

# Preface

This is the second volume of the series of books of problems in  $C_p$ -theory entitled *A  $C_p$ -Theory Problem Book*, i.e., this book is a continuation of the first volume subtitled *Topological and Function Spaces*. The series was conceived as an introduction to  $C_p$ -theory with the hope that each volume will also be used as a reference guide for specialists.

The first volume provides a self-contained introduction to general topology and  $C_p$ -theory and contains some highly nontrivial state-of-the-art results. For example, Sect. 1.4 presents Shapirovsky's theorem on the existence of a point-countable  $\pi$ -base in any compact space of countable tightness and Sect. 1.5 brings the reader to the frontier of the modern knowledge about realcompactness in the context of function spaces.

This present volume introduces quite a few topics from scratch but dealing with topology and  $C_p$ -theory is already a professional endeavour. The objective is to study the behaviour of general topological properties in function spaces and establish the results on duality of cardinal functions and classes with respect to the  $C_p$ -functor. The respective background includes a considerable amount of top-notch results both in topology and set theory; the author's obsession with keeping this work self-contained implied that an introduction to advanced set theory had to be provided in Sect. 1.1. The methods developed in this section made it possible to present a very difficult example of Todorčević of a compact strong  $S$ -space.

Of course, it was impossible to omit the famous Batarov's theorem on coincidence of the Lindelöf number and extent in subspaces of  $C_p(X)$  for any Lindelöf  $\Sigma$ -space  $X$  and the result of Christensen on  $\sigma$ -compactness of  $X$  provided that  $C_p(X)$  is analytic. The self-containment policy of the author made it obligatory for him to give a thorough introduction to Lindelöf  $\Sigma$ -spaces in Sect. 1.3 and to the descriptive set theory in Sect. 1.4.

We use all topological methods developed in the first volume, so we refer to its problems and solutions when necessary. Of course, the author did his best to keep *every* solution as independent as possible, so a short argument could be repeated several times in different places.

The author wants to emphasize that if a postgraduate student mastered the material of the first volume, it will be more than sufficient to understand every problem and solution of this book. However, for a concrete topic, much less might be needed. Finally, let me outline some points which show the potential usefulness of the present work:

- *The only background needed is some knowledge of set theory and real numbers; any reasonable course in calculus covers everything needed to understand this book.*
- *The student can learn all of general topology required without recurring to any textbook or papers; the amount of general topology is strictly minimal and is presented in such a way that the student works with the spaces  $C_p(X)$  from the very beginning.*
- *What is said in the previous paragraph is true as well if a mathematician working outside of topology (e.g., in functional analysis) wants to use results or methods of  $C_p$ -theory; he (or she) will find them easily in a concentrated form or with full proofs if there is such a need.*
- *The material we present here is up to date and brings the reader to the frontier of knowledge in a reasonable number of important areas of  $C_p$ -theory.*
- *This book seems to be the first self-contained introduction to  $C_p$ -theory. Although there is an excellent textbook written by [Arhangel'skii \(1992a\)](#), it heavily depends on the reader's good knowledge of general topology.*

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