Preface

This book is an introduction to the mathematical analysis and approximation of partial differential equations using the finite-element method. It arises in part from third-year undergraduate and graduate lectures given by the author at the University of Pierre and Marie Curie (Paris VI) over the past 15 years, under the headings "Partial Differential Equations" and "Numerical Methods for Mechanics," but also from two books published by the author in 2004 and 2008 [1, 2].

Over the past 60 years, numerical analysis has witnessed a spectacular development. Of course, this has much to do with the unprecedented explosion of computer technology, which has literally taken over the world, generating hitherto undreamt of computational capacity. The whole field of mathematical activity that constitutes numerical analysis might well be described as the mathematics of mathematics, in the same sense in which one speaks of a police of police. For whenever a mathematical technique cannot be implemented in the context of industrial applications for reasons of operational inadequacy, numerical analysis will step into solve the problem by identifying the most suitable approximation process. From this moment on, all other branches of mathematics can be called into "force a way through," estimating a solution, combining tricks of the trade with lucidity, and all this within as rigorous a mathematical framework as possible.

One of the favored areas of application of numerical analysis is the approximation of partial differential equations, a major factor in the modeling of real systems. Indeed, whether we consider applications in physics or mechanics, in economics, the media, marketing, or the field of finance, the mathematical formulation of the system under investigation leads frequently to ordinary or partial differential equations that require solution. For this reason, many methods have been devised to solve such equations. Among the best known are finite-difference methods, finite-volume methods, singularity or integral methods, spectral methods, and variational finite-difference methods. However, there can be no doubt whatever that it is the finite-element method that has literally revolutionized the numerical approximation of partial differential equations.

With its unequaled flexibility, the finite-element method surely constitutes the most widely used method for approximate solution of mathematical models in the engineering sciences. Given the high level of mathematical know-how required to implement this method, several specialists in numerical analysis, such as P. A. Raviart [4], have presented the subject in advanced-level university courses, reserved for students with the necessary mathematical prerequisites, particularly in functional analysis, which is key to a theoretical understanding of the finite-element method. For other students, those not specializing in mathematics, such as undergraduate and graduate students in physics or mechanics, but also engineering students, namely those who consume mathematics at a range of different levels, the book [3] published by D. Euvrard in the 1990s provided a course for those not so familiar with the tools of functional analysis.

The outstanding success of these two books merely reflects the high level of teaching of numerical analysis in the mechanics department at the University of Pierre and Marie Curie, and later in the engineering faculty, by P. A. Raviart and subsequently by D. Euvrard. Concerning the present work, the idea itself and the core of the book result from the significant interaction between the various aspects of my teaching activities, and in particular, undergraduate- and master's-level mechanics courses at the University of Pierre and Marie Curie, including methods of functional analysis and numerical methods applied to mechanics and the mechanics of deformable solids. Indeed, the aim was to pursue the enterprise of the previously cited authors while supplementing the course in such a way as to achieve a new balance between a work devoted to specialists and one "dealing with operational aspects, while merely touching upon the mathematical underpinnings," to quote Euvrard [3 p. 198], who had considerable influence on my thinking during those years of teaching.

My training and my appreciation of the subjects underlying mathematical and numerical analysis, and in particular those relating to partial differential equations, can be largely attributed to Prof. Gérard Tronel, a notably active and devoted member of the numerical analysis teaching group at the Jacques Louis Lions research center of the University of Pierre and Marie Curie. The new balance presented in the present work owes much to the relevance of Prof. Tronel's pedagogical approach, from which I benefited enormously, first as a student, then as a colleague. I extend my warmest thanks to him.

This new form of presentation of 1-dimensional applications to the resistance of materials has been followed by third year undergraduate and master's students in mechanics at UPMC. The present book discusses these examples and extends to other applications. The first step was to identify certain key tools of functional analysis, which are presented without proof unless they exhibit some specific didactic relevance or a potential for implementation in the context of specific applications. The second was to deploy and exploit these results in the examination of problems relating to the existence, uniqueness, and regularity of the weak solutions and their equivalence with the strong solutions.

In this respect, the Part I of present book contains, on the one hand, an introduction to the techniques of functional analysis insofar as they are relevant to partial differential equations, and on the other, a discussion of the finite-element method complementary to that given by Euvrard [3]. The Part II presents and solves several problems that exploit these techniques of functional analysis while constructing the kind of node equations that characterize every numerical implementation of the finite-element method. There is also a detailed comparison of finite-element methods and finite-difference schemes, which correspond to the node equations when the mesh is regular and for certain types of finite-element analysis. Particular attention has been paid to the assembly process as illustrated in problems of resistance of materials.

In this context, I would like to pay homage to the memory of Claude Kammoun, who introduced me to these techniques in the context of the resistance of materials. I would also like to thank my colleagues at the UPMC—Benoit Goyeau, Cédric Croizet, Diana Baltéan, Catherine Weisman, and Hélène Dumontet—and those at the Ecole Nationale Supérieure in Cachan—Ludovic Chamoin and Florent Pled—and also Professor Franck Assous, for many discussions that have done much to extend and deepen my understanding and given me the impetus to write the present book. I extend my warmest thanks to all of them.

Paris, France, May 2013 Joël Chaskalovic

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