Preface

Geometric Control Theory and sub-Riemannian geometry are two areas whose fruitful interaction has been witnessed over the last decades.

On the one hand Geometric Control Theory used the differential geometric and Lie algebraic language for studying controllability, motion planning, stabilizability and optimality for nonlinear and linear control systems. Reflected in one of the contributions to the volume is the fact that the foundational result of optimal control theory – Pontryagin Maximum Principle – has differential geometric/Lie algebraic interpretation. The geometric approach turned out to fruitful in applications to robotics, vision modeling, mathematical physics etc. Current research in geometric control theory is concerned with polydynamic models, described by systems of nonlinear ODEs or PDEs with (control) parameters, or, geometrically speaking, by linear or affine subdistributions of tangent/cotangent bundles of manifolds of finite or infinite dimension.

On the other hand Rimemannian geometry and its generalizations, like sub-Riemannian, semi-Riemannian, Finslerian geometry etc., have been actively adopting methods developed in the scope of geometric control. Application of these methods has led to important results regarding geometry and topology of sub-Riemannian spaces, regularity of sub-Riemannian distances, properties of the group of diffeomorphisms of sub-Riemannian manifolds, local geometry and equivalence of distributions and sub-Riemannian structures, regularity of the Hausdorff volume etc.

Directions of active studies, partially reflected in the present collection are sketched below.

Geometric optimal control. This area is naturally drawn to invariant Hamiltonian formulations and use of the concepts and methods of symplectic geometry. Of particular use are the notions of Jacobi curve and Maslov cycle in Lagrangian Grassmanian, which are central for studying sub-Riemannian length minimization problems. Hamiltonian lifts to cotangent bundle allow for establishing second-order optimality conditions for extremals in optimal control problem with parameters. The same Hamiltonian approach, together with numerical schemes, is used for computation of conjugate and cut loci of metrics on Riemannian surfaces. Some integrability problems are addressed for Pontryagin-Hamilton optimality conditions for high-dimensional generalizations of Euler elastica and of Dubins's minimal path problem. Study of topology of configuration space of complex robotic systems allows to discover topological obstructions to continuous feedback stabilizability.

Geometry of sub-Riemannian manifolds. Natural range of issues is an extension of concepts and results of Riemannian geometry for sub-Riemannian manifolds. Those include, for example, curvature-dimension inequalities and Li-Yau-type estimates, as well as smoothness of length-minimizing curves in sub-Riemannian geometry, which is a long standing open problem. Curvature-type (feedback) invariants (introduced by A.A. Agrachev) can be computed for extremals of "least action principle" for natural mechanical systems on sub-Riemannian manifolds.

Classification problem for distributions is a long time challenge. It has been discovered some decades ago, that the geometry of distributions on manifolds can be characterized via Hamiltonian flows which define abnormal sub-Riemannian geodescis. In several symmetric cases the analysis of the abnormal geodesic flow can lead to a construction of canonical moving frames and to description of the moduli spaces for the rank-2 distributions in $\mathbb{R}^n$. Some conjectures regarding classification of (affine) line fields in $\mathbb{C}^n$ with transitive symmetry algebrae, are confirmed for $n = 2, 3$.

Analysis and topology of Carnot-Caratheodory spaces. Analysis in Carnot-Caratheodory spaces is a well established area, whose interaction with sub-Riemannian geometry and the geometric control theory is natural. Topics, illustrating such interaction include: intrinsic notions of Lipschitz maps and Lipschitz domains in Carnot groups, computation of Hausdorff dimension of sub-Riemannian manifolds and of Hausdorff volume of small balls in sub-Riemannian metrics, local approximation theorem for Carnot-Caratheodory spaces, comparison of various topologies in sub-Lorentzian manifolds.

Controllability and optimal control problems for PDEs. Geometric control for infinite-dimensional systems and PDE's is a rather new research area, whose differential geometric/Lie algebraic apparatus is yet to be created. Recent progress concerns the approximate controllability of the viscous Burger's equation on a line by means of trigonometric polynomial control, the null controllability property for parabolic Grusin equation with singular potential, the optimality of steady state modes for a model of exploitation of size-structured population.

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The Workshop has been dedicated to 60th anniversary of professor Andrei A. Agrachev, whose ideas are deeply influential in geometric control and adjacent areas. Many contributors of the present volume are his coauthors, former and current students, scholars inspired by his work in the above mentioned fields. On request of the editors professor A.Agrachev contributed a survey of some open problems in geometric control theory and sub-Riemannian geometry.

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