

Chapter 2

Undulator Radiation

2.1 Magnetic Field of a Planar Undulator

The motion of an electron in a planar undulator magnet is shown schematically in Fig. 2.1. The undulator axis is along the direction of the beam (z direction), the magnetic field points in the y direction (vertical). The period λ_u of the magnet arrangement is about 20–30 mm. For simplicity we assume that the horizontal width of the pole shoes is larger than λ_u , then one can neglect the x dependence of the field in the vicinity of the tightly collimated electron beam. The curl of the magnetic field vanishes inside the vacuum chamber of the electron beam, hence the field can be written as the gradient of a scalar magnetic potential obeying the Laplace equation

$$\mathbf{B} = -\nabla\Phi_{\text{mag}}, \quad \nabla^2\Phi_{\text{mag}} = 0.$$

The field on the axis is approximately harmonic. Making the Ansatz

$$\Phi_{\text{mag}}(y, z) = f(y) \sin(k_u z) \Rightarrow \frac{d^2 f}{dy^2} - k_u^2 f = 0 \quad \text{with} \quad k_u = \frac{2\pi}{\lambda_u}$$

we obtain the general solution

$$f(y) = c_1 \sinh(k_u y) + c_2 \cosh(k_u y).$$

The vertical field is

$$B_y(y, z) = -\frac{\partial\Phi_{\text{mag}}}{\partial y} = -k_u [c_1 \cosh(k_u y) + c_2 \sinh(k_u y)] \sin(k_u z).$$

B_y has to be symmetric with respect to the plane $y = 0$, hence $c_2 = 0$. We set $k_u c_1 = B_0$ and obtain $B_y(0, z) = -B_0 \sin(k_u z)$. So the potential is

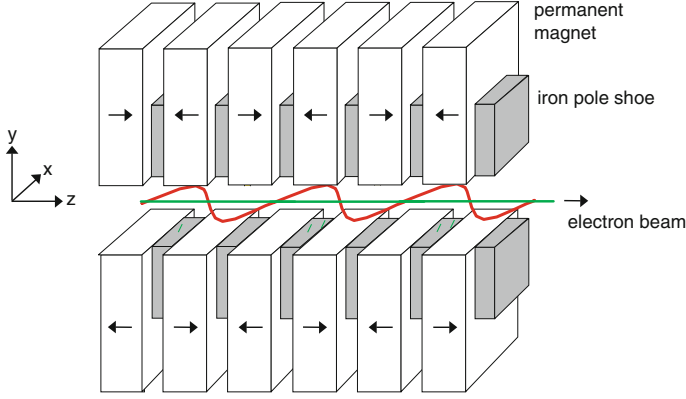


Fig. 2.1 Schematic view of a planar undulator magnet with alternating polarity of the magnetic field and of the sine-like trajectory of the electrons. In the magnet shown here the field is produced by permanent magnets that are placed between iron pole shoes. The distance between two equal poles is called the undulator period λ_u . A typical value is $\lambda_u = 25$ mm.

$$\Phi_{\text{mag}}(x, y, z) = \frac{B_0}{k_u} \sinh(k_u y) \sin(k_u z) . \quad (2.1)$$

For $y \neq 0$ the magnetic field possesses a small longitudinal component B_z :

$$\begin{aligned} B_x &= 0 , \\ B_y &= -B_0 \cosh(k_u y) \sin(k_u z) , \\ B_z &= -B_0 \sinh(k_u y) \cos(k_u z) . \end{aligned} \quad (2.2)$$

In the following we restrict ourselves to the symmetry plane $y = 0$ and use the idealized field

$$\boxed{\mathbf{B} = -B_0 \sin(k_u z) \mathbf{e}_y} , \quad (2.3)$$

where \mathbf{e}_y is the unit vector in y direction.

2.2 Electron Motion in an Undulator

2.2.1 Trajectory in First Order

We call $W = W_{\text{kin}} + m_e c^2 = \gamma m_e c^2$ the total relativistic energy of the electron. The transverse acceleration by the Lorentz force is

$$\gamma m_e \dot{\mathbf{v}} = -e \mathbf{v} \times \mathbf{B} , \quad (2.4)$$

resulting in two coupled equations

$$\ddot{x} = \frac{e}{\gamma m_e} B_y \dot{z}, \quad \ddot{z} = -\frac{e}{\gamma m_e} B_y \dot{x}, \quad (2.5)$$

which are solved iteratively. To obtain the first-order solution we observe that $v_z = \dot{z} \approx v = \beta c = \text{const}$ and $v_x \ll v_z$. Then $\ddot{z} \approx 0$ and the solution for $x(t)$ and $z(t)$ is

$$x(t) \approx \frac{e B_0}{\gamma m_e \beta c k_u^2} \sin(k_u \beta c t), \quad z(t) \approx \beta c t, \quad (2.6)$$

if the initial conditions

$$x(0) = 0, \quad \dot{x}(0) = \frac{e B_0}{\gamma m_e k_u}$$

are realized by a suitable beam steering system in front of the undulator¹ (the undulator magnet starts at $z = 0$). The electron travels on the sine-like trajectory

$$x(z) = \frac{K}{\beta \gamma k_u} \sin(k_u z). \quad (2.7)$$

In this equation we have introduced the important dimensionless *undulator parameter*

$$K = \frac{e B_0}{m_e c k_u} = \frac{e B_0 \lambda_u}{2 \pi m_e c} = 0.934 \cdot B_0 [\text{T}] \cdot \lambda_u [\text{cm}]. \quad (2.8)$$

The transverse velocity is

$$v_x(z) = \frac{K c}{\gamma} \cos(k_u z). \quad (2.9)$$

2.2.2 Motion in Second Order

Due to the sinusoidal trajectory the longitudinal component of the velocity is not constant. It is given by

$$v_z = \sqrt{v^2 - v_x^2} = \sqrt{c^2(1 - 1/\gamma^2) - v_x^2} \approx c \left(1 - \frac{1}{2\gamma^2} (1 + \gamma^2 v_x^2/c^2) \right).$$

¹ In practice the initial conditions can be realized by augmenting the undulator with a quarter period upstream of the periodic magnet structure and by displacing the electron orbit at $z = -\lambda_u/4$ by $\Delta x = -K/(\beta \gamma k_u)$ with the help of two dipole magnets. A similar arrangement at the rear end restores the beam orbit downstream of the undulator. For an illustration see Ref. [1].

Inserting for $v_x = \dot{x}(t)$ the first-order solution and using the trigonometric identity $\cos^2 \alpha = (1 + \cos 2\alpha)/2$, the longitudinal speed becomes

$$v_z(t) = \left(1 - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2}\right)\right) c - \frac{cK^2}{4\gamma^2} \cos(2\omega_u t) \quad (2.10)$$

with $\omega_u = \bar{\beta} c k_u \approx c k_u$. The average longitudinal speed is

$$\bar{v}_z = \left(1 - \frac{1}{2\gamma^2} \left(1 + \frac{K^2}{2}\right)\right) c \equiv \bar{\beta} c. \quad (2.11)$$

The particle trajectory in second order is described by the equations

$$x(t) = \frac{K}{\gamma k_u} \sin(\omega_u t), \quad z(t) = \bar{v}_z t - \frac{K^2}{8\gamma^2 k_u} \sin(2\omega_u t). \quad (2.12)$$

The motion in a helical undulator is treated in Sects. 4.9 and 10.1.

2.3 Emission of Radiation

The radiation emitted by relativistic electrons in a magnetic field is concentrated in a narrow cone with an opening angle of about $\pm 1/\gamma$. The cone is centered around the instantaneous tangent to the particle trajectory. The direction of the tangent varies along the sinusoidal orbit in the undulator magnet, the maximum angle with respect to the axis being

$$\theta_{\max} \approx \left| \frac{dx}{dz} \right|_{\max} = \frac{K}{\beta\gamma} \approx \frac{K}{\gamma}. \quad (2.13)$$

Suppose this directional variation is less than $1/\gamma$. Then the radiation field receives contributions from various sections of the trajectory that overlap in space and interfere with each other. The important consequence is: the radiation spectrum in forward direction is not continuous but nearly monochromatic, more precisely, it is composed of a narrow spectral line at a well-defined frequency and its odd higher harmonics. This is the characteristic feature of undulator radiation. The condition to be satisfied is

$$\theta_{\max} \leq \frac{1}{\gamma} \Rightarrow K \leq 1. \quad (2.14)$$

Incidentally, this condition can be a bit relaxed, and K values of 2 – 3 are still acceptable.

On the other hand, if the maximum angle θ_{\max} exceeds the radiation cone angle $1/\gamma$ by a large factor, which is the case for $K \gg 1$, one speaks of a wiggler magnet.

Wiggler radiation consists of many densely spaced spectral lines forming a quasi-continuous spectrum which resembles the spectrum of ordinary synchrotron radiation in bending magnets. We will not discuss it any further in this book.

2.3.1 Radiation in a Moving Coordinate System

Consider a coordinate system (x^*, y^*, z^*) moving with the average longitudinal speed of the electrons:

$$\bar{v}_z \equiv \bar{\beta} c, \quad \bar{\gamma} = \frac{1}{\sqrt{1 - \bar{\beta}^2}} \approx \frac{\gamma}{\sqrt{1 + K^2/2}} \quad \text{with } \gamma = \frac{W}{m_e c^2}. \quad (2.15)$$

The Lorentz transformation from the moving system to the laboratory system reads

$$\begin{aligned} t^* &= \bar{\gamma} (t - \bar{\beta} z/c) \approx \bar{\gamma} t (1 - \bar{\beta}^2) = t/\bar{\gamma}, \\ x^* &= x = \frac{K}{\gamma k_u} \sin(\omega_u t), \\ z^* &= \bar{\gamma} (z - \bar{\beta} c t) \approx -\frac{K^2}{8\gamma k_u \sqrt{1 + K^2/2}} \sin(2\omega_u t). \end{aligned}$$

The electron orbit in the moving system is thus

$$x^*(t^*) = a \sin(\omega^* t^*), \quad z^*(t^*) = -a \frac{K}{8\sqrt{1 + K^2/2}} \sin(2\omega^* t^*) \quad (2.16)$$

with the amplitude $a = K/(\gamma k_u)$ and the frequency

$$\omega^* = \bar{\gamma} \omega_u = \bar{\gamma} c k_u \approx \frac{\gamma c k_u}{\sqrt{1 + K^2/2}}.$$

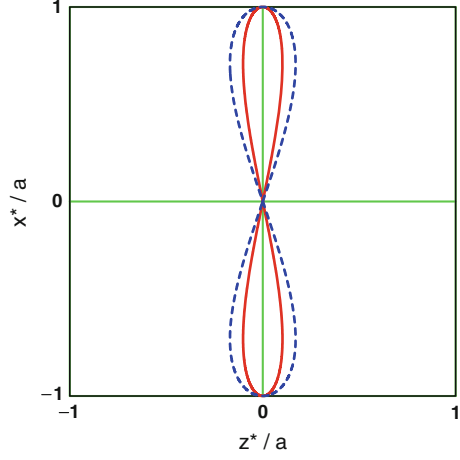
Note that $\omega_u t = \omega^* t^*$. The motion is depicted in Fig. 2.2. It is mainly a transverse harmonic oscillation with the frequency $\omega^* = \bar{\gamma} \omega_u$. Superimposed is a small longitudinal oscillation with twice that frequency. If we ignore the longitudinal oscillation for the time being, the electron will emit dipole radiation in the moving system with the frequency $\omega^* = \bar{\gamma} \omega_u$ and the wavelength $\lambda_u^* = \lambda_u/\bar{\gamma}$.

The radiation power from an accelerated charge is given by the well-known Larmor formula

$$P = \frac{e^2}{6\pi\epsilon_0 c^3} \dot{\mathbf{v}}^2, \quad (2.17)$$

see Ref. [2] or any other textbook on classical electrodynamics. For an oscillating charge, $\dot{\mathbf{v}}^2$ must be averaged over one period. The Larmor formula is applicable for

Fig. 2.2 The electron trajectory in the moving coordinate system for an undulator parameter of $K = 1$ (continuous red curve) or $K = 5$ (dashed blue curve). The curve has the shape of the number eight. For $K \gg 1$ the excursion in longitudinal direction is $z_{\text{max}}^*/a = \sqrt{2}/8 = 0.18$. For $K \rightarrow 0$ the longitudinal width shrinks to zero.



an oscillating dipole which is either at rest or moving at non-relativistic speeds. This condition is satisfied in the moving coordinate system. Ignoring the longitudinal oscillation, the acceleration has only an x component

$$\dot{v}_x^* = \frac{d^2 x^*}{dt^{*2}} = -\frac{K}{\gamma k_u} \omega^{*2} \sin(\omega^* t^*) = -\frac{K \gamma c^2 k_u}{1 + K^2/2} \sin(\omega^* t^*),$$

and the time-averaged square of the acceleration becomes

$$\langle \dot{v}^2 \rangle = \frac{K^2 \gamma^2 c^4 k_u^2}{(1 + K^2/2)^2} \frac{1}{2}.$$

The total radiation power in the moving system is thus

$$P^* = \frac{e^2 c \gamma^2 K^2 k_u^2}{12 \pi \epsilon_0 (1 + K^2/2)^2}. \quad (2.18)$$

2.3.2 Transformation of Radiation into Laboratory System

The radiation characteristics of an oscillating dipole which is either at rest or moving at relativistic speed is depicted in Fig. 2.3. With increasing Lorentz factor γ the radiation becomes more and more concentrated in the forward direction. To compute the light wavelength in the laboratory system as a function of the emission angle θ with respect to the beam axis it is appropriate to apply the Lorentz transformation

$$\hbar \omega^* = \bar{\gamma} (W_{\text{ph}} - \bar{\beta} c p_{\text{ph}} \cos \theta) = \bar{\gamma} \hbar \omega_{\ell} (1 - \bar{\beta} \cos \theta)$$

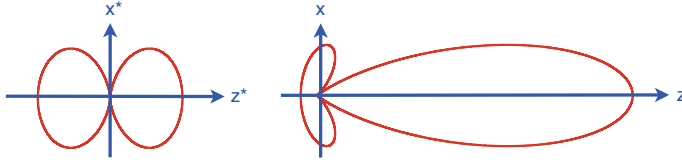


Fig. 2.3 Radiation characteristics in the laboratory system of an oscillating dipole at rest (*left*) or moving horizontally at a speed of $v = 0.9c$ (*right*). The dipole oscillates in vertical direction.

which expresses the photon energy $\hbar\omega^*$ in the moving system in terms of the photon energy $W_{\text{ph}} = \hbar\omega_\ell$ and the photon momentum $p_{\text{ph}} = \hbar\omega_\ell/c$ in the laboratory system. The light frequency in the laboratory system is then

$$\omega_\ell = \frac{\omega^*}{\bar{\gamma}(1 - \bar{\beta}\cos\theta)} \Rightarrow \lambda_\ell = \frac{2\pi c}{\omega_\ell} \approx \lambda_u(1 - \bar{\beta}\cos\theta).$$

Using $\bar{\beta} = 1 - (1 + K^2/2)/(2\gamma^2)$ and $\cos\theta \approx 1 - \theta^2/2$ (the typical angles are $\theta \leq 1/\gamma \ll 1$) we find that the wavelength of undulator radiation near $\theta = 0$ is in good approximation

$$\lambda_\ell(\theta) = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2\theta^2 \right). \quad (2.19)$$

The radiation is linearly polarized with the electric vector in the plane of the wavelike electron trajectory. T. Shintake has written a computer code in which the electric field pattern of a relativistic electron moving through the undulator is computed [3]. The field lines are shown in Fig. 2.4. One can clearly see the optical wavefronts and the dependence of the wavelength on the emission angle.

The total radiation power is relativistically invariant [2]. This can be seen as follows. Since we have ignored the longitudinal oscillation, the longitudinal coordinate and the longitudinal momentum of the electron are zero in the moving system

$$z^* = 0, \quad p_z^* = 0.$$

Then the Lorentz transformations of time and electron energy read

$$t = \bar{\gamma}t^*, \quad W = \bar{\gamma}W^*,$$

so the radiation power in the laboratory system becomes

$$P = -\frac{dW}{dt} = -\frac{dW^*}{dt^*} = P^*.$$

The undulator radiation power per electron in the laboratory system is therefore

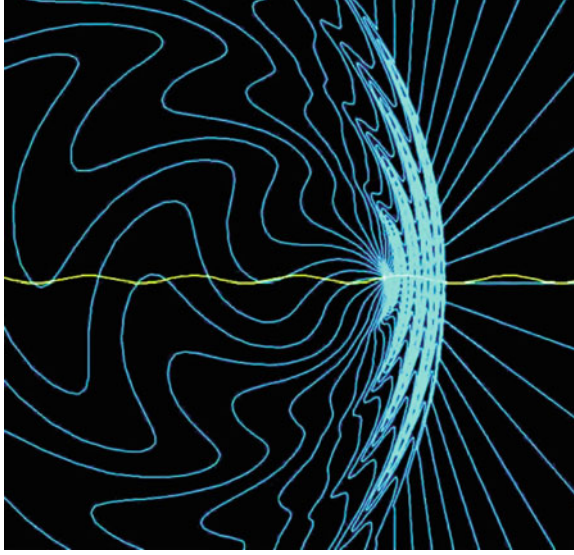


Fig. 2.4 Undulator radiation of an electron with $v = 0.9c$. The undulator parameter is $K = 1$. The wavy curve indicates the electron trajectory in the undulator. (Courtesy of T. Shintake).

$$P_1 = \frac{e^2 c \gamma^2 K^2 k_u^2}{12\pi\epsilon_0(1 + K^2/2)^2} . \quad (2.20)$$

Since this formula has been derived neglecting the influence of the longitudinal oscillation it describes only the power P_1 contained in the first harmonic. The total power of spontaneous undulator radiation, summed over all harmonics and all angles, is found to be equal to the synchrotron radiation power (1.3) in a bending magnet whose field strength is $B = B_0/\sqrt{2}$.

$$P_{\text{rad}} = \frac{e^4 \gamma^2 B_0^2}{12\pi\epsilon_0 c m_e^2} = \frac{e^2 c \gamma^2 K^2 k_u^2}{12\pi\epsilon_0} . \quad (2.21)$$

This is easy to understand: the undulator field varies as $B_y(z) = -B_0 \sin(k_u z)$, and hence $\langle B_y^2 \rangle = B_0^2/2$. Formula (2.21) is valid for any value of the undulator parameter K and therefore also applicable for wiggler radiation. If the undulator parameter is increased much beyond the ideal value of $K = 1$, the power contained in the first harmonic decreases as $1/K^2$.

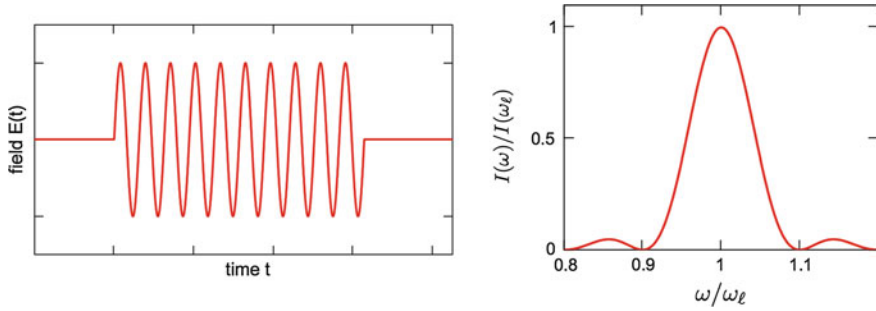


Fig. 2.5 A finite wave train with 10 oscillations and the lineshape function of forward undulator radiation (i.e. the intensity as a function of frequency) in a magnet with $N_u = 10$ periods.

2.4 Lineshape and Spectral Energy of Undulator Radiation

An important property of undulator radiation is that it consists of narrow spectral lines. How wide is such a line? In this section we consider the first harmonic only and look in forward direction. An electron passing through an undulator with N_u periods produces a wave train with N_u oscillations (Fig. 2.5) and a time duration of $T = N_u \lambda_1 / c$. The electric field of the light wave is written as

$$E_\ell(t) = \begin{cases} E_0 \exp(-i \omega_\ell t) & \text{if } -T/2 < t < T/2, \\ 0 & \text{otherwise.} \end{cases} \quad (2.22)$$

Due to its finite length, this wave train is not monochromatic but subtends a frequency spectrum which is obtained by Fourier transformation

$$\begin{aligned} A(\omega) &= \int_{-\infty}^{+\infty} E_\ell(t) e^{i\omega t} dt = E_0 \int_{-T/2}^{+T/2} e^{-i(\omega_\ell - \omega)t} dt \\ &= 2E_0 \cdot \frac{\sin((\omega_\ell - \omega)T/2)}{\omega_\ell - \omega}. \end{aligned} \quad (2.23)$$

The spectral intensity is

$$I(\omega) \propto |A(\omega)|^2 \propto \left(\frac{\sin \xi}{\xi} \right)^2 \quad \text{with} \quad \xi = \frac{(\omega_\ell - \omega)T}{2} = \pi N_u \frac{\omega_\ell - \omega}{\omega_\ell}. \quad (2.24)$$

It has a maximum at $\omega = \omega_\ell$ and a characteristic width of

$$\Delta\omega \approx \frac{\omega_\ell}{N_u}. \quad (2.25)$$

The lineshape function for a wave train with 10 oscillations is shown in Fig. 2.5.

The angular width of the first harmonic around $\theta = 0$ can be estimated as follows. We know from Eq. (2.19) that the frequency decreases with increasing emission angle θ :

$$\omega_\ell(\theta) = \omega_\ell(0) \cdot \frac{1 + K^2/2}{1 + K^2/2 + \gamma^2\theta^2}.$$

The intensity drops to zero when $\delta\omega_\ell = \omega_\ell(0) - \omega_\ell(\theta)$ exceeds the bandwidth following from Eq. (2.24). The root-mean-square (rms) value is found to be [4]

$$\sigma_\theta \approx \frac{1}{\gamma} \cdot \sqrt{\frac{1 + K^2/2}{2N_u}} \approx \frac{1}{\gamma} \cdot \frac{1}{\sqrt{N_u}} \text{ for } K \approx 1. \quad (2.26)$$

Obviously, the first harmonic of undulator radiation is far better collimated than synchrotron radiation: the typical opening angle $1/\gamma$ is divided by $\sqrt{N_u} \gg 1$. It is important to realize that this tight collimation of the first harmonic applies only if one requests that the frequency stays within the bandwidth. If one drops the restriction to a narrow spectral line and accepts the entire angular-dependent frequency range as well as the higher harmonics, the cone angle of undulator radiation becomes for $K > 1$, using Eq. (2.13),

$$\theta_{\text{cone}} \approx \frac{K}{\gamma}. \quad (2.27)$$

2.5 Higher Harmonics

To understand the physical origin of the higher harmonics of undulator radiation we follow the argumentation in the excellent book *The Science and Technology of Undulators and Wigglers* by J. A. Clarke [5]. In the forward direction ($\theta = 0$) only odd higher harmonics are observed while the off-axis radiation ($\theta > 0$) contains also even harmonics. How can one explain this observation?

Consider a detector with a small aperture centered at $\theta = 0$ which is placed in the far-field at large distance from the undulator. The electrons moving on a sinusoidal orbit with maximum angle of K/γ emit their radiation into a cone of opening angle $1/\gamma$. If the undulator parameter is small, $K \ll 1$, the radiation cone points always toward the detector and therefore the radiation from the entire trajectory is detected. One observes a purely sinusoidal electric field which has only one Fourier component at the fundamental harmonic ω_1 , see Fig. 2.6 (top). The situation changes if the undulator parameter is significantly larger than 1, because then the angular excursion of the electron is much larger than the cone angle $1/\gamma$ and the radiation cone sweeps back and forth across the aperture, so the detector receives its light only from short sections of the electron trajectory. The radiation field seen by the detector consists now of narrow pulses of alternating polarity as sketched in the bottom part of Fig. 2.6. The frequency spectrum contains many higher harmonics.

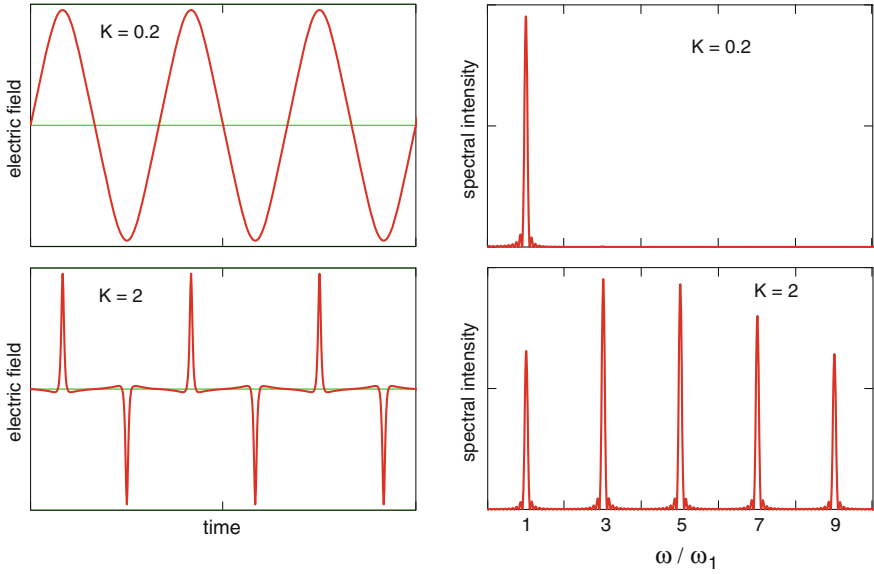


Fig. 2.6 Schematic view of the electric light-wave field seen by a small detector in forward direction and the corresponding frequency spectrum. *Top*: small undulator parameter $K = 0.2$. *Bottom*: fairly large undulator parameter $K = 2$.

Viewed in forward direction, the positive and negative pulses are symmetric in shape and uniformly spaced, and consequently only the odd harmonics occur. When the detector is placed at a finite angle $\theta > 0$, the field pulses are no longer equally spaced and the radiation spectrum contains the even harmonics as well (see [5] for an illustration).

The wavelength of the m th harmonic as a function of the angle θ is

$$\lambda_m(\theta) = \frac{1}{m} \frac{\lambda_u}{2\gamma^2} (1 + K^2/2 + \gamma^2\theta^2), \quad m = 1, 2, 3, 4, \dots \quad (2.28)$$

In forward direction only the odd harmonics are observed with the wavelengths

$$\lambda_m = \frac{1}{m} \frac{\lambda_u}{2\gamma^2} (1 + K^2/2), \quad m = 1, 3, 5, \dots, \quad (2.29)$$

so $\lambda_3 = \lambda_1/3$, $\lambda_5 = \lambda_1/5$. We will present an alternative derivation of Eq. (2.29) in Chap. 3.

The spectral energy density per electron of the radiation emitted in forward direction (emission angle $\theta = 0$) is for the m th harmonic [5]

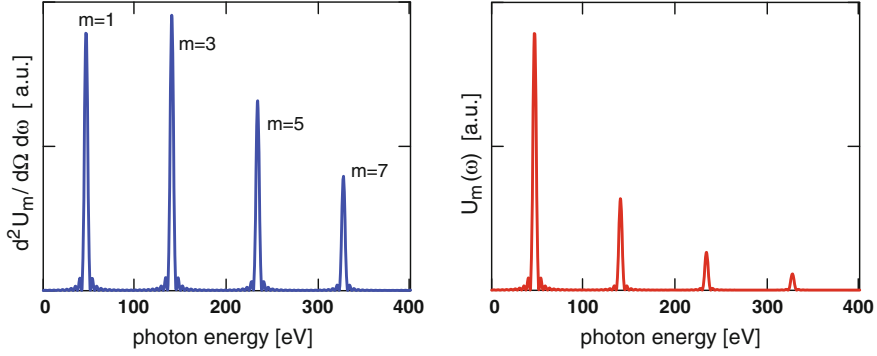


Fig. 2.7 *Left*: Example of a computed photon energy spectrum of undulator radiation for an undulator with 10 periods. Plotted is the differential spectral energy density $d^2 U_m / d\Omega d\omega$ at $\theta = 0$. The units are arbitrary. *Right*: The spectral energy $U_m(\omega)$ of the m th harmonic that is emitted into the solid angle $\Delta\Omega_m$. The electron Lorentz factor is $\gamma = 1000$, the undulator has the period $\lambda_u = 25$ mm and the parameter $K = 1.5$. Note that the energy ratios U_m/U_1 depend only on the harmonic index m and the undulator parameter K , but neither on γ nor on λ_u .

$$\frac{d^2 U_m}{d\Omega d\omega} = \frac{e^2 \gamma^2 m^2 K^2}{4\pi \epsilon_0 c (1 + K^2/2)^2} \cdot \frac{\sin^2(\pi N_u (\omega - \omega_m)/\omega_1)}{\sin^2(\pi (\omega - \omega_m)/\omega_1)} \cdot |JJ|^2 \quad (2.30)$$

with $JJ = J_n \left(\frac{m K^2}{4 + 2K^2} \right) - J_{n+1} \left(\frac{m K^2}{4 + 2K^2} \right)$, $m = 2n + 1$.

Here $\omega_m = m \omega_1 \equiv m \omega_\ell$ is the (angular) frequency of the m th harmonic. The harmonic index m is related to the index n by $m = 2n + 1$ and takes on the odd integer values $m = 1, 3, 5, \dots$ for $n = 0, 1, 2, \dots$. The J_n are the Bessel functions of integer order. In the vicinity of ω_m , the sine function in the denominator of Eq. (2.30) can be replaced by its argument. In this form the equation is presented in Refs. [4, 6].

The absolute bandwidth at $\theta = 0$ is the same for all harmonics

$$\Delta\omega_1 = \Delta\omega_3 = \Delta\omega_5, \dots$$

but the fractional bandwidth shrinks as $1/m$

$$\frac{\Delta\omega_m}{\omega_m} = \frac{1}{m N_u} \quad (2.31)$$

because the wave train comprises now $m N_u$ oscillations in an undulator with N_u periods. The angular width is [4]

$$\sigma_{\theta, m} \approx \frac{1}{\gamma} \cdot \sqrt{\frac{1 + K^2/2}{2m N_u}} \approx \frac{1}{\gamma} \cdot \frac{1}{\sqrt{m N_u}} \quad \text{for } K \approx 1. \quad (2.32)$$

The corresponding solid angle

$$\Delta\Omega_m = 2\pi\sigma_{\theta,m}^2 \approx \frac{2\pi}{\gamma^2} \cdot \frac{1}{m N_u}$$

decreases as $1/m$ with increasing harmonic order. Within the solid angle $\Delta\Omega_m$ the angular-dependent frequency shift is less than the bandwidth. Of practical interest is the spectral energy contained in this solid angle:

$$U_m(\omega) = \frac{d^2 U_m}{d\Omega d\omega} \Delta\Omega_m \quad m = 1, 3, 5, \dots \quad (2.33)$$

This spectral energy is shown in Fig. 2.7 for $m = 1, 3, 5, 7$ for a short undulator with ten periods and $K = 1.5$.

The angular dependence of the spectral energy is derived in Ref. [5]. For emission angles $\theta > 0$ the radiation contains all even and odd higher harmonics, as mentioned above.

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