

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Challenges in Numerical Simulation	1
1.2	Model Reduction: Information Versus Relevant Information.	4
1.2.1	Extracting Relevant Information: The Proper Orthogonal Decomposition.	4
1.2.2	Building the POD Reduced-Order Model.	5
1.2.3	Illustrating the Construction of a Reduced-Order Model	6
1.2.4	Discussion	9
1.3	The Proper Generalized Decomposition at a Glance	10
1.4	Towards a New Paradigm in Computational Science	12
1.5	A Brief Overview of PGD Applications	14
1.5.1	Multidimensional Models.	14
1.5.2	Separating the Physical Space	15
1.5.3	Parametric Models Defining Virtual Charts: A Route to Efficient Optimization, Inverse Identification and Real-time Simulation.	16
1.5.4	Real-time Simulation, DDDAS and More	18
1.5.5	Open Issues and Further Developments	20
	References	21
<b>2</b>	<b>PGD Solution of the Poisson Equation</b>	<b>25</b>
2.1	Progressive Construction of the Separated Representation	26
2.1.1	Alternating Direction Strategy	27
2.1.2	Stopping Criterion for the Enrichment Process	30
2.1.3	Numerical Example.	32
2.2	Taking into Account Neumann Boundary Conditions.	34
2.3	Increasing the Complexity of the Case Study	39
2.3.1	Non-Constant Source Term	39
2.3.2	Non-Homogeneous Dirichlet Boundary Conditions	40
2.3.3	Higher Dimensions and Separability of the Computational Domain.	41

2.4	Numerical Examples . . . . .	42
2.4.1	2D Heat Transfer Problem . . . . .	42
2.4.2	High-Dimensional Problem . . . . .	44
	References . . . . .	46
<b>3</b>	<b>PGD Versus SVD.</b> . . . .	47
3.1	Singular Value Decomposition and On-the-Fly Data Compression Using the PGD . . . . .	47
3.2	The PGD for Constructing Separated Approximations . . . . .	49
3.3	The PGD Approximation Constructor in Action . . . . .	50
3.4	The PGD for Data Post-Compression. . . . .	53
3.5	Numerical Example . . . . .	56
	References . . . . .	56
<b>4</b>	<b>The Transient Diffusion Equation.</b> . . . .	57
4.1	The One-Dimensional Transient Diffusion Equation . . . . .	57
4.1.1	Progressive Construction of the Separated Representation . . . . .	58
4.1.2	Alternating Direction Strategy . . . . .	59
4.1.3	Non-Incremental Versus Incremental Time Integrations . . . . .	62
4.2	Multi-Dimensional Transient Diffusion Equation. . . . .	62
4.3	Numerical Example . . . . .	66
4.4	Concluding Remarks . . . . .	68
	Reference . . . . .	69
<b>5</b>	<b>Parametric Models.</b> . . . .	71
5.1	Material Parameters as Extra-Coordinates. . . . .	71
5.1.1	Numerical Example. . . . .	76
5.2	Boundary Conditions as Extra-Coordinates. . . . .	79
5.2.1	Neumann Boundary Condition as Extra-Coordinate. . . . .	79
5.2.2	Dirichlet Boundary Condition as Extra-Coordinate . . . . .	81
5.2.3	Dirichlet and Neumann Boundary Conditions as Extra-Coordinates . . . . .	82
5.2.4	Non-Constant Neumann Boundary Conditions . . . . .	82
5.2.5	Non-Homogeneous Dirichlet Boundary Conditions . . . . .	83
5.3	Initial Conditions as Extra-Coordinates . . . . .	84
5.4	Computational Domain as Extra-Coordinates . . . . .	87
	References . . . . .	87

<b>6 Advanced Topics</b>	89
6.1 Non-Linear Models	89
6.1.1 PGD Strategies Based on Standard Linearization Schemes	90
6.1.2 Discussion	91
6.1.3 Interpolation of the Non-Linear Term	92
6.1.4 Numerical Example	94
6.2 Convective Stabilization	95
6.2.1 Numerical Example	98
6.3 Discrete Formulation of the PGD	99
6.3.1 Example	100
6.3.2 Alternating Direction Scheme	103
6.4 Residual Minimization for Non-Symmetric Operators	104
6.5 Numerical Example	107
References	109
<b>Appendix A: Standard Discretization Techniques</b>	111

The Proper Generalized Decomposition for Advanced  
Numerical Simulations

A Primer

Chinesta, F.; Keunings, R.; Leygue, A.

2014, XIII, 117 p. 34 illus., 8 illus. in color., Softcover

ISBN: 978-3-319-02864-4