Preface

These notes have been motivated by the interests of the author in variational problems depending on small parameters, for some of which a description based on a global minimization principle does not seem satisfactory. Such problems range from the derivation of physical theories from first principles to numerical problems involving energies with many local minima. Even though an asymptotic description of related global minimization problems can be given in terms of Γ-convergence, the Γ-limit often does not capture the behavior of local minimizers or of gradient flows. This failure is sometimes mentioned as the proof that Γ-convergence is 'wrong'. It may well be so. The author's standpoint is that it might nevertheless be a good starting point that may be systematically 'corrected'.

The author's program has been to examine the (few) results in the literature and try to connect them with his own work in homogenization and discrete systems, where often the local minimization issues are crucial due to the oscillations of the energies. The directions of research have been to:

- find criteria that ensure the convergence of local minimizers and critical points. In case this does not occur, then modify the Γ-limit into an equivalent Γ-expansion (as introduced by the author and L. Truskinovsky) in order to match this requirement. We note that in this way we 'correct' some limit theories, finding (or 'validating') other ones present in the literature.
- modify the concept of local minimizer, so that it may be more 'compatible' with the process of Γ-limit. One such concept is the small-scale stability of C. Larsen.
- treat evolution problems for energies with many local minima obtained by a time-discrete scheme introducing the notion of 'minimizing movements along a sequence of functionals'. In this case the minimizing movement of the Γ-limit can always be obtained by a choice of the space- and time-scale, but more interesting behaviors can be obtained at a critical ratio between them. In many cases a 'critical scale' can be computed together with an effective motion, from which all other minimizing movements are obtained by scaling. Furthermore the choice of suitable Γ-converging sequences in the scheme above allows to address the issues of long-time behavior and backwards motion.

- examine the general variational evolution results that may be related to these minimizing movements, in particular recent theories of quasistatic motion and gradient flow in metric spaces.

The content of the present notes is taken from a series of lectures which formed a PhD course first given at Sapienza University of Rome from March to May 2012 and subsequently at the University of Pavia from November 2012 to January 2013. Those courses were addressed to an audience of students, some of which with an advanced background (meaning that they were already exposed to the main notions of the Calculus of Variations and of Γ-convergence), and researchers in the field of the Calculus of Variations and of Variational Evolution. This was an advanced course in that it was meant to address some current (or future) research issues rather than to discuss some subject systematically. Part of the notes has also been reworked during a 10-h course at the University of Narvik on October 25–30, 2012.

The reader should bear in mind that the scope of the notes has been to foster discussion on the problems presented rather than construct a general detailed theory (a worthy and very interesting objective, though). Hence, we have focused on highlighting the phenomena and issues linked to the interaction of scales, local minimization and variational evolution, rather than on the details of the Γ-convergence process, or the optimal hypotheses for the definition of gradient flows, and so on, for which we refer to the existing literature.

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