivity decreases. Placing of a neutral plane plays the important role for bimorph and trimorph elements. Placing of this plane between the basic and additional piezoelement is ideal case. For asymmetric trimorph element to reach it there is difficulty.

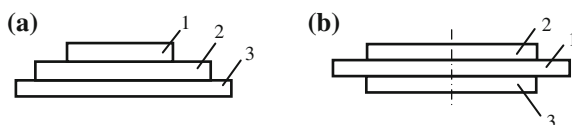


Fig. 2.14 Trimorph piezoelements: **a** asymmetric, **b**—symmetric: 1, 2—piezoelements, 3—metal membrane

Design of symmetric trimorph piezoelement (concerning a neutral plane) satisfies quite to these requirements (Fig. 2.14b).

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