

Chapter 2

Primitive Beginnings, Circa 1667

Abstract: Although the name of the person who prepared this seven-page manuscript, and of the place where it was prepared, are unknown, the last owner indicated that the manuscript was purchased from a very long-established family in Maine, New England. Importantly, the paper is clearly watermarked, and the watermark indicates the paper was made in 1666 or 1667. We have assumed that the paper was brought from Europe to New England during the second half of the 1660s, and that the writing in the manuscript was done in New England. If that was indeed the case then this is easily the oldest of known extant North American cyphering books and, in that sense, is extraordinary. A second notable feature of the manuscript is that the mathematics and spelling are at a very low level. Two other interesting features of the manuscript are: (a) the person who prepared it seemed to want to be able to claim that he or she had cyphered to the rule of three because, all of a sudden, that rule is mentioned on the last page; and (b) one of the topics mentioned was “halving,” which was *not* a normal part of the *abbaco* curriculum sequence for arithmetic, but *was* part of the Sacrobosco curriculum sequence for arithmetic.

Key Words: *Abbaco* curriculum sequence; arithmetic; ciphering book; ciphering tradition; cyphering book; cyphering tradition; rule of three; Sacrobosco.

This chapter will describe, and assess the historical significance of, a seven-page manuscript which might be the oldest extant cyphering book prepared in North America. The document became part of our collection of North American and British cyphering books in June 2013 when we purchased it from an online auction. The manuscript had been advertised in the following way:

This is a curious and quite possibly ultra-rare early (c. 17th century) American mathematics lesson book, handwritten on watermarked, hand-laid paper!

Measuring 8” by 6,” and spanning 16 pages, 7 of which contain manuscript handwriting. Language is Olde English, with handwriting characteristics undoubtedly mid 1600s–early 1700s. Recovered from a Camden, Maine, auction; assumed American through association, as other documents were all American in the sale. No further identifying characteristics other than watermark consisting of a crest with a crown, and a central cross, as shown. Paper with some staining, creasing and folding, held by original strings loosely. A few interesting doodles accompany the mathematical rules, as shown as well. Quite possibly the earliest cypher style manuscript we have seen.

The advertisement was accompanied by photographs of a few pages of the manuscript and, after looking carefully at those photographs, our intuitions told us that, yes, this did seem to be a manuscript that was prepared some time during the period from the mid-1600s to the early 1700s.

After purchasing the manuscript, we were surprised how little handwriting there was on many pages. Taking the lead from the advertisement, we examined the watermarks in the paper, and two clear watermarks enabled us to identify, through the Gravell Watermark Archive, that the paper could be dated to 1666 (referenced as Watermark ARMS.047.1). Unfortunately, the name of the person who prepared the manuscript, and the place where it was prepared, were not given and,

almost certainly, will never be known. However, taking note of the seller's statement that it was purchased at an auction in Camden, Maine, and that it was among other very early American documents, we shall assume that the manuscript was prepared in New England in the late 1660s. We shall also assume that the paper on which the handwriting occurred had been brought to New England from Europe, either by an early settler from Europe or by a commercial exporter. Admittedly, those are large assumptions. However, we recognize that the manuscript would still be interesting if the handwriting was done, totally, in Great Britain before the manuscript was, somehow, and at some unknown time, brought to North America.

If this manuscript was prepared in North America in the 1660s then it is easily the oldest North American cyphering book that we have seen or heard of. And that would justify the use of the word "extraordinary" to describe it. But we have a second reason for regarding the manuscript as "extraordinary." Of all the cyphering books that we have ever seen, this is, mathematically speaking, the *worst*—although, perhaps, "worst" might not be the most appropriate word to describe it. Why we say that will become clear as we examine its pages—which we shall now do.

The Text of the Manuscript

The sections on the pages are numbered, with the first section being numbered 15 and the last 23. Section 21 is missing. Sections 19 and 20 occupy the same page in the handwritten manuscript, and the same is true of Sections 23 and 24. No page has a large amount of handwriting, and several pages have ruled lines (in ink), to guide the writing.

The pages are reproduced in [Figures 2.1](#) through [2.7](#).

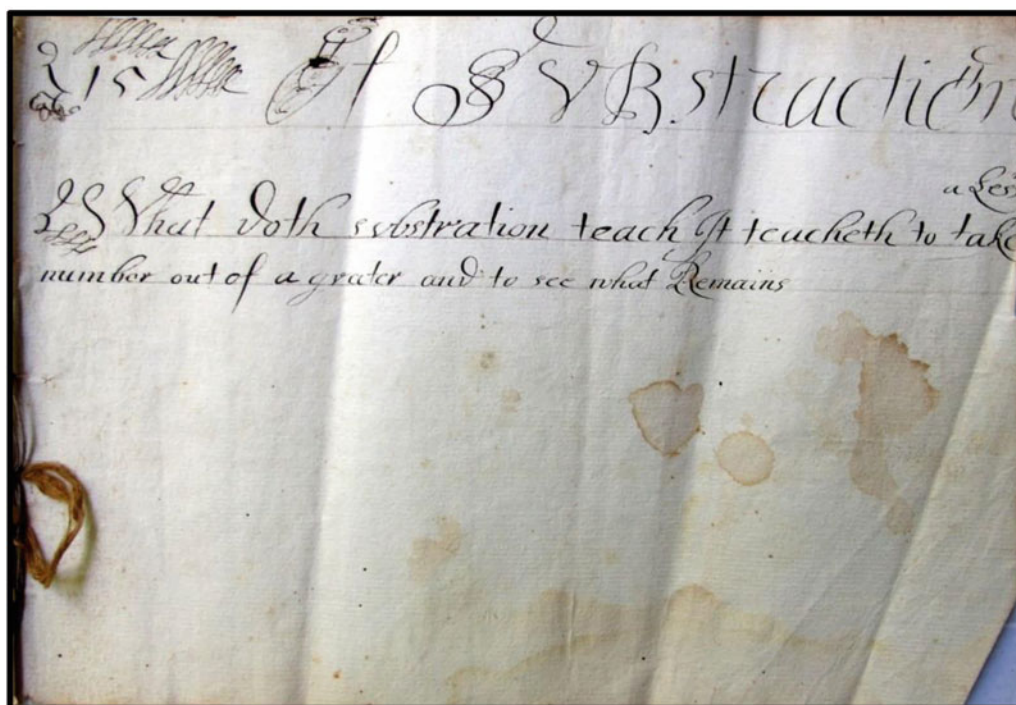


Figure 2.1. "Substraction" c. 1666 (Section 15).

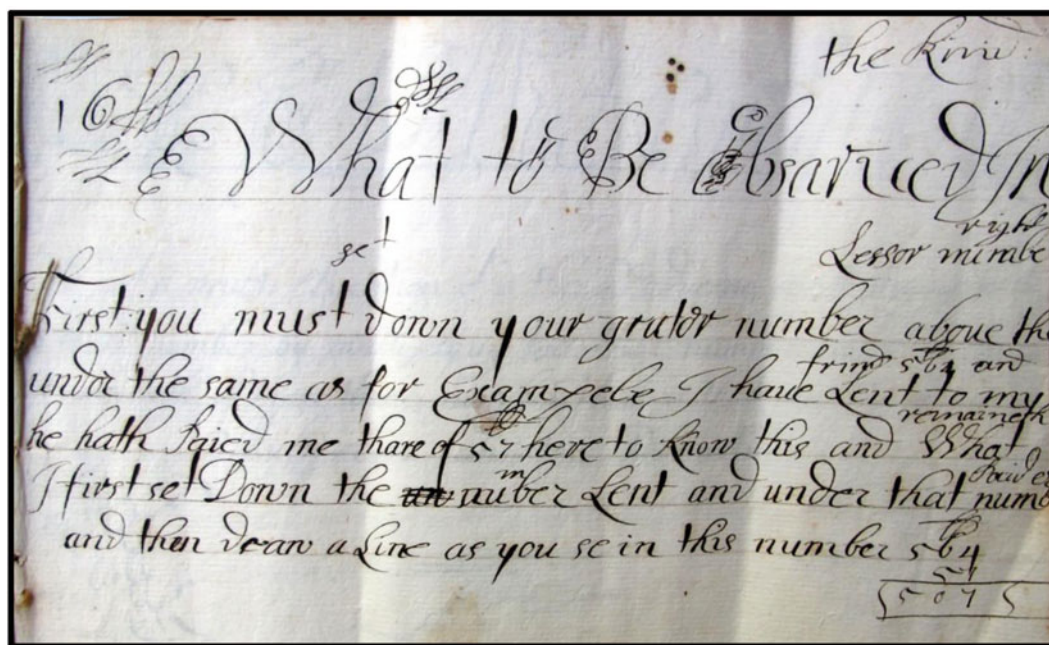


Figure 2.2. "What to be Carried in" (Section 16).

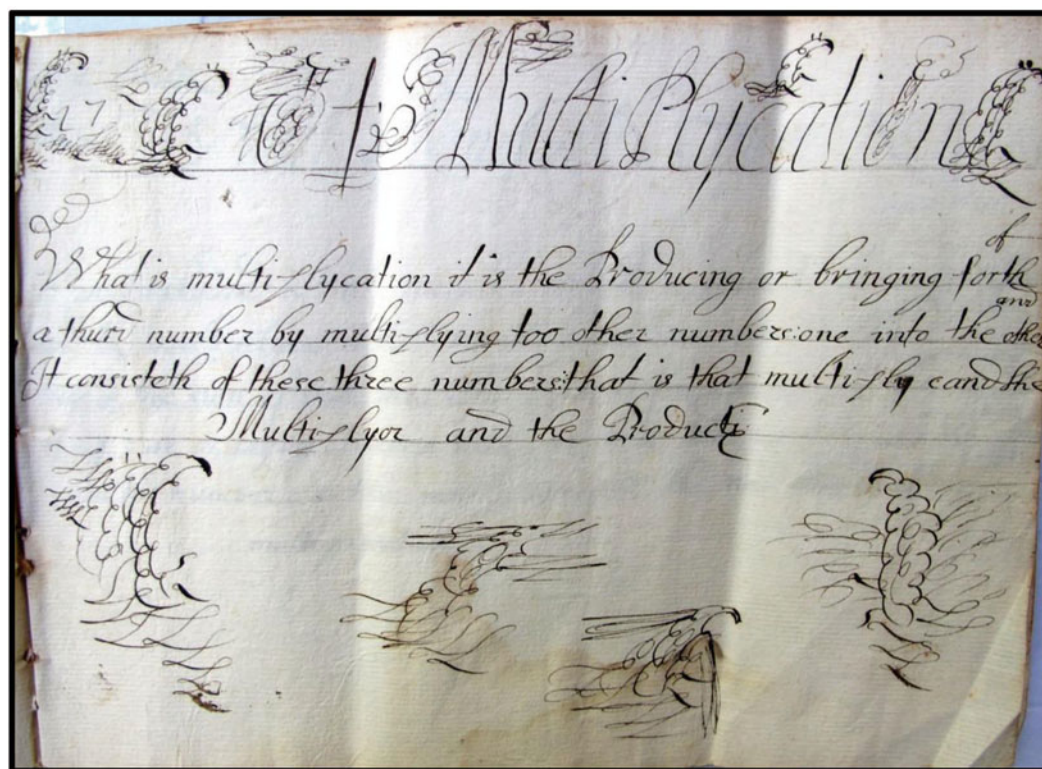


Figure 2.3. "Multiplikation" (Section 17).

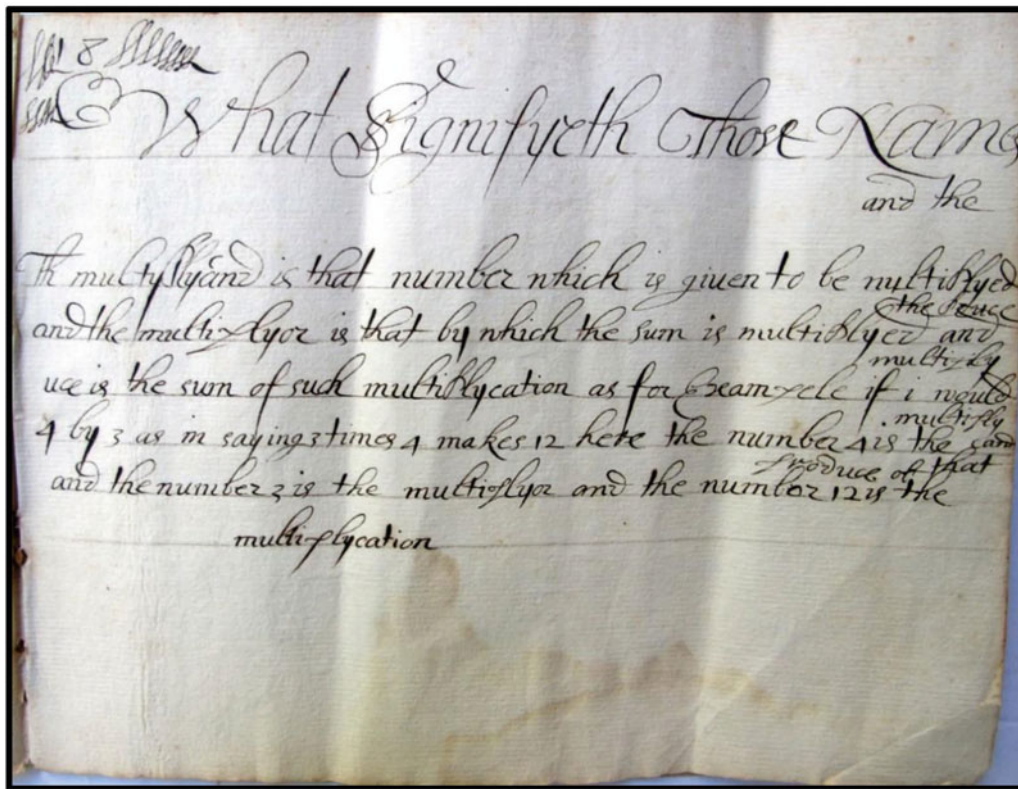


Figure 2.4. "What Signifyeth Those Names" (Section 18).

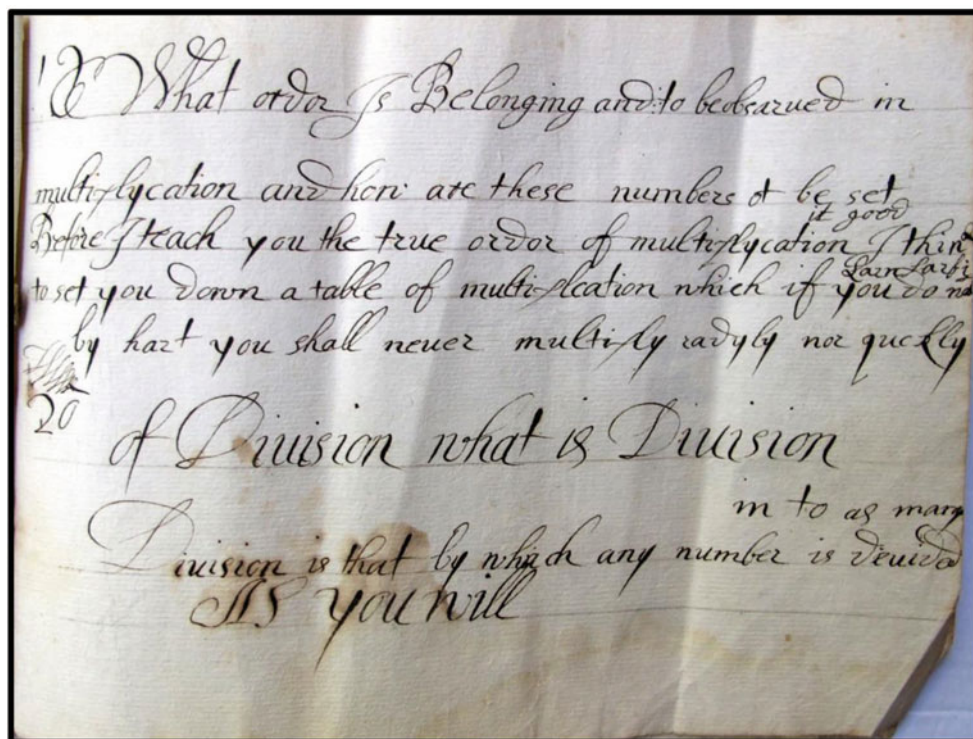


Figure 2.5. "What order is Belonging and to be observed in multiplication and how are these numbers to be set," and "of Division what is Division" (Sections 19 and 20).

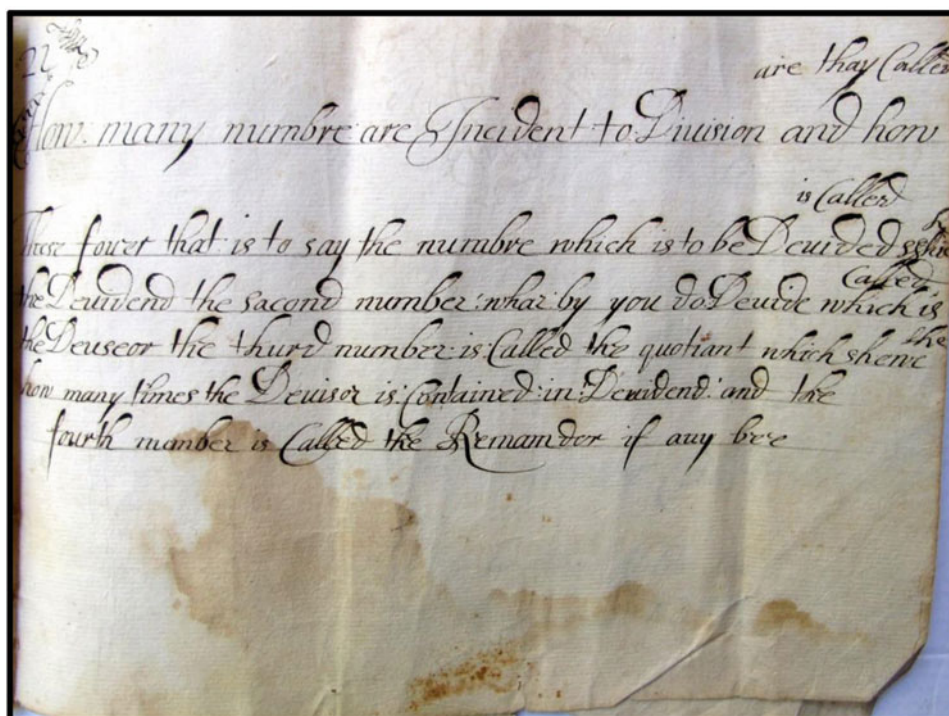


Figure 2.6. "How many numbres are incident to Division and how are they called" (Section 22).

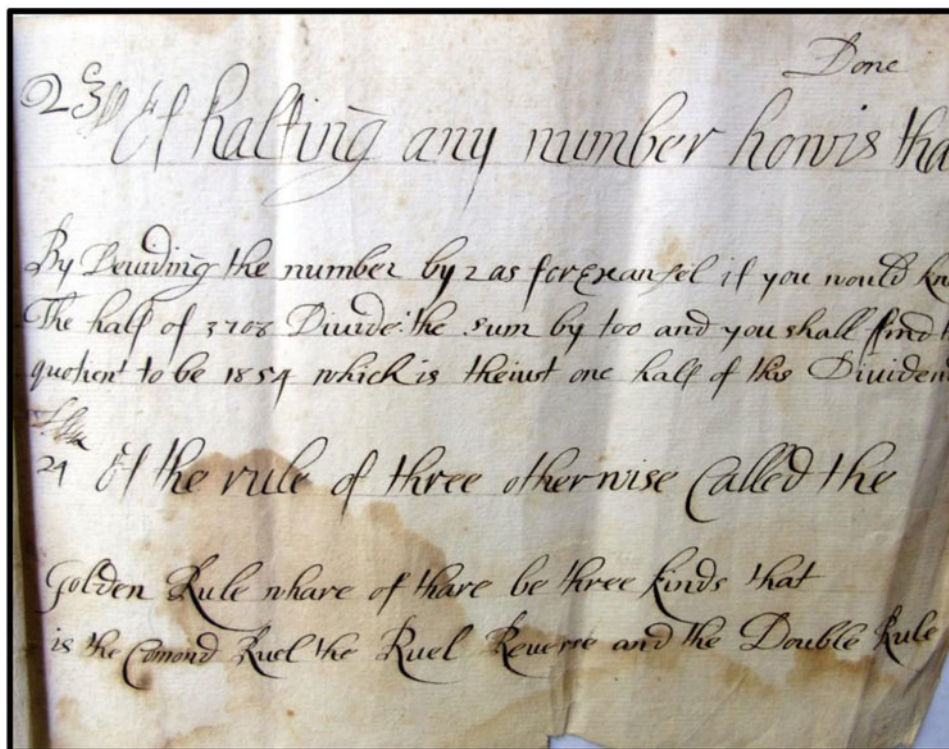


Figure 2.7. "Of halving any number how that is Done," and "Of the rule of three otherwise called the Golden Rule" (Sections 23 and 24).

For readers who found it difficult to read the text in any or all of [Figures 2.1](#) through [2.7](#), the following provides completely unedited versions of what appears on the pages. Occasionally we could not decipher a particular word, and in those cases we have followed the word with (?). Since no question mark was used in the manuscript text, no question mark is included in these transcripts.

Section 15: This was headed “Substraction, a Lesson.” The text asked “What doth subtraction teach,” and answered with “It teacheth to take number out of a grator and to see what remains.”

Section 16: The heading was “What to be Carried in.” The text stated: “First you must down your grator number above the lesser number under the same as for example I have lent to my friend 564 and he hath saied me thare of 57 here to know this and what remaineth. I first set down the number lent and under that number paid, and then draw a line as you se in this number

$$\begin{array}{r} 564 \\ \underline{57} \\ 507 \end{array}$$

Section 17: The heading was “Multiplycation.” The text asked: “What is multiplycation” and the answer given was: “It is the producing or bringing forth of a third number by multiplying too other numbers one into the other. It consisteth of these three numbers: that is that multiplycand, the Multiplier and the Product.” There were numerous unrelated doodles on this page.

Section 18: The heading was “What Signifyeth Those Names.” The text responded to this question with: “The multiplycand is that number which is given to be multiplyed and the multiplier is that by which the sum is multiplyed and the produce is the sum of such multiplycation as for exampele if i would multiply 4 by 3 as in saying 3 times 4 makes 12 here the number 4 is the multiplycand and the number 3 is the multiplier and the number 12 is the produce of that multiplycation.

Section 19: The heading was “What order is Belonging and to be obsearved in multiplycation and how are these numbers to be set.” Then followed: “Before I teach you the true order of multiplycation I think it good to set down a table of multiplication which if you do not Larn [learn] by hart you shall never multiply radyly [readily] nor quckly.”

Section 20 (which is on the same page as *Section 19*, and immediately below the notes for *Section 19*). Following the heading, “Of Division what is Division,” the response was “Division is that by which any number is deuided in to as many as you will.”

Section 22: The heading was “How many numbers are Incident to Division and how are they called.” Then followed: “These fouer [Therefore?] that is to say the numbre which is to be deuided is called the Deuidend, the sacond number whar by you do Deuide which is called the Deuseor the thurd number is called the quotient which shewe the how many times the Deuisor is Contained in Deuidend and fourth number is called the Remamder if any bee.”

Section 23: The heading was “Of halving any number how is that Done.” Then followed—“By Deuiding the number by 2 as for exempel if you would know the half of 3708 Divide the sum by too and you shall find the quotient to be 1854 which is the inst [instantly?] one half of this dividend.” Then followed: *Section 24:* “Of the rule of three otherwise called the Golden Rule whare of thare be three kinds that is the Comond Ruel and Ruel Reverse and Double Rule.” [Note that in this last statement we have “Rule” three times and “Ruel” twice.]

Brief Commentary on the Text

Although Cohen (1982) claimed that in the 18th century local schools in the North American European settlements did not teach arithmetic as part of their regular course of instruction, Monroe (1917) named quite a few 17th-century local schools that offered cyphering, and it is likely that this practice would have continued into the 18th century. Arithmetic skills, and the associated penmanship skills, were obviously useful in colonial society. Money (1993) reported that in 1700 there were at least 17 private tutors within the European settlements in North America who advertised that they were willing to receive students for cyphering, and that between 1720 and 1740 this number grew to 49. So, it is quite feasible that the manuscript under examination in this chapter was prepared in one of the very early New England schools.

The first of the seven pages shows Section 15. It is likely that the notes were copied from, or perhaps dictated from, an existing manuscript—perhaps from another cyphering book—and that the number “15” on the first page corresponded to the number of a statement or section on one of the pages in that other document. That said, if 14, or so, earlier sections did exist, at some time, there can be little doubt that Sections 1 through 14 would have been concerned with “numeration,” and with “addition,” for within cyphering books those topics almost always preceded “subtraction.”

In the heading “Substraction, a Lesson” in Section 15, the word “substraction” rather than “subtraction” was used. Although one can find “substraction”—instead of subtraction—in some cyphering books, the correct word, in English, was then (as it is still) “subtraction.” It seems that sometimes there was confusion regarding the spelling because the corresponding French word was “soustraction,” which had an “r” after the first “t.”

After the heading on Section 15, the text asked “What doth subtraction teach.” The answer given was: “It teacheth to take a number out of a grator and to see what remains.” Such a definition would not be acceptable in the 21st century because for centuries it has been conceptually possible to subtract a larger number from a smaller number. But around 1666 that possibility was not contemplated.

It is difficult to make sense of the notes on Section 16 regarding how to subtract. However, the use of words like “Carried” and “lent” suggest that the person who prepared the manuscript was trying to find the words to describe a standard subtraction algorithm. From that point of view it is interesting that the one example given, $564 - 57$, could involve “carrying.” It seems that even at this early stage a language involving lending and carrying had been developed in an attempt to facilitate the teaching and learning of subtraction.

Sections 17, 18 and 19 were all concerned with elementary multiplication. The emphasis on appropriate language (multiplicand, multiplier, and product) points to an attempt to analyze the process of multiplication. Although no multiplication table is given, the comment that one would need to learn a multiplication table by heart if one wanted to be able to multiply “readily” and “quickly,” would be heard over the centuries—whenever and wherever children were being asked to learn to multiply.

The distinction, in Section 22, between four components of division (i.e., dividend, divisor, quotient, and remainder) was another example of an apparent emphasis on appropriate terminology. The actual process of division was not described, and one wonders whether the person who prepared the cyphering book ever learned how to carry out formal divisions.

In Section 23, the issue of “halving” is raised, and an actual example (half of 3708) is presented, and the answer (1854) given. However, other than to say that one needed to divide by 2, there was no discussion of how the answer would be obtained. The emphasis on finding one-half of a number was part of Sacrobosco’s curriculum—he called the process of finding one-half, “mediation”

(Yeldham, 1926, p. 77). Although that process was never an important component of the *abbaco* curriculum defined by Leonardo of Pisa (Van Egmond, 1976), throughout the period 1200–1800 doubling and halving were dealt with in a minority of arithmetic textbooks across Europe (Jackson, 1906). Jackson (1906) stated that most contemporary commentators regarded doubling as simply another example of multiplication, and therefore did not see any need for it to be introduced as an independent topic. Indeed, one commentator wrote: “I do not understand what influences those stupid ones, since to double is to multiply by 2 and to halve is to divide by 2” (quoted in Jackson, 1906, p. 76). However, although, from a purely logical point of view the commentator made a valid point, from a psychological perspective it is well known that for children there is something special about doubling and halving.

The statement in Section 24 concerned the “golden rule,” or the “rule of three.” As the statement indicated, there was a “common” (or direct) rule of three, a “reverse” rule of three (the more common word in this context was “inverse”), and a double rule of three. No actual statements of these various rules of three were written down in this cyphering book, and one wonders whether the person who prepared the cyphering book ever actually heard, or learned to apply, any of the rules. For centuries it was a mark of prestige to have studied “up to the rule of three,” and one suspects that the person who prepared this cyphering book wanted to be able to say that he (or she) had done that.

The Historical Significance of the Manuscript

Our decision to include this seven-page manuscript as one of 11 “extraordinary” cyphering books for analysis in this book was made on the basis that, despite its brevity, it qualifies as extraordinary for two reasons. First, it is very early—and if indeed it was prepared in Maine in the late 1660s, then it is easily the oldest of the known extant North American cyphering books. And, second, the quality of the mathematical discussion presented in the book is so very low—easily the lowest we have seen in any cyphering book. That is saying something, considering we (Ellerton and Clements) have carefully examined about 1000 cyphering books.

Let us assume, for the moment, that this cyphering book is the oldest extant cyphering book prepared in North America, and that the book was actually prepared somewhere in New England. Because of the British background of most settlers in New England, one would expect that it would not have taken long for the cyphering approach to arithmetic education, so widely used “at home,” to be transplanted in the early New England settlements. However, such an approach was less likely to be as successful in the New World as it had been in Great Britain. In early New England none of the main requirements for success—necessary *mathematical knowledge* among teachers, sufficient *teaching power*, appropriate *student readiness* for learning arithmetic, and adequate *resources*—was satisfied (Ellerton & Clements, 2012). Furthermore, the early settlers in New England had not expected to have to endure the harsh winters which befell them. For them it could have been helpful to find a potentially useful activity—like cyphering—to occupy the minds of boisterous youngsters who found themselves more or less compelled to remain in small homes with not much to do.

Early North American settlements and states were not ready for a problem-solving mathematics regime like that followed by Thomas Dixon (see Chapter 7). Most of the settlers had had very little formal training on anything to do with mathematics and there were relatively few, if any, persons with sufficiently strong mathematical backgrounds to be in a position to teach mathematics effectively. Of the few who might have known something about mathematics, the demands of daily living in a new and difficult world could have precluded their having the time to take seriously the challenge of teaching formal mathematics to their children (Cohen, 2003). This state of affairs would

have had an ongoing effect whereby because children did not learn much mathematics beyond mere counting, the next generation, the children's children, would not have had access to persons who could teach them mathematics beyond counting. The only mathematics textbooks were those brought over from the "mother country," and these were both difficult to get and expensive, in the New World. Little wonder, then, that very few of the children growing up in New England learned more than the absolute rudiments of elementary mathematics—beyond being able to read and write the Hindu-Arabic numerals which appeared on most hornbooks.

That would explain why the quality of mathematics in the cyphering book examined in this chapter was so low. Putting the matter succinctly, it is likely that neither the student nor his or her teacher (if a teacher was available) would have known much about the four operations on numbers, especially multiplication and division. But even in these early times, cyphering "up to the rule of three" was thought to be something desirable to do, and so this cyphering book closed with the student being able to say to himself (or herself) that he (or she) had reached the rule of three.

Undoubtedly, the approach to mathematics in the seven pages emphasized form rather than substance. With division, for example, the emphasis was on knowing the meanings of terminology like "dividend," "divisor," "quotient" and "remainder," but not on learning how to find the actual values of the quotient and the remainder.

It would be unfair to be completely negative with respect to the cyphering book under examination. Whoever prepared the pages of this book wrote in pages which originated in Europe. In the New World paper was expensive, and even the task of getting suitable quills with which to write, and of making suitable ink, was difficult and time-consuming. The pages were watermarked, and likely to have been expensive. The decision within a New England community in the 1660s to use such pages for notes on arithmetic pointed to a community which valued education.

The person who prepared the seven pages had actually learned to write, which of itself was no mean feat within an emerging culture in which opportunity for significant written forms of education was often lacking. Being able to write words like "multiplycand," and "halfing," and "deusior," however imperfect those words might have been, represented an achievement. To cypher "up to the rule of three" was the aim and, most likely, the person who prepared this book would have claimed that he (or she) had done that. In its own way, this cyphering book might represent the beginnings of formal mathematics education in the New World.

There is an old saying: "Well begun half done." It is doubtful whether the quality of the cyphering book examined in this chapter justified the expression "well begun." But at least there had been a beginning. The very existence of this manuscript testified to a determination among early colonists that their children should learn to read, to write, and to cypher. The New World was one of new horizons, and it seems that at least some parents wanted their children to experience forms of education that went beyond the child-minding, dame-school approach, with its hornbook culture. From that viewpoint, this small 1660s manuscript represented an important step forward.

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