

Chapter 2

Thermal Constriction Resistance

It was seen that the contact interface consists of a number of discrete and small actual contact spots separated by relatively large gaps. These gaps may be evacuated or filled with a conducting medium such as gas. In the first case, all of the heat is constrained to flow through the actual contact spots. If the gaps are filled with a conducting medium, however, some of the heat flow lines are allowed to pass through the gaps, that is, they are less constrained and thus the constriction is alleviated to some extent.

Constriction resistance is a measure of the additional temperature drop associated with a single constriction. Let T_0 be the temperature difference required for the passage of heat at the rate Q through a medium when there is no constriction and T the temperature difference required when a constriction is present, all other things remaining the same. Then the constriction resistance R_c is defined as:

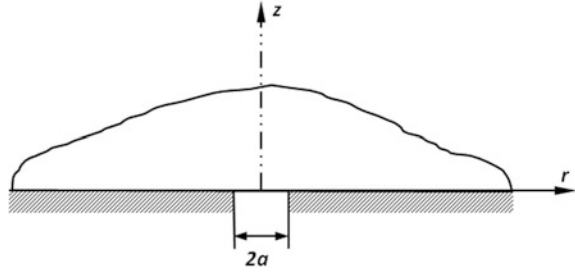
$$R_c = \frac{(T - T_0)}{Q} \quad (2.1)$$

In this chapter, the theory pertaining to the constriction resistance is derived first when the constriction is in isolation, that is, when the effect of the adjacent spots is ignored. Next the constriction resistance of a single spot when it is surrounded by similar spots is determined. The contact conductance is the sum of the conductances of all of the spots existing on the interface. The average radius of these contact spots and their number can be determined by means of surface and deformation analyses so that the conductance may be finally evaluated as a function of the surface parameters, material properties and the contact pressure.

2.1 Circular Disc in Half Space

The logical starting point for the discussion of constriction resistance is to consider the resistance associated with a circular area located on the boundary of a semi-infinite medium. This is equivalent to assuming that:

Fig. 2.1 Disc constriction in half-space



- The constriction is small compared to the other dimensions of the medium in which heat flow occurs.
- The constriction of heat flow lines is not affected by the presence of other contact spots.
- There is no conduction of heat through the gap surrounding the contact spot.

The problem is illustrated in Fig. 2.1. Many solutions to this problem are available (see, for example, Llewellyn-Jones 1957; Holm 1957). We will describe here, in some detail, the method used by Carslaw and Jaeger (1959). It is believed that such detail is necessary in order to appreciate fully the mathematical complexities involved in the analytical solutions of even the simplest configurations. In what follows, frequent reference is made to the work of Gradshteyn and Ryzhik (1980). The formulas of this reference will be indicated by G-R followed by the formula number.

The equation of heat conduction in cylindrical co-ordinates, with no heat generation is:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (2.2)$$

Using the method of separation of variables, we seek a solution of the form:

$$T(r, z) = R(r)Z(z) \quad (2.3)$$

so that Eq. (2.2) may be written as

$$R''Z + \left(\frac{Z}{r}\right)R' + Z''R = 0$$

Dividing through by RZ and separating the variables. We get

$$\frac{(R'' + \frac{R'}{r})}{R} = -\frac{Z''}{Z} = -\lambda^2$$

Thus Eq. (2.2) is reduced to two ordinary differential equations:

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \lambda^2 R = 0 \quad (2.4a)$$

and

$$\frac{d^2 Z}{dz^2} - \lambda^2 Z = 0 \quad (2.4b)$$

Equation (2.4a) is a form of Bessel's differential equation of order zero and a solution of this is $J_0(\lambda r)$ and a solution of Eq. (2.4b) is $e^{-\lambda z}$. Therefore, Eq. (2.2) is satisfied by $e^{-\lambda z} J_0(\lambda r)$ for any λ . Hence

$$T = \int_0^\infty e^{-\lambda z} J_0(\lambda r) f(\lambda) d\lambda \quad (2.5)$$

will also be a solution if $f(\lambda)$ can be chosen to satisfy the boundary conditions at $z = 0$.

At $z = 0$, the solution (2.5) reduces to

$$T_c = \int_0^\infty J_0(\lambda r) f(\lambda) d\lambda \quad (2.6)$$

In the problem being considered, at $z = 0$, there is no heat flow over the region $r > a$. Also, in the same plane, the region $r < a$ could be at constant temperature or, alternatively, at uniform heat flux. These two cases are considered below.

1. *The contact area is maintained at constant temperature T_c over $0 < r < a$*

According to G-R 6.693.1,

$$\begin{aligned} Int_v &= \int_0^\infty J_v(\alpha x) \frac{\sin(\beta x)}{x} dx \\ &= \frac{1}{v} \sin\left(v \arcsin \frac{\beta}{\alpha}\right) \quad \beta \leq \alpha \\ &= \frac{\alpha^v \sin \frac{v\pi}{2}}{v(\beta + \sqrt{\beta^2 - \alpha^2})^v} \quad \beta > \alpha \end{aligned}$$

Taking the limit as $v \rightarrow 0$, (applying L'Hopital's Rule), these integrals turn out to be

$$\begin{aligned} Int_0 &= \arcsin \frac{\beta}{\alpha} \quad \beta < \alpha \\ Int_0 &= \frac{\pi}{2} \quad \beta > \alpha \end{aligned}$$

Hence, if we take

$$\begin{aligned} Int_0 &= \arcsin \frac{\beta}{\alpha} \quad \beta \leq \alpha \\ Int_0 &= \frac{\pi}{2} \quad \beta > \alpha \end{aligned}$$

in Eq. (2.6), we get for the temperature at $z = 0$

$$\frac{2T_c}{\pi} \int_0^\infty J_0(\lambda r) \frac{\sin(\lambda a)}{\lambda} d\lambda = \frac{2T_c}{\pi} \arcsin \frac{\lambda}{\lambda} = \frac{2T_c}{\pi} \frac{\pi}{2} = T_c, \quad r \leq a$$

Since the temperature is independent of z for $r > a$, this satisfies the other boundary condition, namely, no heat flow over rest of the plane at $z = 0$.

Substituting for $f(\lambda)$ in Eq. (2.5)

$$T = \frac{2T_c}{\pi} \int_0^\infty e^{-\lambda z} J_0(\lambda r) \frac{\sin(\lambda a)}{\lambda} d\lambda \quad (2.7)$$

we get, from G-R 6.752.1

$$T = \frac{2T_c}{\pi} \arcsin \frac{2a}{\sqrt{(r+a)^2+z^2} + \sqrt{(r-a)^2+z^2}} \quad (2.8)$$

Note that, for $0 < r \leq a$,

$$-\left(\frac{\partial T}{\partial z}\right)_{z=0} = \frac{2T_c}{\pi} \int_0^\infty J_0(\lambda r) \sin(\lambda a) d\lambda = \frac{2T_c}{\pi} \left(\frac{1}{\sqrt{a^2 - r^2}} \right) \quad (2.9)$$

from G-R 6.671.7.

The heat flow over the circle $0 < r \leq a$,

$$\begin{aligned} Q &= -2\pi k \int_0^a \left(\frac{\partial T}{\partial z} \right)_{z=0} r dr \\ &= -2\pi k \frac{2T_c}{\pi} \int_0^a \left[\int_0^\infty -\lambda e^{-\lambda z} J_0(\lambda r) \frac{\sin(\lambda a)}{\lambda} d\lambda \right]_{z=0} r dr \\ &= 4kT_c \int_0^a \left[\int_0^\infty J_0(\lambda r) \sin(\lambda a) d\lambda \right] r dr \\ &= 4kT_c \int_0^\infty \sin(\lambda a) \left[\int_0^a J_0(\lambda r) r dr \right] d\lambda \\ &= 4kT_c \int_0^\infty \frac{1}{\lambda} \sin(\lambda a) [aJ_1(\lambda a)] d\lambda \end{aligned}$$

from G-R. 6.561.5. Therefore,

$$\begin{aligned} Q &= 4akT_c \int_0^\infty \frac{1}{\lambda} \sin(\lambda a) [J_1(\lambda a)] d\lambda \\ &= 4akT_c (1) \end{aligned}$$

from G-R 6.693.1.

The constriction is finally given by:

$$R_{cd1} = \frac{T_c - 0}{Q} = \frac{1}{4ak} \quad (2.10)$$

2. *The contact area is subjected to uniform heat flux*

In this case, the boundary conditions at $z = 0$ are:

$$-k \frac{\partial T}{\partial z} \begin{cases} = q, & \text{for } 0 \leq r < a \\ = 0, & \text{for } r > a \end{cases} \quad (2.11)$$

where q is the heat flux.

Differentiating Eq. (2.5) with respect to z ,

$$\frac{\partial T}{\partial z} = \int_0^\infty -\lambda e^{-\lambda z} J_0(\lambda r) f(\lambda) d\lambda$$

Applying the first of the two boundary conditions (at $z = 0$) in Eq. (2.11),

$$-\frac{\partial T}{\partial z} = \int_0^\infty \lambda J_0(\lambda r) f(\lambda) d\lambda = \frac{q}{k}$$

Considering the integral (G-R, 6.512.3)

$$\int_0^\infty J_0(\lambda r) J_1(\lambda a) d\lambda \begin{cases} = 0 & \text{for } r > a \\ = \frac{1}{2a} & \text{for } r = a \\ = \frac{1}{a} & \text{for } r < a \end{cases}$$

we see that

$$f(\lambda) = \left(\frac{qa}{k}\right) \frac{J_1(\lambda a)}{\lambda} \quad (2.12)$$

so that the solution is

$$T = \left(\frac{qa}{k}\right) \int_0^\infty e^{-\lambda z} J_0(\lambda r) \frac{J_1(\lambda a)}{\lambda} d\lambda \quad (2.13)$$

The average temperature, T_{av} over $0 < r \leq a$ and $z = 0$ is

$$\begin{aligned} T_{av} &= \frac{1}{\pi a^2} \int_0^a T(2\pi r) dr \\ &= \frac{2q}{ak} \int_0^\infty \frac{J_1(\lambda a)}{\lambda} \left\{ \int_0^a J_0(\lambda r) dr \right\} d\lambda \end{aligned}$$

From G-R 6.561.5

$$T_{av} = \frac{2q}{ak} \int_0^\infty \frac{J_1(\lambda a)}{\lambda} \left\{ a \frac{J_1(\lambda a)}{\lambda} \right\} d\lambda$$

or

$$T_{av} = \frac{2q}{k} \int_0^\infty \left\{ \frac{J_1^2(\lambda a)}{\lambda^2} \right\} d\lambda$$

Using the result from G-R 6.574.2 for the integral in the above expression,

$$T_{av} = \frac{8qa}{3\pi k} \quad (2.14)$$

The heat flow rate is

$$Q = \pi a^2 q$$

Hence the constriction resistance for the uniform heat flux condition is

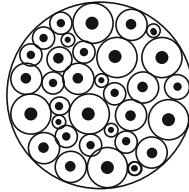
$$R_{cd2} = \frac{T_{av}}{Q} = \frac{8}{3\pi^2 ak} = \frac{0.27}{ka} \quad (2.15)$$

This is about 8 % larger than the constriction resistance R_{cd1} obtained for the uniform temperature condition.

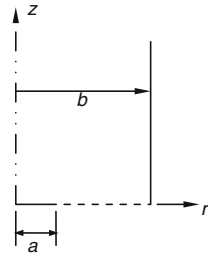
2.2 Resistance of a Constriction Bounded by a Semi-infinite Cylinder

In a real joint there will be several contact spots. Each contact spot of radius a_i may be imagined to be fed by a cylinder of larger radius b_i as shown in Fig. 2.2. Note that the sum of areas of all of the contact spots is equal to the real contact area A_r , while the sum of the cross sectional areas of all of the cylinders is taken as equal to the nominal (apparent) contact area A_n .

Fig. 2.2 Modelling of a single contact spot in a cluster of spots



Idealized View of Contact Plane



Constriction Bounded by a Semi-Infinite Cylinder

2.2.1 Contact Area at Uniform Temperature

It is further assumed that there is no cross flow of heat between the adjacent cylinders. There is also no heat flow across the gap between adjacent contact spots; that is, the contact is surrounded by a vacuum in the contact plane.

There are several solutions available to this problem. The following analysis is based on the solution described Mikic and Rohsenow (1966) and Cooper et al. (1969).

The boundary conditions defined by the problem are:

$$T = \text{constant}; \quad z = 0, \quad 0 < r \leq a \quad (2.16a)$$

$$-k \frac{\partial T}{\partial z} = 0; \quad z = 0, \quad r > a \quad (2.16b)$$

$$-k \frac{\partial T}{\partial z} = \frac{Q}{\pi b^2 k}; \quad z \rightarrow \infty \quad (2.16c)$$

$$-k \frac{\partial T}{\partial r} = 0; \quad r = b \quad (2.16d)$$

$$-k \frac{\partial T}{\partial r} = 0; \quad r = 0 \quad (2.16e)$$

To satisfy the boundary conditions (2.16c) and (2.16e), the solution to Eq. (2.2) should be in the form:

$$T = \frac{Q}{\pi b^2 k} z + \sum_{n=1}^{\infty} C_n e^{-\lambda_n z} J_0(\lambda_n r) + T_0 \quad (2.17)$$

From the boundary condition in Eq. (2.16d), we get

$$J_1(\lambda_n b) = 0 \quad (2.18)$$

Here $b = 3.83171, 7.01559, 10.17347$, etc. (Abramovitz and Stegun 1968a). Also by integrating Eq. (2.17) over the whole of the interfacial area ($0 < r < b$) at $z = 0$ and using Eq. (2.18), we see that the average temperature for this area is T_0 .

In Eq. (2.17), the C_n 's are to be determined from the boundary conditions Eqs. (2.16a) and (2.16b) at $z = 0$. However, these boundary conditions are mixed. To overcome this problem, the Dirichlet boundary condition in Eq. (2.16a) is replaced by a heat flux distribution for the circular disc in half space [see Eq. (2.9)]:

$$-k \frac{\partial T}{\partial z} = \frac{Q}{2\pi a \sqrt{a^2 - r^2}}; \quad z = 0, \quad 0 < r \leq a \quad (2.19)$$

This approximation will lead to a nearly constant temperature distribution over the area specified by $0 < r \leq a$, especially for small values of ε , where $\varepsilon = (a/b)$.

However, from Eq. (2.17), at $z = 0$,

$$-k \frac{\partial T}{\partial z} = k \left[\frac{Q}{\pi b^2 k} + \sum_{n=1}^{\infty} C_n \lambda_n J_0(\lambda_n r) \right] \quad (2.20)$$

From (2.19) and (2.20), therefore

$$\left[\frac{Q}{\pi b^2 k} + \sum_{n=1}^{\infty} C_n \lambda_n J_0(\lambda_n r) \right] = \frac{Q}{2\pi a k \sqrt{a^2 - r^2}}$$

To utilize the orthogonality property of the Bessel function both sides of the above equation are multiplied by $r J_0(\lambda_n r)$ and integrated over the appropriate ranges to yield

$$\frac{Q}{\pi b^2 k} \int_0^b r J_0(\lambda_n r) dr + C_n \lambda_n \int_0^b r J_0^2(\lambda_n r) dr = \frac{Q}{2\pi a k} \int_0^a \frac{r J_0(\lambda_n r)}{\sqrt{a^2 - r^2}} dr$$

(see G-R 6.521.1).

However

$$\int_0^b r J_0(\lambda_n r) dr = \frac{b}{\lambda_n} J_1(\lambda_n b)$$

This is equal to zero by virtue of Eq. (2.18).

From the orthogonality property (Abramovitz and Stegun 1968b)

$$\int_0^b r J_0^2(\lambda_n r) dr = \frac{b^2}{2} J_0^2(\lambda_n b)$$

and

$$\int_0^a \frac{r J_0(\lambda_n r)}{\sqrt{a^2 - r^2}} dr = \frac{\sin(\lambda_n a)}{\lambda_n}$$

(see G-R 6.554.2).

Therefore

$$C_n = \left(\frac{Q}{\pi k a} \right) \frac{\sin(\lambda_n a)}{(\lambda_n b)^2 J_0^2(\lambda_n b)}$$

Substituting for C_n in Eq. (2.17)

$$T = \frac{Q}{\pi b^2 k} z + \left(\frac{Q}{\pi k a} \right) \sum_{n=1}^{\infty} \frac{e^{-\lambda_n z} \sin(\lambda_n a) J_0(\lambda_n r)}{(\lambda_n b)^2 J_0^2(\lambda_n b)} + T_0 \quad (2.21)$$

The mean temperature over the interface ($z = 0$) is then obtained by

$$\begin{aligned}
T_m &= \frac{1}{\pi a^2} \int_0^a \left(\frac{Q}{\pi k a} \right) \sum_{n=1}^{\infty} \frac{e^{-\lambda_n z} \sin(\lambda_n a) J_0(\lambda_n r)}{(\lambda_n b)^2 J_0^2(\lambda_n b)} 2\pi r dr + \frac{1}{\pi b^2} \int_0^b T_0 2\pi r dr \\
&= \frac{Q}{4ka} \left(\frac{8}{\pi a^2} \right) \sum_{n=1}^{\infty} \frac{\sin(\lambda_n a)}{(\lambda_n b)^2 J_0^2(\lambda_n b)} \int_0^a r J_0(\lambda_n r) dr + T_0
\end{aligned}$$

This results in

$$T_m = \frac{Q}{4ka} \left(\frac{8}{\pi} \right) \left(\frac{b}{a} \right) \sum_{n=1}^{\infty} \frac{\sin(\lambda_n a) J_1(\lambda_n a)}{(\lambda_n b)^3 J_0^2(\lambda_n b)} + T_0 \quad (2.22)$$

The factor $1/(4ka)$ in the above expression represents the disc constriction resistance of Eq. (2.10). The thermal resistance between $z = 0$ and $z = L$ (for large L) is given by

$$R_t = \frac{T_m - T_{z=L}}{Q} = \frac{T_m}{Q} + \frac{L}{\pi k b^2} - \frac{T_0}{Q}$$

Hence the *additional* resistance due to constriction is

$$R = R_t - \frac{L}{\pi k b^2} = \frac{T_m - T_0}{Q}$$

Substituting for T_m from Eq. (2.22)

$$R = \frac{1}{4ka} \left(\frac{8}{\pi} \right) \left(\frac{b}{a} \right) \sum_{n=1}^{\infty} \frac{\sin[\lambda_n b(\frac{a}{b})] J_1[\lambda_n b(\frac{a}{b})]}{(\lambda_n b)^3 J_0^2(\lambda_n b)} = R_{cd1} F\left(\frac{a}{b}\right) \quad (2.23)$$

in which

$$F\left(\frac{a}{b}\right) = \left(\frac{8}{\pi} \right) \left(\frac{b}{a} \right) \sum_{n=1}^{\infty} \frac{\sin[\lambda_n b(\frac{a}{b})] J_1[\lambda_n b(\frac{a}{b})]}{(\lambda_n b)^3 J_0^2(\lambda_n b)} \quad (2.24)$$

is called the *constriction alleviation factor*.

Yovanovich (1975) obtained expressions for the constriction alleviation factor for heat flux functions of the form

$$\left[1 - \frac{r^2}{a^2} \right]^m; \quad z = 0, \quad 0 < r \leq a$$

His results for $m = -0.5$ were identical to that of Mikic, as expected.

Other solutions to the above problem include those of Roess (as presented by Weills and Ryder (1949)), Hunter and Williams (1969), Gibson (1976), Rosenfeld and Timsit (1981) and Negus and Yovanovich (1984). The algebraic expressions derived for the constriction alleviation factor by Roess, Gibson, and Negus and Yovanovich are somewhat similar

Table 2.1 Comparison of constriction alleviation factors

a/b	Roess (Eq. 2.25)	Mikic (Eq. 2.24)	Gibson (Eq. 2.26)	N-Y (Eq. 2.27)
0.1	0.8594	0.8584	0.8594	0.8594
0.2	0.7205	0.7202	0.7209	0.7208
0.3	0.5853	0.5851	0.5865	0.5865
0.4	0.4558	0.4557	0.4586	0.4586
0.5	0.3340	0.3341	0.3398	0.3395
0.6	0.2230	0.2231	0.2328	0.2318

$$F_{\text{Roess}} = 1 - 1.4093(a/b) + 0.2959(a/b)^3 + 0.0524(a/b)^5 + \dots \quad (2.25)$$

$$F_{\text{Gibson}} = 1 - 1.4092(a/b) + 0.3380(a/b)^3 + 0.0679(a/b)^5 + \dots \quad (2.26)$$

$$F_{\text{Negus-Yovanovich}} = 1 - 1.4098(a/b) + 0.3441(a/b)^3 + 0.0435(a/b)^5 + \dots \quad (2.27)$$

The constriction alleviation factors obtained by Eqs. (2.24)–(2.27) are compared in Table 2.1. The first 120 terms were used in evaluating the series in Eq. (2.24).

2.2.2 Contact Area Subjected to Uniform Heat Flux

In this case, the boundary condition, represented by Eq. (2.16a) is replaced by

$$-k \frac{\partial T}{\partial z} = \text{constant}; \quad z = 0, \quad 0 < r \leq a \quad (2.28)$$

The constriction alleviation factor for this problem was theoretically derived by Yovanovich (1976):

Note: 1. In the following expression $\varepsilon = a/b$

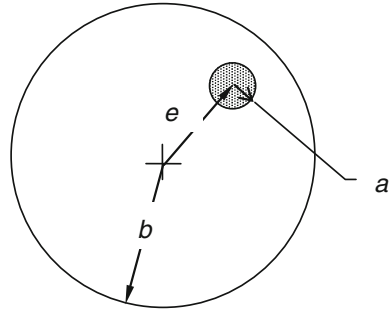
2. The constriction resistance is non-dimensionalized by multiplying it by $k\sqrt{A_c}$, that is, by $ka\sqrt{\pi}$ and not $4ak$

$$F'(\varepsilon) = \frac{16}{\pi\varepsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\lambda_n \varepsilon)}{\lambda_n^3 J_0^2(\lambda_n)} \quad (2.29)$$

Negus, Yovanovich and Beck (1989) provided the following correlation to evaluate $F'(\varepsilon)$

$$F'(\varepsilon) = 0.47890 - 0.62498\varepsilon + 0.11789\varepsilon^3 + \dots \quad (2.30)$$

Fig. 2.3 Eccentric constriction



2.3 Eccentric Constrictions

In Sect. 2.2, the contact spot was assumed to be concentric with the feeding flux tube. The subject of eccentric constrictions has been studied by Cooper et al. (1969), Sexl and Burkhard (1969) and others. A more recent work is by Bairi and Laraqi (2004) who presented an analytical solution to calculate the thermal constriction resistance for an eccentric circular spot with *uniform flux* on a semi-infinite circular heat flux tube. This solution is developed using the finite cosine Fourier transform and the finite Hankel transform (Fig. 2.3).

The authors proposed a dimensionless correlation to calculate the constriction alleviation factor as a function of ε and the eccentricity e :

$$\Psi^* = \frac{\Psi}{\Psi_0} = 1 + \left[1.5816(a/b)^{0.0528} - 1 \right] \left(\frac{e}{b-a} \right)^{1.76} \left(\frac{a}{b-e} \right)^{0.88} \quad (2.31)$$

In this equation the authors took the following correlation for Ψ_0 , the constriction factor for zero eccentricity (Negus et al. 1989):

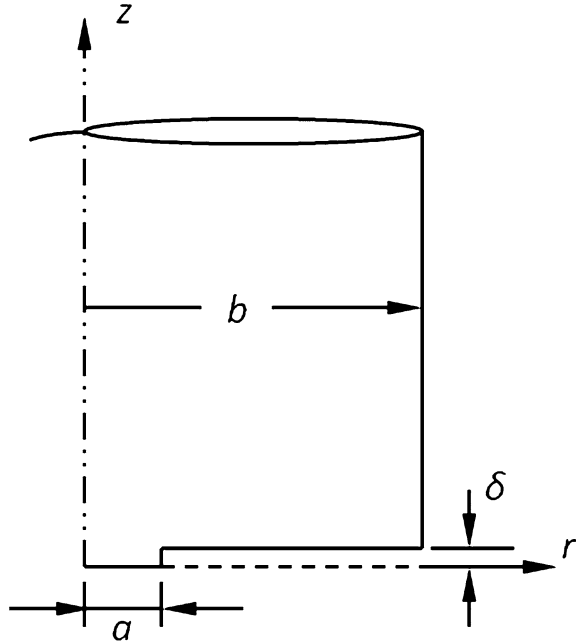
$$\begin{aligned} \Psi_0 = & 0.47890 - 0.62076(a/b) \\ & + 0.114412(a/b)^3 + 0.01924(a/b)^5 + 0.00776(a/b)^7 \end{aligned} \quad (A)$$

However, the expression given in (A) is the *constriction factor for a circular cross section at the end of a square tube!* The correct factor that the authors should have used is in Eq. (2.30). Therefore the accuracy of Eq. (2.31) is open to question.

2.4 Constriction in a Fluid Environment

In this case, the boundary conditions at the contact plane ($z = 0$) are as shown in Fig. 2.4 in which k_f is the thermal conductivity of the fluid (gas) and δ is the effective gap thickness.

Fig. 2.4 Constriction in a fluid environment



Approximate solutions to the above problem have been obtained by Cetinkale and Fishenden (1951), Mikic and Rohsenow (1966) and Tsukizoe and Hisakado (1972). A later work by Das and Sadhal (1998) presents an analytical solution to the problem of a constriction surrounded by an interstitial fluid. An ‘exact’ solution was presented by Sanokawa (1968), but the results of this work were not in a readily usable form. In any case, the model used in these analyses, as illustrated above in Fig. 2.4, is somewhat artificial—the gap thickness is abruptly changed from zero thickness to a finite thickness at $r = a$. The thickness is expected to increase gradually. Any analytical solution is, therefore, likely to be complicated and a digital computer would be still required to evaluate the results. For this reason, a numerical solution is perhaps more suitable for the solution of this type of problems.

In a large number of situations, the heat flow through the gas gap is small compared to the heat flow through the solid contact spots. In such cases, the fluid conductance may be estimated by dividing the fluid conductivity by the effective gap thickness. This may then be added to the solid spot conductance to obtain the total conductance. Factors affecting the gas gap conductance are discussed in detail in Chap. 4.

2.5 Constrictions of Other Types

Apart from the solutions discussed above, problems pertaining to constrictions of other shapes and boundary conditions have been analysed by various researchers. These are listed in Table 2.2.

Table 2.2 Constriction resistance—representative works

No.	Reference	Configuration	Approach
1	Mikic and Rohsenow (1966)	Strip of contact and rectangles in vacuum	Analytical
2	Yip and Venart (1968)	Single and multiple constrictions in vacuum	Analogue
3	Veziroglu and Chandra (1969)	Two dimensional, symmetric and eccentric constrictions in vacuum	Analytical and analogue
4	Williams (1975)	Conical constrictions in vacuum	Experimental
5	Yovanovich (1976)	Circular annular constriction at the end of a semi-infinite cylinder in vacuum	Analytical
6	Gibson and Bush (1977)	Disc constriction in half space in conducting medium	Analytical
7	Major and Williams (1977)	Conical constrictions in vacuum	Analogue
8	Schneider (1978)	Rectangular and annular contacts in vacuum half space	Numerical
9	Yovanovich et al. (1979)	Doubly connected areas bounded by circles, squares and triangles in vacuum	Analytical
10	Madhusudana (1979a, b), (1980)	Conical constrictions at the end of a long cylinder, in vacuum and in conducting medium	Numerical and experimental
11	Major (1980)	Conical constrictions in vacuum	Numerical
12	Negus et al. (1988)	Circular contact on coated surfaces in vacuum	Analytical
13	Das and Sadhal (1992)	Two dimensional gaps at the interface of two semi-infinite solids in a conducting environment	Analytical
14	Madhusudana and Chen (1994)	Annular constriction at the end of a semi-infinite cylinder in vacuum	Analytical and analogue
15	Olsen et al. (2001a, b) (2002)	Coated conical constrictions in vacuum, gas and with radiation	Numerical

References

- Abramovitz M, Stegun IA (1968a) Handbook of mathematical functions. Dover, New York, p 409
- Abramovitz M, Stegun IA (1968b) Handbook of mathematical functions. Dover, New York, p 48
- Bairi A, Laraqi N (2004) The thermal constriction resistance for an eccentric spot on a circular heat flux tube. *Trans ASME J Heat Transf* 128:652–655
- Carslaw HS, Jaeger JC (1959) Conduction of heat in solids, 2nd edn. Clarendon Press, Oxford, pp 214–217
- Cetinkale TN, Fishenden M (1951) Thermal conductance of metal surfaces in contact. In: Proceedings of the *general* discussion on *heat* transfer, Institute of Mechanical Engineers, London, pp 271–275
- Cooper MG, Mikic BB, Yovanovich MM (1969) Thermal contact conductance. *Int J Heat Mass Transf* 12:279–300
- Das AK, Sadhal SS (1992) The effect of interstitial fluid on thermal constriction resistance. *Trans ASME J Heat Transf* 114:1045–1048
- Das AK, Sadhal SS (1998) Analytical solution for constriction resistance with interstitial fluid in the gap. *Heat Mass Transf* 34:111–119
- Gibson RD (1976) The contact resistance for a semi-infinite cylinder in vacuum. *Appl Energy* 2:57–65
- Gibson RD, Bush AW (1977) The flow of heat between bodies in gas-filled contact. *Appl Energy* 3:189–195
- Gradshteyn IS, Ryzhik M (1980) Tables of integrals, series and products. Academic Press, New York
- Holm R (1957) Electric contacts, theory and application, 4th edn. Springer, New York, pp 11–16
- Hunter A, Williams A (1969) Heat flow across metallic joints—the constriction alleviation factor. *Int J Heat Mass Transf* 12:524–526
- Llewellyn-Jones F (1957) The physics of electric contacts. Oxford University Press, New York, pp 13–15
- Madhusudana CV, Chen PYP (1994) Heat flow through concentric annular constrictions. In: Proceedings of the 10th international heat transfer conference, paper 3-Nt 18, Institution of Chemical Engineers, Rugby
- Madhusudana CV (1979a) Heat flow through conical constrictions in vacuum and in conducting media. In: AIAA 14th thermophysics conference, paper 79-1071, Orlando, Florida
- Madhusudana CV (1979b) Heat flow through conical constrictions. *AIAA J* 18:1261–1262
- Madhusudana, CV (1980) Heat flow through conical constrictions, *AIAA Journal*, 18: 1261–1262
- Major SJ, Williams A (1977) The solution of a steady conduction heat transfer problem using an electrolytic tank analogue, *Inst Engrs (Australia)*. *Mech Eng Trans* 7–11
- Major, SJ (1980) The finite difference solution of conduction problems in cylindrical co-ordinates, *IE (Aust)*, *Mech Eng Trans*, Paper M 1049
- Mikic BB, Rohsenow WM (1966) Thermal contact resistance, Mechanical Engineering Department report no. DSR 74542-41. MIT, Cambridge
- Negus KJ, Yovanovich MM (1984) Constriction resistance of circular flux tubes with mixed boundary conditions by superposition of neumann solutions. In: *ASME Paper 84-HT-84*, American Society of Mechanical Engineers
- Negus KJ, Yovanovich MM, Thompson JC (1988) Constriction resistance of circular contacts on coated surfaces: effect of boundary conditions. *J Thermophys Heat Transf* 12(2):158–164
- Negus KJ, Yovanovich MM, Beck JV (1989) On the nondimensionalization of constriction resistance for semi-infinite heat flux tubes. *Trans ASME J Heat Transf* 111:804–807
- Olsen E, Garimella SV, Madhusudana CV (2001a) Modeling of constriction resistance at coated joints in a gas environment. In: 2nd international symposium on advances in computational heat transfer, Palm Cove, Qld, Australia, 20–25 May 2001

- Olsen E, Garimella SV, Madhusudana CV (2001b) Modeling of constriction resistance at coated joints with radiation. In: 35th national heat transfer conference, Anaheim, California, 10 June 2001
- Olsen EL, Garimella SV, Madhusudana CV (2002) Modeling of constriction resistance in coated joints. *J Thermophys Heat Transf* 16(2):207–216
- Rosenfeld AM, Timsit RS (1981) The potential distribution in a constricted cylinder: an exact solution. *Q Appl Maths* 39:405–417
- Sanokawa K (1968) Heat transfer between metallic surfaces. *Bull JSME* 11:253–293
- Schneider, G.E. (1978), Thermal constriction resistances due to arbitrary contacts on a half space—numerical solution. In: AIAA Paper 78-870, Amer Inst Aeronautics and Astronautics, New York
- Sextl RU, Burkhard DG (1969) An exact solution for thermal conduction through a two dimensional eccentric constriction. *Prog Astro Aero* 21:617–620
- Tsukizoe T, Hisakado T (1972) On the mechanism of heat transfer between metal surfaces in contact—Part 1, heat transfer. *Jpn Res* 1(1):104–112
- Veziroglu TN, Chandra S (1969) Thermal conductance of two dimensional constrictions. *Prog Astro Aero* 21:591–615
- Weills ND, Ryder EA (1949) Thermal resistance of joints formed by stationary metal surfaces. *Trans ASME* 71:259–267
- Williams A (1975) Heat flow through single points of metallic contacts of simple shapes. *Prog Astro Aero* 39:129–142
- Yip FC, Venart JES (1968) Surface topography effects in the estimation of thermal and electrical contact resistance. *Proc I Mech Eng* 182(3):81–91
- Yovanovich, M.M. (1975), General expressions for constriction resistance due to arbitrary flux distributions, AIAA Paper 75-188, Amer Inst Aeronautics and Astronautics, New York
- Yovanovich, MM (1976) General thermal constriction parameter for annular contacts on circular flux tubes, *AIAA Journal*, 14:822–824
- Yovanovich MM, Martin KA, Schneider GE (1979) Constriction resistance of doubly-connected contact areas under uniform heat flux, AIAA Paper 79-1070, The American Institute of Aeronautics and Astronautics

<http://www.springer.com/978-3-319-01275-9>

Thermal Contact Conductance

Madhusudana, C.V.

2014, XVIII, 260 p. 133 illus., 17 illus. in color.,

Hardcover

ISBN: 978-3-319-01275-9