Chapter 2
Generalities and State-of-the-Art on the Control of Underactuated Mechanical Systems

> *... I only wanted to expose, in this work, what I managed to do at this moment in time and which may be used as a starting point for other research of the same kind.*
>
> *M.A. Lyapunov*

Ever since time began, mankind has never stopped dreaming about traveling from one continent to another and about flying like a bird, exploring the depths of the ocean and conquering space. His ambitions have compelled him to search for and to realize and to even improve the means that will permit him to realize his objectives. Furthermore, it would be difficult or even impossible to achieve such objectives without having recourse to mechanical systems. Even though the research interest in mechanical systems goes far back to the time of Newton, Lagrange, Kepler, Hamilton, and many other famous researchers, actually this area of research is even more active due to its diverse applications in real life and in the industrial domain.

In fact, during the last few decades, a number of scientific, industrial, and military applications have instigated the analysis and the rigorous derivation of control algorithms for mechanical systems. This area of research has also attracted the attention of mathematicians since the majority of the systems possess a global nonlinear characteristic, and their linear approximation seems to be inadequate. In combining their efforts, the engineers and scientists have developed several control design methodologies that include linear control, optimal control, adaptive, and nonlinear control, and more recently robust control in order to take into account uncertainties in a practical and real life context. In fact, the interest in mechanical systems became even stronger when researchers realized that the latter can be underactuated.

2.1 Underactuated Mechanical Systems: Generalities and Motivations

A mechanical system is said to be underactuated when the number of control inputs is less than the number of degrees of freedom to be controlled. This class of systems has a varied and rich applications, at both the practical and the theoretical level, in various fields such as in robotics, aeronautical and spatial systems, marine and

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underwater systems, and flexible and mobile systems. In contrast to systems that have direct practical applications, the pendulum systems, the Acrobot, the Pendubot, the Tora and the ball and beam systems have a meaning in terms of benchmarks for nonlinear control where classical procedures cannot be applied.

The underactuation can be due to one of the following reasons [40]:

(i) It can be natural due to the dynamics of the systems such as those of aircrafts, helicopters, and underwater vehicles.
(ii) It can be imposed by design in order to reduce the costs and weight such as satellites with two thrusters and flexible-link robots.
(iii) It can be due to actuators' failure such as in aeroplanes and ships.
(vi) It can be artificially imposed in order to generate low-order complex nonlinear systems so as to gain insight on the control of high-order UMSs such as the inverted pendulum and all the above benchmark examples mentioned above.

The restriction of the control authority renders the control of these systems rather complicated. In some sense, the underactuation characteristics are even more difficult to handle than the nonlinear characteristics of the underlying system. As a matter of fact, some well-established results and properties for nonlinear systems such as linearization by feedback, passivity and matching condition are not generally valid in the case of UMSs. Furthermore, these systems show other undesirable properties like an undetermined relative degree or non-minimal phase behavior.

On the other hand, several UMSs present a structural obstruction to the existence of smooth and time-invariant stabilizing feedback control laws, since they do not satisfy the well-known and necessary condition of Brockett [11] for smooth time-invariant feedback stabilization, which is one of the most remarkable contributions in this area. Typically, a first indication of this obstruction comes from the fact that the linearization of these systems around any equilibrium point is uncontrollable, particularly in the absence of gravity terms. Hence, false conclusions on the controllability can be easily drawn.

Although these control difficulties suggest that the objective of asymptotic stabilization is, without any doubt, too demanding for the control of UMSs, the very existence of these systems and the theoretical challenges they present have forced many researchers to fully investigate that topic. In addition, mastering the control of these systems can transform their shortcomings into advantages. In effect, for the same configuration space, a fully actuated system requires more controls than if it were underactuated. This increases the weight and cost of the system. Finding the means to control a version of an underactuated system allows to eliminate certain control devices, improves global performances, and reduces the cost of realization.

Additionally, underactuation provides a control solution for the safety of systems. For example, if a fully actuated system becomes faulty and if we have an underactuated control system, then we can use the latter in critical situations (as for example in the case of a fault in one of the thrusters of an aeroplane, rocket or space engine) in order to avoid complete failure of the system or mission. Obviously, such a solution is more economical than the addition of redundant actuators.

On the other hand, UMSs has been studied on a case by case basis due to the difficulty in putting forward sufficiently general and exploitable structural properties in order to classify them according to their corresponding properties, and, at the same time, to be able to choose the appropriate control strategy according to their classification. Hence, there have been various research works on the control synthesis and strategies of control for these systems.

2.2 Brief State-of-the-Art on the UMSs Control

The aim of this section is not to give a complete account on the literature on the control of UMSs but to highlight the main contributions in this area.

Among the most recognized works, there are those based on the energy point of view. These are mainly the works of Astrom, Bloch, Furuta, Spong, and others [3, 5, 7, 8, 10, 17, 27, 29, 34, 60–62].

In these works the general control strategy is to swing the systems (mainly of pendular types such as the Acrobot, the Pendubot, inertial wheel pendulum) in order to bring them to the neighborhood of their linearity domain. Once this domain is attained, a switch towards a linear control of LQR type or pole placement is realized.

In a similar fashion, certain passivity-based methods also consist in swinging or steering the previous systems but this time in order to bring them to their homocline orbits. After that, a switch towards a linear control is realized such as in the works of Fantoni, Ortega and Spong in [18, 41, 43, 60]. Other work on passivity due to Janković and Sepulchre relates to the transformation of the systems in a cascaded form [31, 56] such as for the Tora system or for the Pendubot, as in the work of Kolesnichenko [32].

Most of the time, the authors do not deem it necessary to establish a stability proof of the system with switch. Additionally, the application domain of these methods are quite restrictive in real applications.

Because of its complexity, the ball and beam system has been the subject of several studies, namely by using: methods of approximate linearization by Hauser et al. [25], saturation for stabilization of cascaded system in feedforward by Teel [65], stabilization by output feedback of Teel and Praly [66], small gains synthesis by Sepulchre [55] and sliding mode control by Voytsekhovsky and Hirschorn [68].

The VTOL (vertical take-off and landing aircraft) is another example of UMS that is largely studied, namely for its industrial applications and for its non-minimum phase property [18, 26, 36] and [14, 39].

Due to their wide application in industry, cranes, and inertia wheel pendulums have been studied extensively. Reviews on models, applications, and control strategies are presented and discussed, respectively, in [1] and in [9].

Marine and underwater vehicles have also been the subject of numerous research. For instance, a smooth and continuous control allowing to exponentially reach a desired position and orientation has been introduced by Egeland [15]. A periodic control that asymptotically stabilizes the vehicle to the origin has been presented by Pettersen and Egeland [45]. In addition, inspired by the work of Morin and Samson [37], Pettersen and Egeland [46] have proposed a periodic and non-stationary control allowing to obtain an exponential stability of the underactuated marine vehicle. Then, Pettersen and Nijmeijer [47] have proposed a time-varying control law that led to a global and practical tracking and stabilization of the underactuated marine vehicle. The work of Ghommam [21] formulates and solves dynamic control positioning problems and trajectory tracking of underactuated marine vehicles.

In addition to the problem of stabilization of UMSs, the problem of trajectory tracking has also been tackled in the works of Bullo, Hu, Reyhanoglu and Sandoz, [35, 44, 48, 52, 72]. On the other hand, some researchers focused their attention to the case where the condition of Brockett (on the stabilizability of nonlinear systems using time-invariant continuously differentiable state feedbacks) is not satisfied and have proposed discontinuous control algorithms. Among these works, we can cite those of Oriolo and Nakamura and those of Reyhanoglu [42, 49, 50].

Other control strategies have also be derived such as: backstepping and forwarding procedures by Gronard, Sepulchre and Seto [23, 56, 57, 71]; sliding mode control by Fridman, Fahimi, Khalil and Su [2, 16, 38, 64, 68, 70]; hybrid and switching control by Fierro, Tomlin and Zhang [19, 48, 67, 73], optimization-based design by [53, 54, 63], inverse dynamics control and differential flatness by [4, 6, 20, 51, 58], and fuzzy logic and neural networks by Han, Lin and Wai [24, 33, 69].

Recently, some researchers have been interested in the control of biped robots. For this one can cite the work of Chevallereau [13], Chemori [12], and that of Spong [22, 28, 30, 59].

2.3 Scope and Objectives of This Book

One can clearly notice that all the previous aforementioned systems have been studied on a case by case basis. Based on that observation, the main objective of this book is to attempt to find and present the means that will permit the synthesis of control laws in a systematic manner for all UMSs but not necessarily with the same type of control. To meet this objective, it is quite intuitive to look for common (or even different) properties of UMSs that will permit to classify them.

This book also aims to gather existing classifications for UMSs in the literature. In fact, there exist two such classifications. The first classification is due to *Dambing Seto and John Baillieul* [57], which is of a graphical nature. It consists in tracing the Control Flow Diagram (CFD) of the given system and describes the ways the control inputs are transmitted through the degrees of freedom. According to this approach, three main structures are identified, namely: the chain structure, the tree structure, and the isolated vertex (or point) structure.

The combination of these structures yields seven structures for this classification. The authors of this classification have proposed a systematic control procedure of backstepping type that can globally and asymptotically stabilize the systems belonging to the chain structure. The stabilization problem for the other two classes are still open problems according to them.

The second classification is due to *Reza Olfati-Saber* [40] and is rather of an analytical nature. It considers structural properties of mechanical systems such as the actuation of certain degrees of freedom, the coupling between the inputs and the integrability of generalized momentums. Thus, eight classes are generated among which three are considered to be the principal ones, namely: the strict feedback normal form, feedforward normal form, and the non-triangular normal form.

The author of this classification has proposed a control design procedure in two steps for the first two normal forms: first to stabilize the reduced system and then to extend the stabilization to the global system by a backstepping or by a forwarding procedure depending on the considered normal form.

Some control design suggestions have been given for the third form. However, the procedure proposed for the stabilization of the reduced system requires the verification of a rather restrictive hypothesis.

This book tries to give some answers to the stabilization of the tree and isolated vertex structures based on the *Seto and Baillieul* classification. These two structures are more difficult to control but have the advantage (or shortcoming, depending on one's viewpoint) of representing the majority of UMSs.

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