Preface

This book evolved from our experiences over several years teaching abstract algebra to mixed audiences of mathematics majors and majors in secondary mathematics education at New Mexico State University (the course is required for both groups at NMSU as it is in many institutions of higher learning in the USA) along with our outreach work with Las Cruces area middle and high school mathematics students and teachers. These undertakings left us with a dilemma. While sympathetic to the frustrations expressed by pre-service and in-service teachers with the abstract nature of the standard presentations of the subject matter, and the perception of its irrelevance to pre-college teaching, we maintain that a rigorous grounding in the conceptual framework of algebra is absolutely critical to a high school or middle mathematics teacher's success, both in conveying content to their students and in fostering their enthusiasm and self-confidence for future careers in STEM fields and even public policy. The latter is particularly timely given the ubiquitous use of social media and current controversies over corporate and governmental surveillance. Our solution was to develop the structures and basic theorems of modern algebra through applications that have relevance to daily life (e.g., Identification Schemes, Error Correcting Codes, Cryptography, Wallpaper Patterns) and that directly inform topics that arise in high school or middle school mathematics classes (e.g., Number Theory, Symmetry, Ruler and Compass Constructions).

The result is a text intended for a one semester course in modern algebra that can be used in a variety of contexts. For an audience composed primarily of mathematics majors, the material on identification numbers, modular arithmetic, and linear algebra over arbitrary fields can be covered quickly, so that the chapters on codes defined over finite fields, isometries of the real plane, and ruler and compass constructions (and the associated abstract ring, field, and group theory) can be covered in depth. For an Applied Algebra course, with computer science majors in mind, the material on ruler and compass constructions can be given a lighter treatment so that emphasis can be placed on error detection and correction, cryptography, and isometries (important for computer-aided design). For courses designed for secondary mathematics teachers, the chapters on identification numbers, linear codes, ruler and compass constructions, and isometries (at least through the classification of frieze patterns) introduce groups, rings, and fields through accessible applications and provide ample rigor. A course based on these chapters would also serve programs offering a Master's degree in middle school mathematics education or a Master of Arts in Teaching Mathematics.

Numerous exercises are given after appropriate subsections. An exception is in Chap. 6 on ruler and compass constructions, where some steps in proofs are given as exercises within the text. This is done not only because the requisite drawings take up a lot of text space but also, more importantly, because they're fun. Exercises range from routine verifications and computations to more serious applications of the text material and conceptual issues. Proofs of a few propositions are left as exercises because

they give opportunities to employ important techniques that have been used earlier and will arise again. Some of the exercises refer to electronic supplementary materials (ESM) in the form of MAPLE worksheets. The worksheets, which give the reader practice with computations in modular arithmetic, RSA encryption and decryption, and error correction for Reed–Solomon codes, are accessible from this book's page at http://link.springer.com.

While the text is self-contained, references to supplementary sources solely for more background or further study are given at the end of each chapter.

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