Preface

This volume contains original research articles, survey articles and lecture notes related to the *Computations with Modular Forms 2011* Summer School and Conference that was held at the University of Heidelberg in August and September 2011. Organized by Gebhard Böckle, John Voight and Gabor Wiese, the Summer School and Conference were supported by the Mathematics Center Heidelberg (MATCH), the DFG priority program Algorithmic and Experimental Methods in Algebra, Geometry and Number Theory (SPP 1489) and the Number Theory Foundation.

The study of modular forms can be traced back to the work of Jakob Bernoulli and Leonhard Euler in the 18th century, in which certain theta functions appear. Later, in the 19th century, the concept of a modular form was formalized, and the term *Modulform* (modular form) seems to have been coined by Felix Klein. Whereas in the classical period modular forms were studied through function theoretic methods, a deep algebraic and algebro-geometric theory emerged in the 20th century. Moreover, spectacular links with objects from other disciplines were conjectured and some of these conjectures were recently proved. The link most well-known to the general public is the one between elliptic curves over the rationals and weight two newforms with rational coefficients, going under the name Taniyama-Shimura-Weil Conjecture: its partial proof by Andrew Wiles in 1995 made headlines since it implies the famous Fermat's Last Theorem. Currently, a whole framework of conjectural links between modular forms (and automorphic forms in various generalizations), representation theoretic objects and number theoretic ones is being developed, intensively studied, and partially proved, going under the names (p-adic) Langlands Program, Fontaine-Mazur Conjecture, (generalizations of) Serre's Modularity Conjecture, etc.

Modular forms, elliptic curves and related objects have proven to be very amenable to explicit computer calculations. Such calculations have played a very prominent role for many years in the building and refining of theory. For instance, the famous Birch and Swinnerton-Dyer conjecture, one of the seven Millennium Prize Problems of the Clay Mathematics Institute, was discovered through computer calculations in the early 1960s. At about the same time the Sato-Tate Conjecture was made, to which Sato was inspired through a numerical study, whereas Tate was apparently led to it by a theoretical reasoning. Also, concerning Serre's very influential modularity conjecture: it was some computer calculations of Mestre that led Serre to be "convinced to take the conjecture seriously" ("suffisant pour me convaincre que la conjecture méritait d'être prise au sérieux"[1]).

Nowadays, computer computations are standard for classical modular forms and can be done for instance using Sage. Implementations of Hilbert modular forms and others are included, for instance, in the computer algebra system Magma. It is impossible to list all research articles in which the authors have made use of Cremona's database of elliptic curves (developed since the end of the 1980s), Stein's database of modular forms, or the various modular forms and elliptic curves related functionality implemented in computer algebra systems. Needless to say, computational data is particularly useful in the development of the conjectural building encompassed in the Langlands Program and related conjectures. From a different, real-life applications oriented perspective, one cannot stress enough the role of algorithms for elliptic curves (and generalizations thereof) used in cryptography (smart cards, mobile phones, passport authentication, etc.).

The main focus of the Computation with Modular Forms 2011 Summer School and Conference was the development and application of algorithms in the field of modular and automorphic forms. The Summer School held prior to the Conference was aimed at young researchers and PhD students working or interested in this area.[2]

During the Summer School, lecture series were given by: Paul Gunnells, *Lectures on computing cohomology of arithmetic groups*, David Loeffler, *Computing with algebraic modular forms* and Robert Pollack, *Overconvergent modular symbols*, with two guest lectures by Henri Darmon. These three themes offer an introduction to modern algorithms including some theoretical background for computations of and with modular forms. In addition, the most basic and widely used algorithm, *modular symbols*, was briefly covered. The organizers would like to take this opportunity to thank again the speakers of the Summer School for their excellent presentations and their willingness to write notes that are included in these proceedings.

The research part of the Conference covered a wide range of themes related to computations with modular forms. Not all talks are included in this volume but also not all contributions to this volume were presented at the Conference. A number of articles are concerned with modular forms and related cohomology classes over imaginary quadratic fields, that is, the situation of Bianchi groups. Haluk Şengün surveys arithmetic aspects of Bianchi groups, particularly focussing on their cohomology, its computation and its partly conjectural arithmetic significance. For speeding up modular symbols calculations, Adam Mohamed explicitly describes Hecke operators on Manin symbols over imaginary quadratic fields in his article. In a higher generality, Dan Yasaki provides a survey on computing Hecke eigenclasses on the cohomology of Bianchi groups, GL_2 over totally real fields, totally complex quartic and mixed signature cubic fields, based on Voronoĭ-Köcher reduction theory

[1] Jean-Pierre Serre, *Sur les représentation modulaires de degré 2 de* $\mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$, Duke Math. J. 54, no. 1, 1987, 179–230.

[2] See also http://www1.iwr.uni-heidelberg.de/conferences/modularforms11/.

and the Sharbly complex. Paul Gunnels provides general background on this theory in his lecture series. Also motivated by modular symbols over imaginary quadratic fields, John Cremona and M. T. Aranés provide algorithms for treating cusps and Manin symbols over arbitrary number fields.

Still in the realm of computing modular forms, Kevin Buzzard outlines an algorithm for calculating weight one cusp forms in characteristic zero and over finite fields and reports on computational findings. A talk at the Conference by George Schaeffer described further developments and extensions in this domain, which Schaeffer obtained in his PhD thesis. Other computations with classical modular forms appear in the work of John Voight and John Willis who describe an algorithm for series expansions of such forms around CM points. An algorithm for computing algebraic modular forms for certain rather general classical higher rank groups is treated in the work of Matthew Greenberg and John Voight using lattice methods, which has close links to the lecture series of David Loeffler. Finally, Alan Lauder describes some explicit calculations of triple product p-adic L-functions. It is related to the talks given by Henri Darmon that complemented the lecture series by Robert Pollack.

Other articles cover experimental and theoretical results: Explicit computations of modular forms led Panagiotis Tsaknias to discover a natural generalization of Maeda's conjecture on Galois orbits of newforms to higher levels, on which he reports in his article. Bartosz Naskręcki investigates congruences modulo prime powers between cusp forms and Eisenstein series theoretically and on the computer. Loïc Merel gives a description of the component group of the real points of modular Jacobians and addresses the computational question of how to determine whether complex conjugation in a modular mod 2 representation is trivial or not.

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