Preface

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The CAST (*Contact and Symplectic Topology*) Research Networking Programme has been established in 2010 as one of the ESF (European Science Foundation) sponsored networks. The network is financed by the support of 13 contributing European countries, embracing researchers from all over the globe. The main profile of the network is to foster collaboration throughout institutions in Europe. This aim has been achieved by supporting conferences, workshops, Summer Schools focusing on various aspects of contact and symplectic topology and by supporting research collaborations and exchanges of doctoral students and postdoctoral researchers within the field of symplectic and contact topology.

In particular, the network partially sponsored (together with the Pays de la Loire region, the ANR agency and the Institut Universitaire de France) the *Trimester on Contact and Symplectic Topology* in Nantes (March-June 2011), and (together with the *Lendület program* of the Hungarian Academy of Sciences, through the *Lendület* group *ADT* of the Rényi Institute) supported the CAST Summer School and Conference in Budapest (July 2012). Nantes' program has gathered, during five focused weeks, a summer school and an international conference, a total of 160 mathematicians. The Budapest event attracted more than 130 graduate students, postdoctoral researchers and senior mathematicians from around the globe. Both events provided lecture series in various current topics in contact and symplectic topology. The present volume is the compilation of the notes of these lecture series written by the lecturers. These notes provide a gentle introduction to topics which have developed in an amazing speed in the recent past. The surveys target both graduate students with solid contact and symplectic backgrounds, as well as senior researchers interested in certain aspects of the field. The topics of the lecture series include:

- contact topological questions in dimensions three and in dimensions greater than three,
- open book decompositions and Lefschetz fibrations in contact topology through asymptotically holomorphic techniques,
- Fukaya categories,
- Heegaard Floer homologies and embedded contact homologies (ECH) of 3-dimensional manifolds,
- Stein structures on manifolds of dimension at least six, and
- knot contact homologies.

We dedicate this volume to the memory of V.I. Arnold, whose ideas and results shaped the developement of symplectic and contact topology. The opening paper (by Michèle Audin) is a tribute to the influence of Arnold on symplectic topology, providing an account of the early days of the subject. It is followed by the contributions of the speakers of the Nantes and Budapest Summer Schools. Below we provide short abstracts of each of the contributions.

Orsay, France — Frédéric Bourgeois
Nantes, France — Vincent Colin
Budapest, Hungary — András Stipsicz

- **Patrick Massot (Université Paris Sud, Orsay, France) Topological methods in 3-dimensional contact geometry**

  These notes provide an introduction to Giroux's theory of convex surfaces in contact 3-manifolds and its simplest applications. They put a special emphasis on pictures and discussions of explicit examples. The first goal is to explain why all the information about a contact structure in a neighborhood of a generic surface is encoded by finitely many curves on the surface. Then we describe the bifurcations that happen in generic families of surfaces. As applications, we explain how Giroux used this technology to reprove Bennequin's theorem saying that the standard contact structure on S^3 is tight and Eliashberg's theorem saying that all tight contact structures on S^3 are isotopic to the standard one.

- **Denis Auroux (University of California, Berkeley, USA): A beginner's introduction to Fukaya categories**

  In these notes, we give a short introduction to Fukaya categories and some of their applications. We first briefly review the definition of Lagrangian Floer homology and its algebraic structures. Then we introduce the Fukaya category (informally and without a lot of the necessary technical detail), and discuss algebraic concepts such as exact triangles and generators. Finally, we outline a few applications to symplectic topology, mirror symmetry and low-dimensional topology.

- **Francisco Presas (ICMAT, Madrid, Spain): Geometric decompositions of almost contact manifolds**

  These notes are intended to be an introduction to the use of approximately holomorphic techniques in almost contact and contact geometry. We develop the setup of the approximately holomorphic geometry. Once done, we sketch the existence of the two main geometric decompositions available for an almost contact or contact manifold: open books and Lefschetz pencils. The possible use of the two decompositions for the problem of existence of contact structures is briefly explained.

- **Klaus Niederkrüger (Université de Toulouse, France): Higher dimensional contact topology via holomorphic disks**

  We will focus on fillability questions of higher dimensional contact manifolds. We start with an overview of some basic examples and theorems known so far, comparing them with analogous results in dimension three. We will also describe an easy construction of non-fillable manifolds by Fran Presas. Then we will explain how to use holomorphic curves with boundary to prove the fillability results stated earlier. No *a priori* knowledge of holomorphic curves will be required, and many properties will only be quoted.

- **Gordana Matić (University of Georgia): Contact invariants in Floer Homology**

  In a pair of seminal papers Peter Ozsváth and Zoltan Szabó defined a collection of homology groups associated to a 3-manifold they named

Heegaard-Floer homologies. Soon after, they associated to a contact structure ξ on a 3-manifold, an element of its Heegaard-Floer homology, the contact invariant $c(\xi)$. This invariant has been used to prove a plethora of results in contact topology of 3-manifolds. In this series of lectures we introduce and review some basic facts about Heegaard Floer Homology and its generalization to manifolds with boundary due to Andras Juhász, the Sutured Floer Homology. We use the open book decompositions in the case of closed manifolds, and partial open book decompositions in the case of contact manifolds with convex boundary to define contact invariants in both settings, and show some applications to fillability questions.

- **Robert Lipshitz (Columbia University, USA), Peter Ozsváth (Princeton University, USA) and Dylan Thurston (University of California, Berkeley, USA): Notes on bordered Floer homology**

  Bordered Heegaard Floer homology is an extension of Ozsváth-Szabó's Heegaard Floer homology to 3-manifolds with boundary, enjoying good properties with respect to gluings. In these notes we will introduce the key features of bordered Heegaard Floer homology: its formal structure, a precise definition of the invariants of surfaces, a sketch of the definitions of the 3-manifold invariants, and some hints at the analysis underlying the theory. We also talk about bordered Heegaard Floer homology as a computational tool, both in theory and practice.

- **Kai Cieliebak (Augsburg University, Germany) and Yakov Eliashberg (Stanford, USA): Stein structures: existence and flexibility**

  This survey on the topology of Stein manifolds is an extract from our book "From Stein to Weinstein and Back". It is compiled from two short lecture series given by the first author in 2012 at the Institute for Advanced Study, Princeton, and the Alfréd Rényi Institute of Mathematics, Budapest.

  The first part of this survey is devoted to the topological characterization of those smooth manifolds of real dimension greater than four that admit the structure of a Stein complex manifold. The second part discusses more recent results on the topology of Stein structures such as a Stein version of the h-cobordism theorem, a uniqueness theorem for subcritical Stein structures, and a remarkable class of "flexible" Stein structures that also satisfy uniqueness.

- **Michael Hutchings (University of California, Berkeley, USA): Lecture notes on embedded contact homology**

  These notes give an introduction to embedded contact homology (ECH) of contact three-manifolds, gathering together many basic notions which are scattered across a number of papers. We also discuss the origins of ECH, including various remarks and examples which have not been previously published. Finally, we review the recent application to four-dimensional symplectic embedding problems. This article is based on lectures given in Budapest and Munich in the summer of 2012, a series of accompanying blog postings at `floerhomology.wordpress.com`, and related lectures at UC Berkeley in Fall 2012. There is already a brief introduction to ECH in the article[1], but the present notes give much more background and detail.

- **Lenhard Ng (Duke University, USA): A topological introduction to knot contact homology**

  Knot contact homology is a Floer-theoretic knot invariant derived from counting holomorphic curves in the cotangent bundle of $\mathbb{R}^3$ with Lagrangian boundary condition on the conormal bundle to the knot. Among other things, this can be used to produce a three-variable polynomial that detects the unknot and conjecturally contains many known knot invariants; a different part of the package yields an effective invariant of transverse knots in $\mathbb{R}^3$.

  In these notes we will describe knot contact homology and the topology and algebra behind it, as well as connections to other knot invariants, transverse knot theory, and physics. Topics to be treated along the way include: Legendrian contact homology; the conormal construction; a combinatorial formulation of knot contact homology in terms of braids; the cord algebra, a topological interpretation of part of the invariant; transverse homology, a filtered version associated to transverse knots; and relations to the A-polynomial, the HOMFLY polynomial, and recent work in string theory.

[1] M. Hutchings: *Embedded contact homology and its applications*, Proceedings of the International Congress of Mathematicians, Volume II, 1022–1041, Hindustan Book Agency, New Delhi, 2010.

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