Preface

In this monograph we discuss the C^∞ well-posedness of the Cauchy problem for hyperbolic systems. We are mainly concerned with the following two questions for differential operators of order q with smooth $m \times m$ matrix coefficients:

(A) Under which conditions on lower order terms is the Cauchy problem C^∞ well posed?
(B) When is the Cauchy problem C^∞ well posed for any lower order term?

For scalar case, that is $m = 1$, the question (B) has been answered. As for the question (A), in particular for second order scalar equations, that is $m = 1$ and $q = 2$, so many works are devoted to this question and the situation is fairly well understood. Contrary to the scalar case, for systems that is if $m \geq 2$ we have no satisfactory result.

Even for differential operators with characteristics of constant multiplicity with real analytic matrix coefficients, the question (A) has been solved very recently.

So in this monograph, assuming that the coefficients are real analytic in a neighborhood of the origin, we study these two questions. Of course this analyticity assumption is rather restrictive but which allows us to make detailed studies on the Cauchy problem. We hope that this study can throw light on the studies of the Cauchy problem for hyperbolic systems with less regular, in particular C^∞ coefficients.

The contents are organized as follows. In Chap. 1 after giving the definition of C^∞ well-posedness of the Cauchy problem we show that the Cauchy problem for symmetric hyperbolic systems is C^∞ well posed for any lower order term. Then we give an example of first order 2×2 system which is not symmetrizable but for which the Cauchy problem is C^∞ well posed for any lower order term. Actually there is a class of non-symmetrizable systems for which the Cauchy problem is C^∞ well posed for any lower order term. This is a main objection when we try to answer to the problem (B). We prove the Lax-Mizohata theorem exhibiting naive ideas which are used in Chaps. 2 and 3. For first order systems with characteristics of constant multiplicities, the necessity of the Levi condition for the C^∞ well-posedness is proved which is used in Chap. 2. In Chap. 2, we study necessary conditions about the problem (B) for $m \times m$ first order systems with real analytic coefficients. We prove rather general necessary conditions in terms of minors of the principal symbols. Contrary to the scalar case the multiplicity of characteristics is irrelevant for the problem (B) since for symmetric or symmetrizable hyperbolic systems (of first order) the Cauchy problem is always C^∞ well posed for any lower order term. Here the maximal size of the Jordan blocks, which is supposed to measure the distance from diagonal matrices, plays an important role in the problem (B).

In Chap. 3, we study two questions (A) and (B) for first order 2×2 systems with two independent variables with real analytic coefficients. For this special case we can give a necessary and sufficient condition for the questions (A) and (B), that is, in this case we have complete answer for (A) and (B). The results provide many instructive examples. For instance, we can exhibit a first order 2×2 system with analytic coefficients which is strictly hyperbolic outside the initial line for which no lower order term could be taken so that the Cauchy problem is C^∞ well posed. This cannot happen for second order hyperbolic scalar operators with two independent variables with analytic coefficients.

In Chap. 4, we introduce a new class of hyperbolic systems, that is hyperbolic systems with nondegenerate characteristics which generalizes strictly hyperbolic systems. Strictly hyperbolic systems are hyperbolic systems with nondegenerate characteristics of order one. The theory of strictly hyperbolic systems is rich, but first order strictly hyperbolic system hardly exists. We prove that the Cauchy problem for hyperbolic systems with nondegenerate characteristics is C^∞ well posed for any lower order term. We also show that nondegenerate characteristics are stable, that is any hyperbolic system which is close to a hyperbolic system with a nondegenerate characteristic of order r has a nondegenerate characteristic of the same order nearby. This shows, in particular, that near any hyperbolic system with a nondegenerate characteristic of order $r \geq 2$ there is no strictly hyperbolic system, which gives a great difference from the scalar case and shows a complexity of hyperbolic systems.

We also discuss hyperbolic systems which are perturbations of symmetric systems and prove that if the dimension of the linear space that the symbol of the symmetric system spans is large enough, then generically such hyperbolic system is similar to a symmetric system.

Osaka, Japan
March 2013

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Hyperbolic Systems with Analytic Coefficients
Well-posedness of the Cauchy Problem
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2014, VIII, 237 p., Softcover
ISBN: 978-3-319-02272-7