

Contents

1	Introduction to Metric Fixed Point Theory	1
	M.A. Khamsi	
1.1	Introduction	1
1.2	Metric Fixed Point Theory	2
1.3	Caristi–Ekeland Extension	3
1.4	The Converse Problem	4
1.5	Some Applications.....	5
1.5.1	ODE and Integral Equations.....	5
1.5.2	Cantor and Fractal Sets	6
1.6	Historical Note.....	6
1.7	Metric Fixed Point Theory in Banach Spaces	7
1.7.1	Classical Existence Results.....	8
1.7.2	The Normal Structure Property.....	10
1.7.3	More on Normal Structure Property	11
1.7.4	Normal Structure and Smoothness	13
1.7.5	Karlovitz–Goebel Lemma	14
1.8	Nonstandard Techniques	14
1.8.1	Extending Mappings to Ultrapowers.....	15
1.9	More on Metric Fixed Point Theory in Metric Spaces.....	17
1.9.1	Menger Convexity in Metric Spaces	17
1.9.2	Uniformly Convex Metric Spaces.....	19
1.9.3	Fixed Point Property in Uniformly Convex Metric Spaces	22
1.10	The Convexity Structures	24
1.10.1	Hyperconvex Metric Spaces	25
	References	29
2	Banach Contraction Principle and Its Generalizations	33
	Abdul Latif	
2.1	Introduction	33
2.2	Contractions: Definition and Examples.....	33

2.3	The Banach Contraction Principle with Some Applications	35
2.4	Some Other Extensions of BCP for Single-Valued Mappings	40
2.5	Caristi's Fixed Point Theorem	47
2.6	Some Extensions of BCP Under Generalized Distances	48
2.7	Multivalued Versions of BCP	51
	References	61
3	Ekeland's Variational Principle and Its Extensions with Applications	65
	Qamrul Hasan Ansari	
3.1	Introduction	65
3.2	Ekeland's Variational Principle in Complete Metric Spaces	66
3.3	Applications to Fixed Point Theorems	75
3.4	Applications to Optimization	80
3.5	Applications to Weak Sharp Minima	84
3.6	Equilibrium Problems and Extended Ekeland's Variational Principle	87
	3.6.1 Equilibrium Problems	87
	3.6.2 Extended Ekeland's Variational Principle	88
	Appendix A	96
	References	96
4	Fixed Point Theory in Hyperconvex Metric Spaces	101
	Rafael Espínola and Aurora Fernández-León	
4.1	Introduction and Basic Definitions	101
4.2	Some Basic Properties of Hyperconvex Metric Spaces	108
4.3	Hyperconvexity, Injectivity, and Retractions	114
4.4	Isbell's Hyperconvex Hull	120
4.5	Topological Fixed Point Property and Hyperconvexity	127
4.6	Metric Fixed Point Property and Hyperconvexity	134
4.7	Metric Fixed Point Property for Multivalued Mappings and Nonexpansive Selections	137
4.8	KKM Theory and Hyperconvex Spaces	140
4.9	Fixed Point Theory and \mathbb{R} -Trees	144
4.10	New Trends in Hyperconvexity	151
	4.10.1 Two Long-Standing Open Problems	151
	4.10.2 Ultrametries and Hyperconvex Metric Spaces	152
	4.10.3 Diversities and Hyperconvexity	152
	4.10.4 Q-hyperconvexity	154
	References	155
5	An Introduction to Fixed Point Theory in Modular Function Spaces	159
	W.M. Kozłowski	
5.1	Introduction	159

5.2	Modular Function Spaces and Modular Geometry	161
5.2.1	Introduction to Modular Function Spaces	162
5.2.2	Geometrical Properties of Modular Function Spaces	170
5.2.3	Nonlinear Mappings in Modular Function Spaces	183
5.3	Existence of Fixed Points	186
5.3.1	Generalized Contractions in Modular Function Spaces	186
5.3.2	Generalized Nonexpansive Mappings.....	193
5.4	Convergence of Fixed Point Iterative Algorithms.....	200
5.5	Semigroups of Mappings in Modular Function Spaces.....	214
	References	219
6	Fixed Point Theory in Ordered Sets from the Metric	
	Point of View	223
	M.Z. Abu-Sbeih and M.A. Khamsi	
6.1	Introduction	223
6.2	Generalized Metric Spaces	224
6.3	The Retraction Property	228
6.4	Fixed Point Property	230
6.5	Externally Complete Sets	233
	References	235
7	Some Fundamental Topological Fixed Point Theorems	
	for Set-Valued Maps	237
	Hichem Ben-El-Mechaiekh	
7.1	Introduction and Preliminaries	237
7.1.1	Elementary Proofs of the Brouwer and Schauder–Tychonoff Fixed Point Theorems	239
7.2	Ky Fan and Kakutani Maps	243
7.2.1	Ky Fan Maps	244
7.2.2	Kakutani Maps	245
7.3	Continuous Selections and Approximations	246
7.3.1	Continuous Selections for Ky Fan Maps.....	246
7.3.2	Continuous Approximations of Kakutani Maps	247
7.4	The Browder–Ky Fan and the Kakutani–Ky Fan Fixed Point Theorems and Their Consequences	249
7.4.1	The Browder–Ky Fan Fixed Point Theorem	249
7.4.2	The Kakutani–Ky Fan Fixed Point Theorem	250
7.4.3	Coincidence Theorems	251
7.5	Relaxing Compactness and Related Results.....	254
7.5.1	The Coercivity Condition (κ)	254
7.5.2	Intersection Theorems: KKM and Matching	256
7.5.3	Browder–Ky Fan Theorem on Star-Shaped Domains	258
7.5.4	A Leray–Schauder Alternative for Kakutani Maps.....	259
7.6	Systems of Nonlinear Inequalities and Applications.....	262
7.7	Concluding Remarks	269
	References	270

8	Some Iterative Methods for Fixed Point Problems	273
	Q.H. Ansari and D.R. Sahu	
8.1	Introduction	273
8.2	Preliminaries	274
8.2.1	The LE Property and the AF Point Property for Nonlinear Operators	277
8.2.2	Nearly Lipschitzian Mappings	278
8.2.3	Asymptotically κ -strict Pseudocontractive Mappings in the Intermediate Sense	281
8.3	Picard Iterative Method.....	283
8.4	Mann Iterative Method	285
8.5	Ishikawa Iterative Method.....	288
8.6	Helpertn Iterative Method.....	290
8.7	CQ Iteration Method.....	293
8.8	Browder Iterative Method	296
	References	298
Index		301

Topics in Fixed Point Theory

Almezel, S.; Ansari, Q.H.; Khamisi, M.A. (Eds.)

2014, XI, 304 p., Hardcover

ISBN: 978-3-319-01585-9