Preface

Linear algebra can be regarded as a theory of the vector spaces, because a vector space is a set of objects or elements that can be added together and multiplied by numbers (the result remaining an element of the set), so that the ordinary rules of calculation are valid. An example of a vector space is the geometric vector space (the free vector space), presented in the first chapter of the book, which plays a central role in physics and technology and illustrates the importance of the vector spaces and linear algebra for all practical applications.

Besides the notions which operates mathematics, created by abstraction from environmental observation (for example, the geometric concepts) or quantitative and qualitative research of the natural phenomena (for example, the notion of number) in mathematics there are elements from other sciences. The notion of vector from physics has been studied and developed creating vector calculus, which became a useful tool for both mathematics and physics. All physical quantities are represented by vectors (for example, the force and velocity).

A vector indicates a translation in the three-dimensional space; therefore we study the basics of the three-dimensional Euclidean geometry: the points, the straight lines and the planes, were in the second chapter.

The linear transformations are studied in the third chapter, because they are compatible with the operations defined in a vector space and allow us to transfer algebraic situations and related problems in three-dimensional space.

Matrix operations clearly reflect their similarity to the operations with linear transformations; so the matrices can be used for the numerical representation of the linear transformations. The matrix representation of linear transformations is analogous to the representation of the vectors through n coordinates relative to a basis.

The eigenvalue problems (also treated in the third chapter) are of great importance in many branches of physics. They make it possible to find some coordinate systems in which changes take the simplest forms. For example, in mechanics the main moments of a solid body are found with the eigenvalues of a symmetric matrix representing the vector tensor. The situation is similar in continuous mechanics, where the body rotations and deformations in the main directions are found using the eigenvalues of a symmetric matrix. Eigenvalues have a central importance in quantum mechanics, where the measured values of the observable physical quantities appear as eigenvalues of operators. Also, the eigenvalues are useful in the study of differential equations and continuous dynamical systems that arise in areas such as physics and chemistry.

The study of the Euclidean vector space in the fourth chapter is required to obtain the orthonormal bases, whereas relative to these bases the calculations are considerably simplified. In a Euclidean vector space, scalar product can be used to define the length of vectors and the angle between them. In the investigation of the Euclidean vector spaces very useful are the linear transformations compatible with the scalar product, i.e. the orthogonal transformations. The orthogonal transformations in the Euclidean plane are: the rotations, the reflections or the compositions of rotations and reflections.

The theory of bilinear and quadratic form are described in the fifth chapter. These are used with analytic geometry to get the classification of the conics and of the quadrics, presented in the Chap. 8.

In Analytic Geometry we replace the definitions and the geometrical study of the curves and the surfaces, by the algebraic correspondence: a curve and a surface are defined by algebraic equations, and the study of the curve and the surface is reduced to the study of the equation corresponding to each one (see the seventh chapter).

The above are used in physics, in particular to describe physical systems subject to small vibrations. The coefficients of a bilinear form behave for certain transformations like the tensor coordinates. The tensors are useful in theory of elasticity (deformation of an elastic medium is described through the deformation tensor).

In the differential geometry, in the study of the geometric figures, we use the concepts and methods of the mathematical analysis, especially the differential calculus and the theory of differential equations, presented in the sixth chapter. The physical problems lead to inhomogeneous linear differential equations of order n with constant coefficients.

In this book we apply extensively the software SAGE, which can be found free online http://www.sagemath.org/.

We give plenty of SAGE applications at each step of our exposition.

This book is usefull to all researchers and students in mathematics, statistics, physics, engineering and other applied sciences. To the best of our knowledge this is the first one.

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