

Chapter 1

Flow Through Capillary

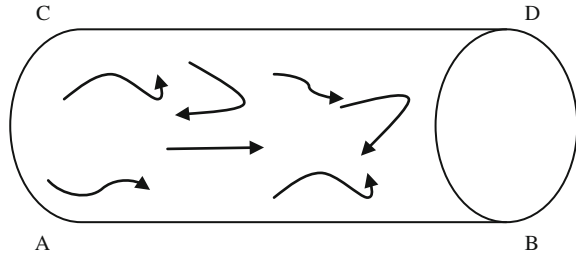
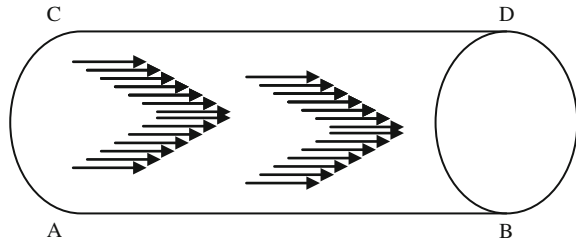
Abstract With introductory remarks about laminar and turbulent flow, expression for rate of discharge of liquid for a laminar flow has been derived. The expression does contain kinetic energy term. Different confusing numerical factors appearing in various literatures in the kinetic energy term have been explained. SI units of dynamic and kinematic viscosities together with their corresponding CGS units have been given. Kinematic viscosity in terms of efflux time for given volume of liquid is given in terms of efflux time. Corrections applicable to viscometer constants to capillary viscometers due to various parameters have been derived. The parameters are gravity, buoyancy, thermal expansion of viscometer bulb, different temperatures of loading and use, surface tension, temperature coefficient of viscosity etc.

1.1 Types of Flow

1.1.1 Turbulent Flow

The type of fluid flow, in which local velocities and pressures that fluctuate randomly, is known as turbulent flow. In turbulent flow the speed of the fluid at a point is continuously changing both in magnitude and direction. The flow of wind and rivers is generally turbulent, even if the currents are gentle. The air or water swirls and eddies while its overall bulk moves along a specific direction.

Most kinds of **fluid flow** are turbulent, except for laminar flow at the leading edge of solids moving relative to fluids or extremely close to solid surfaces, such as the inside wall of a pipe, or in cases of fluids of high viscosity (relatively great sluggishness) flowing slowly through small channels. Common examples of turbulent flow are **blood flow** in arteries, oil transport in pipelines, **lava flow**, atmosphere and **ocean currents**, the flow through pumps and turbines, and the flow in boat wakes and around aircraft-wing tips. The velocity profile of different layers in turbulent flow is shown in Fig. 1.1.

Fig. 1.1 Turbulent flow**Fig. 1.2** Laminar flow

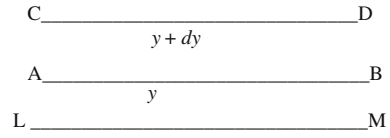
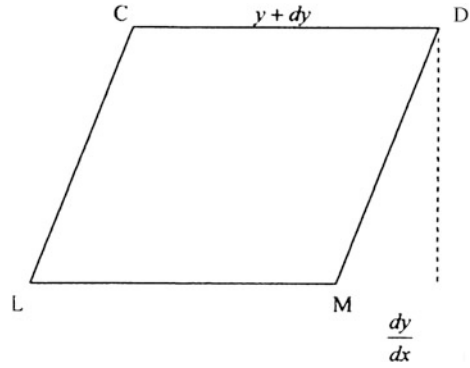
1.1.2 Laminar Flow

The laminar flow is the type of fluid flow in which the fluid moves in smooth paths or layers. All the fluid is flowing in the same direction. Fluid in contact of the solid stationary objects like the walls of a channel has zero velocity and velocity of consecutive layers increases away from the walls. In a closed channel say a cylindrical pipe the velocity of the layer of fluid along the axis of the pipe is maximum. The liquids of high viscosity moving in relatively small bore tube have the laminar flow. The velocity profile of different layers of liquids with laminar flow in a circular tube is shown in Fig. 1.2.

1.2 Motion in Laminar Flow

In a laminar flow, due to relative motion of layers with respect of each other, a resistive force acts between any two consecutive layers. This resistive force per unit area (shearing stress) divided by rate of change of velocity (shearing strain) is known as coefficient of dynamic viscosity or simply dynamic viscosity. This is normally represented by η .

Let us consider the laminar flow of a liquid along a fixed surface LM, Fig. 1.3. It is experimentally found that a layer CD at a distance $y + dy$ from LM flows faster than the layer AB distance y from the fixed surface LM. If the difference between the velocities of two layers is dv , the velocity gradient along y - the direction perpendicular to the direction of flow is $\frac{dv}{dy}$. Due to this relative motion of

Fig. 1.3 Motion in laminar flow**Fig. 1.4** Shearing of the flowing liquid

the layers, resistive force due to internal friction or viscosity acts and whole liquid will shear in the direction of laminar flow and its vertical section will look like as shown in the Fig. 1.4. If F is the force between any two consecutive layers having an area A , then shearing stress is $\frac{F}{A}$ and shearing strain is $\frac{dy}{dx}$. Hence for liquids flowing with the laminar flow, the coefficient of dynamic viscosity η is given as

$$\eta = \frac{F/A}{\frac{dy}{dx}} \quad (1.1)$$

Giving F the frictional force as

$$F = \eta A \frac{dv}{dy} \quad (1.2)$$

This is the Newton's Law of viscous flow for stream line (laminar) flow.

1.3 Unit of Dynamic Viscosity

To obtain the unit of dynamic viscosity (symbol η), write the units of quantities in (1.1), in which it has been defined.

$$\text{SI Unit of dynamic viscosity} = \frac{\text{N/m}^2}{(\text{m/s})/\text{m}} = \frac{\text{pascal}}{1/\text{s}} = \text{Pa}\cdot\text{s} \quad (1.3)$$

Similarly the unit of dynamic viscosity η expressed in terms of stress (F/A) and strain

$$(\text{velocity gradient}) \text{ in CGS system} = \frac{\text{dyn/cm}^2}{(\text{cm/s})/\text{cm}} = \frac{\text{dyn}\cdot\text{s}}{\text{cm}^2} = \text{poise} \quad (1.4)$$

The poise is the unit of dynamic velocity in CGS system.

Expressing the CGS units in terms of SI units in (1.4), we get the relation between CGS and SI units.

$$\text{poise} = \frac{10^{-5}\text{N}\cdot\text{s}}{10^{-4}\text{m}^2} = 10^{-1}\text{Pa}\cdot\text{s} = 0.1 \text{ Pa}\cdot\text{s} \quad (1.5)$$

The symbol of CGS unit of dynamic viscosity poise is P, giving

$$1 \text{ P} = 0.1 \text{ Pa}\cdot\text{s} \quad (1.6)$$

or

$$\begin{aligned} 10^{-2} \text{ P} &= 1 \text{ cP} = 10^{-3} \text{ Pa}\cdot\text{s} = \text{mPa}\cdot\text{s} \\ 1 \text{ cP} &= 1 \text{ mPa}\cdot\text{s} \end{aligned} \quad (1.7)$$

In the older literature, the values of dynamic viscosity of different fluids are given in terms of centi-poise symbol (cP). Hence from (1.7) the values of dynamic viscosities in cP would remain unchanged when expressed in millipascal second (mPa·s) a sub-multiple of SI unit.

1.4 Rate of Flow in a Capillary

Assuming that there is a laminar flow that means (1) pressure over any cross-section is constant, (2) there is no radial flow and (3) the liquid in contact of the wall of the circular tube is stationary, we can arrive at a theoretical formula for rate of flow of the liquid.

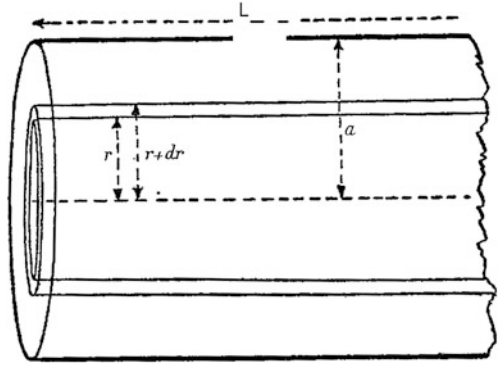
Referring to Fig. 1.5, let the velocity of the liquid layer at a distance r from the axis of the tube be v , and that of the layer $r + dr$ be $v + dv$, then the velocity gradient will be $-\frac{dv}{dr}$ giving tangential stress from (1.1) as

$$-\eta \frac{dv}{dr} \quad (1.8)$$

If the pressure difference between two points, distance L apart, is p , then the driving force on the liquid will be given as

$$\pi r^2 p \quad (1.9)$$

Fig. 1.5 Flow through a capillary



While the viscous force will be

$$-\eta \frac{dv}{dr} 2\pi r L \quad (1.10)$$

Here $2\pi r L$ is surface area of the cylindrical layer at a distance r from the axis. If this entire driving force is utilised in overcoming the viscous force, then

$$p\pi r^2 = -\eta \frac{dv}{dr} 2\pi r L \quad (1.11)$$

Giving

$$\frac{dv}{dr} = -\frac{rp}{2\eta L} \quad (1.12)$$

We know that v is zero at the wall i.e. at $r = a$. Integrating the (1.12) for r between 0 to a , we get

$$v = \frac{p}{4\eta L} (a^2 - r^2) \quad (1.13)$$

The velocity profile across the cross-section is, therefore, a parabola.

Here $2\pi r dr$ is the cross-sectional area of the ring bounded by the two layers dr apart and v is the velocity so dQ the rate of flow of the liquid is the product of area of cross-section and velocity v of the liquid. Hence dQ is given by

$$dQ = 2\pi r v dr$$

Substituting the value of v from (1.13) and integrating it with respect of r for limits 0 to a , and, we get

$$\begin{aligned} Q &= 2\pi \frac{p}{4\eta L} \int_0^a r(a^2 - r^2) dr \\ Q &= \frac{p\pi a^4}{8\eta L} \end{aligned} \quad (1.14)$$

If V is the volume of liquid flown in the time T then

$$\frac{V}{T} = Q = \frac{p\pi a^4}{8\eta L} \quad (1.15)$$

giving η as

$$\eta = \frac{p\pi a^4}{8VL} T. \quad (1.16)$$

1.4.1 Kinetic Energy Correction

In driving (1.11) it is assumed that entire pressure difference p is used for overcoming the viscous forces, however it is a fact that the liquid has acquired a certain velocity hence some pressure difference is required for imparting this velocity. The rate of kinetic energy acquired by the liquid is derived as follows:

Volume flow rate of the liquid between the two layers dr apart and at a distance r from the axis is

$$2\pi r v dr$$

If ρ is the density of the liquid then mass flow rate is given by

$$2\pi \rho r v dr$$

Hence rate of kinetic energy acquired by the liquid $\frac{1}{2} 2\pi \rho r v dr v^2$

Using (1.13), total rate of kinetic energy K is given as

$$K = \int_0^a \pi \rho r \left(\frac{p}{4\eta L} (a^2 - r^2) \right)^3 dr$$

$$\begin{aligned} \text{Put } a^2 - r^2 &= z \\ -2r dr &= dz \end{aligned}$$

The limits of the variable z are a^2 to 0, giving us

$$\begin{aligned} K &= - \int_{a^2}^0 \frac{\pi \rho}{2} \left(\frac{p}{4\eta L} \right)^3 z^3 dz = \frac{\pi \rho}{2} \left(\frac{p}{4\eta L} \right)^3 \frac{a^8}{4} \\ &= \left(\frac{\pi p a^4}{8\eta L} \right)^3 \frac{\rho}{\pi^2 a^4} = \frac{Q^3 \rho}{\pi^2 a^4} \end{aligned} \quad (1.17)$$

The energy spent per second in moving the liquid through capillary is pQ . If p_1 is total external pressure then $p_1 Q$ is total rate of loss of energy giving us

$$p_1 Q = p Q + \frac{Q^3 \rho}{\pi^2 a^4} \quad (1.18)$$

Giving

$$p = p_1 - \frac{Q^2 \rho}{\pi^2 a^4} \quad (1.19)$$

Substituting this value of p in (1.16) and replacing Q by $\frac{V}{T}$, we get

$$\begin{aligned} \eta &= \frac{\pi a^4}{8VL} T \left[p_1 - \frac{V^2 \rho}{T^2 \pi^2 a^4} \right] \\ \eta &= \frac{\pi p_1 a^4}{8LV} T - \frac{V \rho}{8\pi L} \frac{1}{T} \end{aligned} \quad (1.20)$$

In driving (1.20), the energy required is only for the axial flow, however there is some radial flow also, so we use a coefficient say m to compensate for radial kinetic energy. Hence more complete equation is

$$\eta = \frac{\pi p_1 a^4}{8LV} T - \frac{V \rho}{8\pi L} \frac{m}{T} \quad (1.21)$$

In case of a capillary viscometer

$$p_1 = h\rho(1 - \sigma/\rho)g \quad (1.22)$$

h being the fall of the menisci when fixed volume of the liquid has flown through the capillary viscometer.

Substituting the value of p_1 from (1.22) and dividing both sides of (1.21) by ρ , we get

$$\frac{\eta}{\rho} = \frac{\pi h(1 - \sigma/\rho)ga^4}{8VL} T - \frac{mV}{8\pi L} \frac{1}{T} \quad (1.23)$$

For a given viscometer $\frac{V}{8\pi L}$ is constant, so writing B for it, and

C for $\frac{\pi ha^4(1-\sigma/\rho)g}{8LV}$, (1.23) may be written as

$$\frac{\eta}{\rho} = CT - \frac{mB}{T} \quad (1.24)$$

In earlier days m was taken as constant however Cannon and Manning [1] proved that m is not constant. For a trumpet shaped capillary ends m can be represented best as

$$m = 0.037(\text{Re})^{1/2} \quad (1.25)$$

Multiplying factor is 0.037 for trumpet shape ends of the capillary tube, for square cut ends this factor is more than 0.037. However capillary tubes with square

cut ends are not recommended as there is no specific advantage of the use of such tubes but with a distinct disadvantage of having more kinetic energy correction.

Here Re is the Reynolds number given by

$$Re = \frac{\rho v L_1}{\eta} \quad (1.26)$$

Here

ρ is the density of the liquid

v is the average velocity

L_1 is distance over which velocity is averaged out and

η is the dynamic viscosity.

In the case of flow through a capillary, average velocity $v = \frac{Q}{\pi D^2/4} = \frac{4v}{\pi D^2 T}$ and the L_1 in (1.26) is the diameter D as the average is taken over the diameter.

Substituting the values of v and L_1 in the definition of Reynolds number, (1.25) becomes

$$m = 0.037 \left[\frac{4VD\rho}{\pi D^2 T \eta} \right]^{1/2} \quad (1.27)$$

For a properly made viscometer the second term in (1.24) is only 3 % of the first term, hence replacing η/ρ by CT in (1.27) will cause an error of not more than 0.09 %.

Hence (1.27) becomes

$$m = 0.074 \left[\frac{V}{\pi CDT^2} \right]^{1/2} \quad (1.28)$$

The second term in (1.24) becomes

$$\begin{aligned} \frac{mB}{T} &= \frac{1}{T} 0.074 \left[\frac{V}{\pi CD} \right]^{1/2} \frac{1}{T} \frac{V}{8\pi L} \\ &= \frac{0.074 V^{3/2}}{8\pi^{3/2} L (CD)^{1/2}} \frac{1}{T^2} \end{aligned} \quad (1.29)$$

$$\frac{mB}{T} = \frac{0.00166 V^{3/2}}{L(CD)^2} \cdot \frac{1}{T^2} \quad (1.30)$$

Substituting the value of $\frac{mB}{T}$ from (1.29) in (1.24), we get

$$\frac{\eta}{\rho} = CT - \frac{E}{T^2} \quad (1.31)$$

Here

$$E = \frac{0.00166V^{3/2}}{L(CD)^{1/2}} \quad (1.32)$$

But $\frac{\eta}{\rho}$ is the kinematic viscosity of the liquid with symbol ν . So the ν can be expressed as

$$\nu = CT - \frac{E}{T^2} \quad (1.33)$$

Here

$$C = \frac{\pi ha^4(1 - \sigma/\rho)g}{8LV} \quad \text{and} \quad E = \frac{0.00166V^{3/2}}{L(CD)^{1/2}} \quad (1.34)$$

It can be easily seen that the dimension of each term on right hand side of (1.33) is (Length)²/time. Hence SI unit of ν is m² s⁻¹, of C is m² s⁻² and that of E is m² s.

It has been noticed that the numerical factor of E has been given different in various national and International documents. The reason is the use of mixed system of units. The value of the multiplying factor for E in (1.32) is 0.00166 when all the quantities are expressed in coherent units of measurement. Say if V is in m³, L and D should be in m and C in m²/s² then unit of E is m²s and that of ν is m²/s. Similarly if we take V in cm³, L and D in cm and C in cm²/s² then unit of E shall be cm² s and that ν of cm²/s.

However if V is taken in cm³, L and D are in cm but C in mm²/s² (mixed system of units) then the factor in E becomes

$$\begin{aligned} \frac{0.00166(10^{-6}\text{m}^3)^{3/2}}{10^{-2}\text{m}(10^{-6}\text{m}^2\text{s}^{-2}10^{-2}\text{m})^{1/2}} &= 0.00166 \frac{10^{-9}}{10^{-6}} \text{m}^2 \text{s} \\ &= 0.00166 \cdot 10^{-3} \cdot 10^6 \text{mm}^2 \text{s} \\ &= 1.66 \text{mm}^2 \text{s} \end{aligned} \quad (1.35)$$

This is the value of the factor given by Fujita et al. in Metrologia paper 2009 [2]. If V is taken in cm³, L and D are in mm and C in mm²/s², then the factor in E is

$$\begin{aligned} \frac{0.00166(10^{-6}\text{m}^3)^{3/2}}{10^{-3}\text{m}(10^{-6}\text{m}^2\text{s}^{-2} \cdot 10^{-3}\text{m})^{1/2}} &= \frac{0.00166 \cdot 10^{-9}\text{m}^9/2}{10^{-3}\text{m} \cdot 10^{-9/2}\text{m}^{3/2}\text{s}^{-1}} \\ &= 0.00166 \cdot 10^{-3/2}\text{m}^2 \text{s} \\ &= 0.00166 \cdot 10^{-3/2} \cdot 10^6 \text{mm}^2 \text{s} \\ &= 52.5 \text{mm}^2 \text{s} \end{aligned} \quad (1.36)$$

ASTM D446-07 [3] has given this value of the factor in E .

1.4.2 End Correction

An additional correction is required to account for the peculiar phenomena at the ends of the tube. It is the work done against the viscous forces in the re-arrangement of velocity distribution. The effect of this is the resistance offered in converging at the entrance and diverging of steam at the exit ends of the tube. The effect is proportional to the radius of the tube and is expressed as a hypothetical addition to the length. The effective length L of the tube then becomes $L + na$. For further details of end corrections (change in effective length of the flow tube) one may consult [4]. So finally (1.21) becomes:

$$\eta = \frac{\pi a^4 p_1}{8V(L + na)} T - \frac{m\rho V}{8\pi(L + na)T}. \quad (1.37)$$

1.5 Units of Kinematic Viscosity

By definition the kinematic viscosity of a fluid is the ratio of dynamic viscosity to its density.

Units of dynamic viscosity is

$$\text{Pa}\cdot\text{s} = \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-2}\cdot\text{s} = \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$$

$$\text{Unit of kinematic viscosity} = \frac{\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}}{\text{kg}\cdot\text{m}^{-3}} = \text{m}^2\cdot\text{s}^{-1} \quad (1.38\text{A})$$

But metre square per second is quite big unit so it's sub-multiple namely mm^2/s is used. For example $1.0034 \text{ mm}^2/\text{s}$ is the value of kinematic viscosity of double distilled water at 20°C .

$$\text{m}^2/\text{s} = 10^6 \text{ mm}^2/\text{s} \quad (1.38\text{B})$$

CGS unit of kinematic viscosity is stoke (St) = $[\{\text{dyne}\cdot\text{cm}^{-2}\}\cdot\text{s}]/\text{g}\cdot\text{cm}^{-3}$

Expressing the right hand side in SI units, we get

$$\begin{aligned} &= [10^{-5}\text{N}/10^{-4}\text{m}^2\cdot\text{s}] / 10^3\text{kg}/\text{m}^3 \\ &= 10^{-4}\text{m}^2/\text{s} = 10^2\text{mm}^2/\text{s} \end{aligned}$$

Giving us

$$1 \text{ centi-stoke (cSt)} = 1 \text{ mm}^2/\text{s} \quad (1.39)$$

The values of kinematic viscosities expressed in centi-stokes remain unchanged when quoted in mm^2s^{-1} .

1.6 Corrections to C Due to Various Parameters

Here we see that C the so called viscometer constant depends upon

1. the local acceleration due to gravity
2. the buoyancy:- densities of liquids and air used
3. the effective value of h which depends on several factors such as the difference in rise of liquid due to different surface tension of the liquids used and the difference in temperatures at which the viscometer was calibrated and used.
4. the volume of liquid V which in turn depends upon the temperature and expansion coefficients of liquid and glass used and essentially affect the effective pressure head h .

1.6.1 Correction Due to Gravity

If a viscometer is calibrated at a place with g_c as the acceleration due to gravity and is used at a place having acceleration due to gravity as g_u then the values of C at the places of calibration and use respectively are given as

$$C_c = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV} \quad (1.40)$$

$$C_u = \frac{\pi a^4 (1 - \sigma/\rho) g_u}{8LV} \quad (1.41)$$

Dividing (1.41) by (1.40), we get

$$C_u = C_c \frac{g_u}{g_c} \quad (1.42)$$

1.6.2 Buoyancy Correction

The c_b is air buoyancy correction on liquids of different density at the time of calibration and use. It is relative to calibration constant of viscometer C_c .

The viscometer constant C determined by calibrating the viscometer, using the standard oil of density ρ_c and with air density σ_c , is expressed as

$$C_c = \frac{\pi a^4 (1 - \sigma_c/\rho_c) g_c}{8LV} \quad (1.43)$$

Similarly when the same viscometer is used to determine the viscosity of an oil of density ρ_u when air density is σ_u , C_u will be expressed as

$$C_u = \frac{\pi a^4 (1 - \sigma_u / \rho_u) g_c}{8LV} \quad (1.44)$$

Dividing the two, we get

$$\frac{C_u}{C_c} = \frac{(1 - \sigma_u / \rho_u)}{(1 - \sigma_c / \rho_c)} = (1 - \sigma_u / \rho_u + \sigma_c / \rho_c) \quad (1.45)$$

$$C_u = C_c \{1 + (\sigma_c / \rho_c - \sigma_u / \rho_u)\} = C_c (1 + c_b)$$

Giving c_b the buoyancy correction relative to C_c as

$$c_b = (\sigma_c / \rho_c - \sigma_u / \rho_u). \quad (1.46)$$

1.6.3 Correction Due to Thermal Expansion of Viscometer Bulb

The c_e is the correction for the effect of thermal expansion of the viscometer due to different temperatures at which the viscometer was calibrated and used for measurement of viscosity of a liquid.

Let volume of the timing bulb be V_o at $^\circ\text{C}$ then volume V_c at the temperature of calibration t_c will be

$$V_c = V_o (1 + 3\alpha t_c) \quad (1.47)$$

Here α is the coefficient of linear expansion of the material of the viscometer. Similarly V_u at the temperature of use (measurement) t_m is given by

$$V_u = V_o (1 + 3\alpha t_m) \quad (1.48)$$

Now viscometer constants at the temperatures of calibration and use respectively are:

$$C_c = \frac{\pi a^4 (1 - \sigma / \rho) g_c}{8LV_c} \quad (1.49)$$

$$C_u = \frac{\pi a^4 (1 - \sigma / \rho) h g_c}{8LV_u} \quad (1.50)$$

Substituting the value of V_u from (1.48) in (1.50) and dividing (1.50) by (1.49) we get

$$C_u = C_c \frac{(1 + 3\alpha t_c)}{(1 + 3\alpha t_m)} \quad (1.51)$$

or

$$C_u = C_c \{1 - 3\alpha(t_m - t_c)\} \quad (1.52)$$

The correction c_e due to thermal expansion of bulb relative to C_c is given by

$$c_e = -3\alpha(t_m - t_c). \quad (1.53)$$

1.6.4 Correction to C Due to Different Temperatures of Loading and Use

Some viscometers like Cannon–Fenske routine, Pinkevitch, Cannon–Manning semi-micro and Cannon–Fenske opaque are charged (loaded) at room temperature but are calibrated or used at another fixed temperature of bath. The temperature of bath, depending upon the kinematic viscosity of the liquid under-measurement, may be any thing between -30 and 100 °C. As the capillary extends on both sides of the two fixed marks, a small change in volume will have larger change in the pressure head h . The change in volume of the under-measurement liquid will cause a change in the effective pressure head h .

If the viscometer is loaded with volume V_f at temperature t_f and measurement is carried out at a different temperature t_m then a correction c_f for thermal expansion is to be applied. If t_f is higher than t_m , then V_f the volume of liquid will be reduced on attaining the bath temperature and also the capacity of the timing bulb will reduce. The difference in two reductions will be the net reduction in volume. The net reduction in volume will affect the effective pressure head h . The net reduction divided by the effective area of the capillary at the timing mark will be the reduction in the effective pressure head h .

We know the mass of the loaded sample remains unchanged, hence

$$V_f \rho_f = V_m \rho_m$$

Giving

$$V_m = V_f \frac{\rho_f}{\rho_m} \quad (1.54)$$

The change in volume of the liquid, therefore, is

$$V_m - V_f = V_f \left[\frac{\rho_f}{\rho_m} - 1 \right] \quad (1.55)$$

The reduction in capacity v_c of the viscometer is given by

$$v_c = 3V_f \alpha(t_m - t_f) \quad (1.56)$$

$$\text{Net change in volume} = V_f \left[\frac{\rho_f}{\rho_m} - 1 \right] - 3\alpha V_f (t_m - t_f) \quad (1.57)$$

But reduction in h is net reduction in volume divided by the area of cross-section at the lower timing mark, giving us

$$\begin{aligned} \text{Reduction in } h &= \frac{V_f \{ (\rho_f / \rho_m - 1) - 3\alpha (t_m - t_f) \}}{\pi a^2} \\ &= h_c - h_u \end{aligned} \quad (1.58)$$

Now

$$C_c = \frac{\pi a^4 (1 - \sigma / \rho) h_c g_c}{8LV_u} \quad (1.59)$$

And

$$C_u = \frac{\pi a^4 (1 - \sigma / \rho) h_u g_c}{8LV_u} \quad (1.60)$$

Subtracting

$$C_c - C_u = \frac{\pi a^4 (1 - \sigma / \rho) g_c (h_c - h_u)}{8LV_u} \quad (1.61)$$

Giving

$$C_u = C_c - C_c \frac{(h_c - h_u)}{h_c} \quad (1.62)$$

$$C_u = C_c (1 + c_f) \quad (1.63)$$

From (1.58) and (1.63) we get

$$c_f = \frac{V_f \{ (\rho_f / \rho_m - 1) - 3\alpha (t_m - t_f) \}}{\pi a^2 h_c} \quad (1.64)$$

Here V_f , a and h should be measured in coherent units, otherwise an appropriate factor will appear. If h and a are expressed in millimetres the V_f should also be expressed in cubic millimetres. Normally, however, a and h are taken in millimetres and volume V_f is taken in centimetre cube then new value of c_f is multiplied by 1000 giving us

$$c_f = 1000 \frac{V_f \{ (\rho_f / \rho_m - 1) - 3\alpha (t_m - t_f) \}}{\pi a^2 h} \quad (1.65)$$

Normally instead of radius a , the diameter d is measured hence a may be replaced by $d/2$ giving us

$$c_f = 4000 \frac{V_f \{ (\rho_f / \rho_m - 1) - 3\alpha(t_m - t_f) \}}{\pi d^2 h} \quad (1.66)$$

This is the expression used in ASTM D446-07. This correction is applied to the viscometer constant at temperatures other than the calibration temperature for Cannon–Fenske routine, Pinkevitch and Cannon–Manning semi-micro viscometers. The correction for Cannon–Fenske opaque (Reverse flow) viscometer given in (1.66) will be negative.

1.6.5 Correction to C Due to Change in Surface Tension

If the two menisci have different average radii and the surface tension of liquid under measurement differs substantially from the liquid by which viscometer was calibrated then a surface tension correction c_s relative to C_c is necessary.

We know that

$$\delta h \text{ the rise of liquid due to surface tension} = \frac{2\Gamma}{\rho g r} \quad (1.67)$$

Here Γ is surface tension, ρ is density of the liquid, g is acceleration due to gravity and r is radius of the capillary at the point of rise in liquid.

Let r_u and r_L be the radii of the timing bulb at the upper and lower graduation mark respectively; Γ_c be surface tension and ρ_c be the density of liquid used at the time of calibration.

Rise of liquid at the upper graduation mark δh_u will be

$$\delta h_u = \frac{2\Gamma_c}{\rho_c g r_u} \quad (1.68)$$

Similarly rise due to surface tension at the lower graduation mark δh_L is given by

$$\delta h_L = \frac{2\Gamma_c}{\rho_c g r_L} \quad (1.69)$$

The net change in effective height between the two graduation marks δh_c is given as

$$\delta h_c = \delta h_u - \delta h_L$$

or

$$\delta h_c = \frac{2\Gamma_c}{\rho_c g} \left(\frac{1}{r_u} - \frac{1}{r_L} \right) \quad (1.70)$$

Similarly if a different liquid of density ρ_t having surface tension Γ_t is used then, net change in effective height δh_{uu} between the two graduation marks is given as

$$\delta h_{uu} = (\delta h_u - \delta h_L)_{use}$$

or

$$\delta h_{uu} = \frac{2\Gamma_t}{\rho_t g} \left(\frac{1}{r_u} - \frac{1}{r_L} \right) \quad (1.71)$$

But

$$C_c = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV_u} (h_c + \delta h_c) \quad (1.72)$$

And

$$C_u = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV_u} \cdot (h_c + \delta h_{uu}) \quad (1.73)$$

Subtracting

$$C_u - C_c = \frac{\pi a^4 (1 - \sigma/\rho) g_c}{8LV_u} (\delta h_{uu} - \delta h_c) \quad (1.74)$$

$$C_u - C_c = C_c \frac{(\delta h_{uu} - \delta h_c)}{h_c} \quad (1.75)$$

Or

$$C_u = C_c \left[1 + \frac{(\delta h_{uu} - \delta h_c)}{h_c} \right] \quad (1.76)$$

$$C_u = C_c [1 + c_s] \quad (1.77)$$

But $\frac{(\delta h_u - \delta h_c)}{h_c} = \delta h_{eff}$, which from (1.70) and (1.71) is given as

$$\delta h_{eff} = \frac{2}{gh_c} \left(\frac{1}{r_u} - \frac{1}{r_L} \right) \left(\frac{\Gamma_c}{\rho_c} - \frac{\Gamma_t}{\rho_t} \right) \quad (1.78)$$

Hence

$$c_s = \frac{2}{gh_c} \left(\frac{1}{r_u} - \frac{1}{r_L} \right) \left(\frac{\Gamma_c}{\rho_c} - \frac{\Gamma_t}{\rho_t} \right) \quad (1.79)$$

Here all parameters like g , h , r_u , r_L , Γ_c , Γ_t , ρ_c and ρ_t must be expressed in SI units or in a set of coherent units.

1.6.6 Temperature Correction to Kinematic Viscosity

The kinematic viscosity of all liquids is temperature dependent. Hence there is a correction c_t due to the actual measurement temperature and the standard temperature at which viscosity value is stated. If α_v is coefficient of kinematic viscosity of the liquid under test and t_m and t_s are the measurement and standard temperatures then c_t is given as

$$c_t = \alpha_v(t_m - t_s). \quad (1.80)$$

References

1. Cannon MR, Manning RE (1960) The kinetic energy correction and a new viscometer. Anal Chem 32:355–360
2. Fujita Y, Kurano Y, Fujii K (2009) Evaluation of uncertainty in viscosity measurements by capillary master viscometers. Metrologia 46:237–248
3. ASTM D446-07 (2007) Standard Specifications and operating instructions for glass capillary kinematic viscometers
4. Barr G (1931) A monograph of viscometry, Chapter II. Humphrey Milford, New York

Viscometry for Liquids

Calibration of Viscometers

Gupta, S.V.

2014, XV, 256 p. 81 illus., 3 illus. in color., Hardcover

ISBN: 978-3-319-04857-4