Preface

Mathematical modeling plays a relevant role in many different fields: for instance meteorology, population dynamics, demography, control of industrial plants, financial markets, retirement funds. The few examples just mentioned show how, not only the expert in the field but also the common man, in spite of himself, has to take part in the debate concerning models and their implications, or even he is summoned to polls for the choice between alternative solutions that may achieve some reasonable control of the complex systems described by these models. So it is desirable that some basic knowledge of mathematical modeling and related analysis is spread among citizens of the "global village".

When models refer to time dependent phenomena, the modern terminology specifies the **mathematical model** with the notion of **dynamical system**.

The word **discrete**, which appears in the title, points out the class of models we deal with: starting from a finite number of snapshots, the model allows to compute recursively the state of the system at a discrete set of future times, e.g. the positive integer multiples of a fixed unit of time (the **time step**), without the ambition of knowing the phenomenon under study at every subsequent real value of time, as in the the case of **continuous-time models**.

Sometimes also the physical quantities under measure turn out to be discrete (in this cases they are called "quantized") due to experimental measurement limits or to the aim of reducing the volume of data to be stored or transmitted (signal compression). Since we focus our attention on models where the quantization has no relevant effects, we will neglect it.

In recent years a special attention has been paid to **nonlinear dynamical systems**. As a consequence, many new ideas and paradigms stemmed from this field of investigation and allowed a deeper understanding of many theoretical and applied problems. In order to understand the term **nonlinear**, it is useful recalling the mathematical meaning of **linear problem**: a problem is called linear if there is a direct proportionality between the initial conditions and the resulting effects; in formulae, summing two or more initial data leads to a response of the system which is equal to the sum of the related effects.

In general the theory provides a complete description (or at least a satisfactory numerical approximation) of solutions for linear problems, but the relevant problems in engineering, physics, chemistry, biology and finance are more adequately described by **nonlinear models**.

Unfortunately, much less is known about solutions of nonlinear problems. Often nonlinearity leads to qualitatively complex dynamics, instability and sensitive dependence on initial conditions, to such an extent that some extreme behavior is described by the term **chaos.**

This spellbinding and evocative but somehow misleading expression is used to express the possible extreme complexity in the whole picture of the solutions. This may happen even for the simplest nonlinear deterministic model, as the iteration of a quadratic polynomial. So we are lead to an apparent paradox: in such cases the sensitive dependence on initial conditions (whose knowledge is always affected by experimental error) leads to very weak reliability of any long-term anticipation based on deterministic nonlinear models. The phenomenon is well exemplified by weather forecast; nevertheless short-term forecasting has become more and more affordable now, as everybody knows.

These remarks do not call into question the importance and the effectiveness of nonlinear models analysis, but they simply advise not to extrapolate in the long term the solutions computed by means of nonlinear models, because of their intrinsic instability.

The present volume aims to provide a self-contained introduction to discrete mathematical modeling together with a description of basic tools for the analysis of discrete dynamical systems.

The techniques for studying discrete dynamical models are scattered in the literature of many disciplines: mathematics, engineering, biology, demography and finance. Here, starting by examples and motivation and then facing the study of the models, we provide a unitary approach by merging the modeling viewpoint with the perspective of mathematical analysis, system theory, linear algebra, probability and numerical analysis.

Several qualitative techniques to deal with recursive phenomenon are shown and the notion of explicit solution is given (Chap. 1); the general solutions of multi-step linear difference equations are deduced (Chap. 2); global and local methods for the analysis of nonlinear systems are presented, by focussing upon the issue of stability (Chap. 3). The logistic dynamics is studied in detail (Chap. 4).

The more technical vector-valued case is studied in the last chapters, by restricting the analysis to linear problems.

The theory is introduced step-by-step together with recommended exercises of increasing difficulty. Additional summary exercises are grouped in specific sections. The worked solutions are available for most of them in a closing chapter. Hints and algorithms for numerical simulations are proposed.

We emphasize that discrete dynamical systems essentially consist in the iteration of maps, while computers are very efficient in the implementation of iterative computations. The reader is invited to do the exercises of the text, either by looking for a closed solution (if possible), or by finding qualitative properties of the solution via graphical method and exploitation of the theoretic results provided by the text, or by numerical simulations.

Presently, many software packages oriented to symbolic computation and computer graphics are easily available; this allows to produce on the computer screen those entangling and fascinating images which are called fractals. It can be achieved by simple iterations of polynomials, even without knowing the underlying technicalities of Geometric Measure Theory, the discipline which describes and categorizes these non-elementary geometrical objects.

At any rate some basic notions for the study of these objects are given in Chap. 4 and Appendix F: fractal dimension and fractal measure.

For every proposed exercise, the reader is asked to ponder not only on the formal algebraic properties of the wanted solution, but also on the modeled phenomenon and its meaning, on the **parameter domain** where the model is meaningful, on the **reliability** of model predictions and on the numerical computations starting from initial data which are unavoidably affected by **measurement errors**.

In this perspective Mathematics can play as a feedback loop between the study and the formulation of models. If an appropriate abstraction level and a rigorous formulation of the model are achieved for one single problem, then innovative ideas may develop and be applied to different problems, thanks to the comprehension of the underlying general paradigms in the dynamics of solutions. In short: understanding the whole by using the essential.

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Ernesto Salinelli
Franco Tomarelli

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