Preface

The main goal of these lectures is to give an introduction to sub-Riemannian geometry and optimal transport, and to present some of the recent progress in these two fields. This set of notes is divided into three chapters and two appendices. Chapter 1 is concerned with the notions of totally nonholonomic distributions and sub-Riemannian structures. The concepts of End-Point mappings and singular horizontal paths which play a major role through these lectures are introduced here. Chapter 2 deals with sub-Riemannian geodesics. We study first- and second-order variations of the End-Point mapping to derive necessary and sufficient conditions for an horizontal path to be minimizing. In Chap. 3, we study the Monge problem for sub-Riemannian quadratic costs. We give a crash-course in optimal transport theory and explain how the sub-TWIST condition together with the Lipschitz regularity of a "variational" cost implies the well-posedness of Monge's problem. Then, we study the fine regularity properties of sub-Riemannian distances to obtain existence and uniqueness of optimal transport maps in the sub-Riemannian context. We recall basic facts on ordinary differential equations in Appendix A and less classical results of differential calculus in normed vector spaces in Appendix B. The latter plays a key role in Chap. 2.

The reader of these notes should be familiar with the basics in differential geometry and measure theory. For further reading, we strongly encourage the reader to look at other texts in sub-Riemannian geometry and optimal transport. Multiple viewpoints always lead to deeper understanding and may open new directions for research. Among them, we may suggest the textbooks by Montgomery [2], Agrachev, Barilari and Boscain [1], and Villani [3].

This set of notes grew from a series of lectures that I gave during a CIMPA school in Beyrouth, Lebanon, on the invitation of Fernand Pelletier. I take the opportunity of this preface to warmly thank Ali Fardoun, Mohamad Mehdi, and Fernand Pelletier who organized the school, Ahmed El Soufi for his support and friendship, and through him the "Centre International de Mathématiques Pures et Appliquées." My gratitude goes also to all faculties and students who attended this sub-Riemannian CIMPA school in making it a success.

Nice, June 2013 Ludovic Rifford

References

1. Agrachev, A., Barilari, D., Boscain, U.: Introduction to Riemannian and sub-Riemannian geometry. To appear
2. Montgomery, R.: A tour of subriemannian geometries, their geodesics and applications. In: Mathematical Surveys and Monographs, vol. 91. American Mathematical Society, Providence, RI (2002)
3. Villani, C.: Optimal transport, Old and new. Springer-Verlag, Heidelberg (2008)

http://www.springer.com/978-3-319-04803-1

Sub-Riemannian Geometry and Optimal Transport

Rifford, L.

2014, VII, 140 p. 14 illus., Softcover
ISBN: 978-3-319-04803-1