

# Chapter 2

## Lab Exercises for a Course on Mechanical Vibrations

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**Abstract** This paper presents some exercises designed to teach fundamental aspects of mechanical vibrations in general, and experimental techniques for vibration measurements in particular. Teaching students to become good experimentalists is a very difficult task, and is perhaps not even possible inside standard curricula. However, some fundamental aspects of experimental work must be taught, and can be included in a course on vibrations as well as in other courses. The first exercise is designed to teach the student how careful one has to be when applying vibration sensors to a structure, and is based on the repeatability of mass calibration measurements. This makes it a good exercise to base a discussion on experiment setup and repeatability issues etc. The second exercise is an exercise where an approximate single-degree-of-freedom (SDOF) system is investigated by some simple analytical calculations as well as by an experimental measurement. This exercise serves to demonstrate the rather abstract notion of SDOF, and also illustrate the applicability of a simplified model in a limited frequency range. Finally, a third exercise is made where modal analysis of a slalom ski using impact testing is used as a demonstration of more advanced vibration analysis.

**Keywords** Vibration measurement • Teaching vibration • Mass calibration • Accelerometer mounting • Experimental modal analysis

### 2.1 Introduction

Making accurate vibration measurements is known to be rather difficult due to the many challenges of sensor choice, sensor mounting, and, in the case of investigation of eigenfrequencies (modal analysis), for example, the problems of suspending the structure properly. As this paper describes some exercises designed to teach students some good practice for vibration measurements, additional difficulties for students are often that they are not yet mature when it comes to perform good measurements; i.e. they have not yet learnt to be patient, to double check everything, etc.

The course, for which the exercises described in the present paper are used, is a graduate (master) level course of a third semester length. The students have had a course on general signal processing, but are unfamiliar with the particular measurement and analysis techniques for vibration analysis. In the course, both wave theory of continuous structures and discrete mechanical systems are taught; the exercises covered in the present paper, however, focuses on the discrete system description of mechanical systems.

The particular difficulties of vibration measurements addressed by the exercises described here are

- to learn some experimental methodology to ensure good experimental results, such as checking repeatability, double checking everything that could potentially affect the measurement, etc.,
- to yield respect for the particular difficulties of applying accelerometers correctly, and
- to correctly suspend a structure under free-free conditions for experimental modal analysis

An additional point which is very important for these exercises is, of course, to tie the theory taught in the course to experimental results.

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Perhaps a good way to summarize the spirit of the exercises described here is to use the famous quote by Albert Einstein; “A theory is something nobody believes, except the person who made it. An experiment is something everybody believes, except the person who made it.” The exercises described in this paper, addresses first and foremost the second part of this quote; the importance of being in constant doubt over ones experimental results. The exercises are, however, also touching on the essence of also questioning *theoretical results*, and the importance of verifying ones theories (which in our engineering terminology are, of course, usually called *models*).

## 2.2 Exercises

The exercises we are about to describe here, require some basic vibration measurement equipment and sensors. The measurement hardware and software can be essentially any vibration measurement system. We are using a National Instruments 9234 USB box, driven by homemade MATLAB software using the Data Acquisition Toolbox, and the free ABRVIBE [1] toolbox for the analysis. This means that the students record time data, for subsequent analysis in MATLAB. We have found that this ensures that the students understand every step in the processing of the data; something more “automatic” commercial systems often make more difficult.

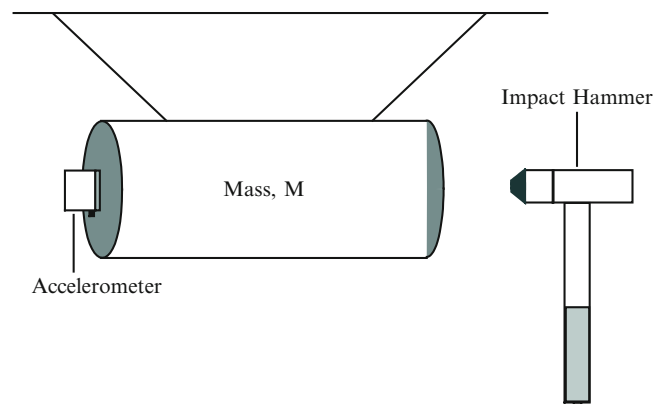
On the sensor side, in the exercises we use two accelerometers, a force sensor, an impact hammer, and a shaker with amplifier and random noise generator. Which particular sensors used are not particularly important, although the accelerometers should weigh less than 10 g, the force sensor should be of suitable sensitivity, and the same is true for the impact hammer. We currently use Dytran 3097A2T accelerometers with a sensitivity of 100 mV/g and mass of 4.3 g. The force sensor is a Kistler 9712B50 with a sensitivity of approximately 20 mV/N, and the impulse hammer is a Dytran 5800B3T with a sensitivity of approximately 10 mV/N.

In addition to this, some relatively inexpensive measurement objects are needed, which will be described in conjunction with each exercise. The first three exercises described in Sects. 2.2.1–2.2.3 are made at one instance in the laboratory, in approximately 2 h, followed by 4–6 h of analysis and report writing. The last exercise, described in Sect. 2.2.4, is accomplished in a second laboratory session, in approximately 4 h, followed by 6–8 h of analysis and report writing.

### 2.2.1 Mass Calibration

The first exercise we present, is based on calibration of an accelerometer (or a force sensor in an impact hammer) using a simple mass, as depicted in Fig. 2.1. This well-known technique, see for example [2], should be familiar to everyone making vibration measurements. It is a good technique not only for calibration, but also for checking that accelerometers are functional through the full frequency range, and for checking which frequency range a particular accelerometer can be used for (see Sect. 2.2.2), etc.

The calibration procedure is simple; the accelerometer is attached to a mass, typically a piece of round rod of steel. The weight of the mass is accurately measured, and should be different depending on the impact hammer used, so that a soft hit with the hammer produces a suitable acceleration level. For this exercise we use a weight of approximately 1 kg, made of a piece of rod with approximately 40 mm diameter. The mass is hanging in two strings from a supporting rig, so that it can



**Fig. 2.1** Setup for mass calibration for the exercise described in Sect. 2.2.1

move as a pendulum in the direction of the accelerometer as illustrated in Fig. 2.1, or alternatively placed on a soft foam pad. The mass is then excited by the impact hammer, while the force and accelerometer signals are recorded. The hardest tip for the hammer is chosen, to yield a frequency range as high as possible, typically up to 10 kHz. To give better accuracy, several impacts can be averaged, although this is generally not necessary if the sensitivities of the force sensor in the impact hammer and the accelerometer are appropriate, so that the measurement noise is minimal.

After the time signals with the impact force and resulting acceleration are recorded, the students calculate the frequency response (FRF) of acceleration with force. Since Newton's well-known formula for a mass is  $F = Ma$ , this FRF should be a constant  $H = A/F = 1/M$ , i.e. the FRF forms a straight line, independent of frequency. The exercise is made so that the students are given the sensitivity of the force sensor in the impact hammer, but not the sensitivity of the accelerometer: They are then asked to calculate the sensitivity of the accelerometer, given the measured frequency response at, for example, 159.2 Hz, which is equal to 1,000 rad/s, a common frequency for this purpose.

### 2.2.2 Accelerometer Mounting

The next exercise is using the mass calibration method described in Sect. 2.2.2 to investigate the effects of different mounting techniques for mounting accelerometers. This exercise has several important objectives; first of all it obviously discusses different means of attaching an accelerometer to the test structure and, as we will see, what performance these mounting techniques result in. Second of all, it demonstrates the difficulty of getting repeatable measurements, as in most cases the students do not get the same result even if they use the same mounting technique twice. This also makes a good point of discussing the concept of repeatability, and the importance of this concept in engineering (or science in general). Third, I am using this exercise to teach the students not to trust their measurements, until they have investigated that the accelerometer they use with a particular mounting technique, actually has a frequency range high enough for the measurements they want to make; this is very easily investigated by using the mass calibration method. Fourth, this exercise also illustrates that accelerometers, like all measurement sensors, are not perfect, but vary rather much with frequency.

In this exercise, the students use some different techniques to mount an accelerometer on the calibration mass, and perform measurements as described in Sect. 2.2.2. For each measurement the FRF is calculated and stored. In our case we mount the accelerometer with the following techniques:

- a thin layer of wax,
- a thick layer of wax,
- a thin layer of hot glue (hot melt adhesive, using a "hot glue gun"), and
- super glue (cyanoacrylate adhesive)

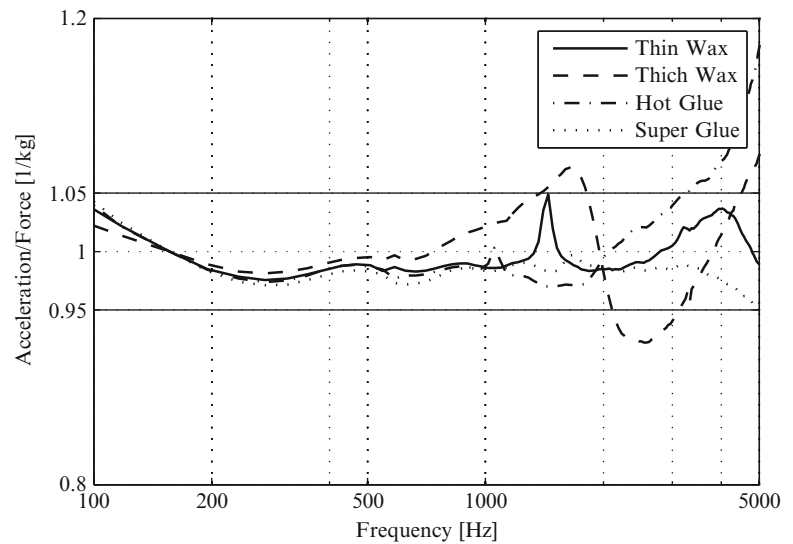
Other techniques such as screw mount and magnetic base could also be used. They are, however, somewhat difficult to make identical to the techniques above, as they change the mass of the accelerometer unit.

Since the students already have made a measurement with a thin layer of wax in the first exercise, this means that they obtain a total of five different FRFs, of which the two first should be identical (or very similar). These two FRFs based on a thin layer of wax are first compared. Regardless of whether they are identical or not (often they are not; usually because of too much wax) a fruitful discussion on repeatability issues is held, and the students who gets almost identical results are told that they have applied a sufficiently thin layer of wax. Many students are surprised how thin this layer has to be to yield identical FRFs.

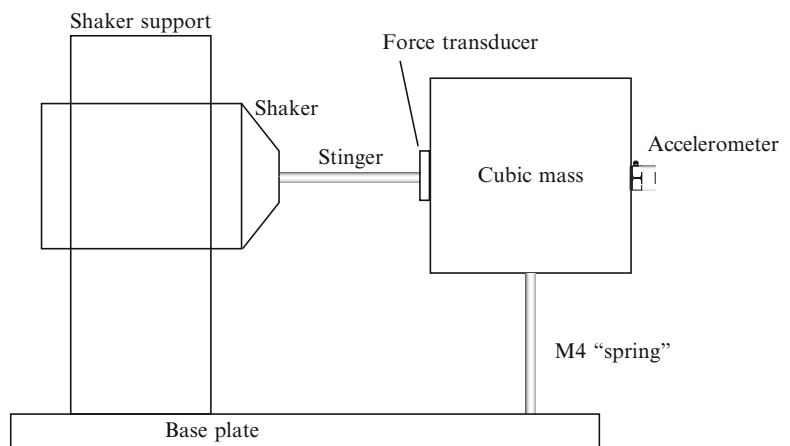
As the next step in this exercise, the students are asked to plot all five FRFs in one plot, and determine which of the mounting techniques works best. This produces a plot similar to Fig. 2.2, where the FRFs have first been normalized to have the same value at 159.2 Hz, and limits of  $\pm 5\%$  around this value are plotted, to indicate the accuracy limits specified by the sensor manufacturer. Note that only one of the two thin wax measurements is included in the figure, which is for clarity only. The students are finally asked to find the frequency where the error reaches 5 % for all mounting techniques, and find the technique giving the highest frequency limit. This often results in a dead-end between superglue and wax.

It should be stressed, and we do this during the lab exercise, that the frequency range obtained by the method described here, is not necessarily the frequency range obtained when the accelerometer is mounted in a point with less stiffness, such as on a lightly damped structure. It is therefore necessary to consider the obtained frequency range found on the calibration mass as a maximum frequency range, and to use some margin when using the accelerometer on a real structure.

**Fig. 2.2** Comparison of the FRFs from four measurements on a mass, using four different mounting techniques as described in Sect. 2.2.2. The FRFs are normalized to the same value at 159.2 Hz (1,000 rad/s) (from [2], Copyright 2011, John Wiley and Sons; reprinted with permission)



**Fig. 2.3** Schematic illustration of the approximate SDOF system used for the exercise described in Sect. 2.2.3



### 2.2.3 SDOF Measurement and Analysis

The single degree of freedom (SDOF) system is a key component in vibrations and structural dynamics. In this exercise, a simple system behaving as an approximate SDOF system is investigated, and used to illustrate the connection between theory and real world to the students. The system used is shown schematically in Fig. 2.3, and consists of a base plate of steel, approximately  $100 \times 300 \times 10$  mm; a steel cube with 30 mm side length; and a M4 bolt, approximately 50 mm long. There are two locking nuts locking the M4 bolt against the base plate and the mass. The mass is excited by a random force applied by a shaker through a stinger and a force sensor, and an accelerometer is mounted on the opposite to the force sensor. The FRF between the force and the acceleration is measured and compared to analytical results.

This exercise has several objectives; first it illustrates that the SDOF system used in theory can, at least in a limited frequency range, be found in “real life”. Second, it allows for discussion between model and reality, since the results obtained are rarely identical to the analytical results calculated by the students.

The stiffness of the M4 bolt is readily calculated using known formulas for moment of inertia and stiffness of a beam, and is omitted here so that professors can include this step as an exercise for the students with having direct access to the answer. The students are asked to measure the various components, and estimate the mass of the cube, including the accelerometer, and the stiffness, and from this estimate the “SDOF” natural frequency.

From the measured FRF, the students are asked to approximately estimate the natural frequency and damping (through the  $-3$  dB bandwidth and natural frequency). Once these parameters are obtained, the students should calculate the mass, stiffness, and damping coefficients of the system. The mass and stiffness coefficients thus obtained are compared with the mass calculated from the measurements of the cube, and the stiffness calculated for the beam. Since the results rarely compare particularly closely due to the lack of “precision” in the setup, it forms a good discussion point for differences between model and reality.

### 2.2.4 Full-Scale Modal Analysis Test

After the initial exercises described in Sects. 2.2.1–2.2.3, the students are well suited for the second exercise; a full experimental modal analysis test of a slalom ski using impact excitation with roving hammer. The students are asked to read the text [3] prior to the lab exercise, so that they are acquainted with the theory, and particularly with the practical aspects of an experimental modal analysis test.

The choice of the slalom ski is, of course, rather arbitrary, and almost any linear structure could be used. The idea with using the slalom ski instead of a more simple beam or plate, is that the students, in our experience, find it more interesting to measure a “real” object. Therefore an automotive component or any other simple structure would work equally well.

The exercise is rather straight forward. First, the structure is suspended, and a discussion is held on how to best suspend this long, slender, structure. The “correct” answer is, that it should be suspended hanging vertically, as this ensures the rotational rigid body mode is low, which is very difficult to obtain if it is supported horizontally. This is the case for all long, slender structures with small moments of inertia around the long axis. In our case, the ski has a small drilled hole in the back end, in the center of the ski in the “short” direction. Through this hole, a fishing line is threaded, forming a loop, to which a rubber cord is attached. The reason for this is that the rubber cord could add damping to the ski if it was in direct contact with the ski. Second, the ski is instrumented with accelerometers in two corners, for a minimum multi-reference test. We thus discuss the potential use of even more sensors, and the advantages with that. Third, the impact hammer tip, suitable frequency range, etc. are investigated by some rough measurements. Proper FRF estimation settings such as trigger level, pretrigger condition, block size, and force and exponential windows, are then obtained by the procedures laid out in [2, 4], which are supported by the command `impsetup` in the ABRVIBE toolbox [1].

After these optimal settings are obtained, the experimental setup is investigated for two things: (1) is the suspension affecting the structure?, and (2) are the measurements free from mass loading effects from the accelerometers? The first point, the suspension effects, are investigated by changing the suspension, which in this case is a rubber cord, by extending the length so as to almost double it (or some sufficient change). FRFs measured before and after this change are compared, and if they differ in the frequency range of interest, the reasons for this are discussed; obviously the suspension is then wrong (Is it too short? Or is there friction between the ski and the fishing line? Or are the cables for the accelerometers “pulling” the structure, adding damping? Etc.) The second point, mass loading, is investigated by mounting an additional accelerometer right next to one of the existing accelerometers, and making a new measurement. If the new FRF is different from the previous without the extra mass next to the accelerometer, there is apparently mass loading with the double mass of two accelerometers. The risk of having mass loading even when using a single accelerometer is then imminent. Actually, avoiding mass loading on the slalom ski requires very low weight accelerometers, due to the low damping, so with the 4.3-g accelerometers used in this exercise there is some mass loading, affecting the frequency range of the higher modes. This becomes a point of discussion, and I still have not had a single student who has thought that mass loading would occur with this small light sensor on this ski.

Finally, when everything is checked and the students are satisfied that everything is in order, the ski is excited in all points, one by one, in a  $3 \times 7$  grid, and time data are stored at each point for later processing by the ABRVIBE command `impproc` [1, 4]. Before leaving the lab, the students are encouraged to post process all their time data and make a first curve fitting using a MATLAB script given to the students prior to the lab exercise, to ensure that they get some reasonable good looking stabilization diagrams. This step only takes 10–15 min, and ensures that the students leave with good data that allows for the rest of the analysis to be performed outside the lab.

## 2.3 Conclusions

We have described four exercises whereof three fundamental exercises which teach students some good experimental practices as applied to vibration measurements, and illustrate the concept of model versus reality. The three exercises comprises an exercise to learn to apply mass calibration using and impact hammer and an accelerometer, and an exercise to compare different mounting techniques such as wax, hot glue, and super glue. The third exercise is demonstrating measurements on an approximate single degree of freedom (SDOF) system, and to identify the mechanical properties of this system using an experimentally obtained frequency response function (FRF). A fourth exercise, a complete experimental modal analysis of a slalom ski, is then performed using impact testing, giving the students experience of this important measurement technique, and some of the issues one has to watch for applying it. The exercises are suitable for students which have had some introduction to signal processing, but not necessarily vibration analysis procedures.

Although it is difficult, if not impossible, to teach students to be good vibration experimentalists in a few hours of lab exercises, some key points can surely be taught. The main points thus taught in the exercises presented in this paper are

- Summarizing the message, it is: do not “trust” anything that can be tested easily. This applies for example to
  - if the useful frequency range of your accelerometer is sufficient, using the same mounting technique you are going to use in your experiment; do not trust the data sheet frequency range!
  - if there are mass loading effects from your accelerometers; do not trust your intuition!
  - if the suspension affects your measured FRFs, do not believe it does not—investigate it!
  - if there are effects, on damping for example, from accelerometer cables; again—investigate it!
- A model is always limited and should be verified by experiments
- Measuring FRFs with impact testing, the measurement settings need to be optimized to ensure best possible FRFs

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