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★Janus-faced probability.

With a preface by Andrei Khrennikov.
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While probability theory finds fruitful applications in many domains of human knowledge—such as physics, biology, economics and sociology among others—the interpretation of this discipline remains challenging and a great deal of debate still continues among the different schools. An example of this is the debate between the frequentist school of von Mises and the subjective school of de Finetti, just to mention two of them among others.

In this book, Paolo Rocchi addresses the problem of the interpretation and the foundations of probability theory. He endorses a perspective in which the various conceptual facets of probability theory can be considered in a unified framework. In this way, he presents a pluralistic approach in which the subjectivist and the objectivist schools have different well-defined domains of application. This novel perspective certainly opens an interesting debate. The working scientist applies different approaches to probability theory and statistics depending on the particular problem he is dealing with, and the most important schools seem to work perfectly in practice (provided that they are restricted to a suitable domain of application). In view of this state of affairs, it seems natural to look for an interpretation of probability theory which stands closer to the real situations that the working scientist faces everyday in his research. And this is just what Paolo Rocchi looks for in this book: he presents a perspective in which all these different applications can be considered in a unified framework.

The book is also very readable and constitutes an excellent choice for those who look for an introduction to the foundational problems of probability theory and statistics. It is also important to mention that the bibliographic list is delightful and very complete. It contains a nice appendix (Appendix A) in which the different schools of probability theory and the criticisms raised against them are reviewed, both from a historical and a conceptual point of view, which is a contribution by itself. I highly recommend to the non-specialized reader to take a look at Appendix A before reading the whole book.

The proposal of the book consists in making a clear cut between two very different situations: when the number of trials at hand is very large and when we only have one trial. In the first scenario, the von Mises approach seems more adequate and in the second one, the de Finetti schema applies. In other words, it puts a clear cut between objectivist and subjectivist approaches to probability theory.

The book is separated into two parts. The first one relates to the meanings and interpretations of probability. And the second one continues with the problems related to the axiomatic approach, where further developments are investigated.

The first part starts by posing the problem of interpretations of probability theory, putting the emphasis in the pluralistic approach and showing how the different schools are applied successfully in many different domains of human knowledge. An important list of references in which related pluralistic views are discussed is shown in Appendix B. In Chapter 2 the author presents arguments for a mathematical approach to the interpretation problem. The problem of probability validation is addressed in Chapter 3. The compatibility of the diverging interpretations is discussed in Chapter 4. In Chapter 5, named "Criticism on the philosophical pollution", the author advocates

for a formulation of the problem which is independent of philosophical discussions. The book separates between "speculative inquiries of pure mathematicians" and the "problem of testing probability by means of experiments". Personally, I find this choice a weak point rather than an advantage. This becomes clear if it is taken into account that there are philosophical approaches which take ontological pluralism as a starting point (see, for example, [O. Lombardi and M. Labarca, Found. Chem. **7** (2005), no. 2, 125–148, doi:10.1007/s10698-004-0980-6] and references therein for a discussion on the subject). It is certainly a pity that the author does not go further into these matters: the task of developing an interpretation of probability theory based in ontological pluralism could help to improve the arguments, providing a solid ground for the "Janus faced" approach presented in this book. I believe that this certainly constitutes a field to explore in further investigations.

The second part of the book centers on the problems related to the probability axiomatization, putting emphasis on the probability argument (the assortment of variables to which the probability P refers), which is largely discussed in Chapter 6. The classical approach to the modeling of the probability argument is discussed in Chapter 7. Chapters 8, 9 and 10 are devoted to the structural modeling of the probability argument illustrating how events can be described using this framework by providing some examples and variants. An entity relationship language is presented, in which the main elements are entities related by directed relationships connecting them. The problem of the probability argument is studied under the light of this approach.

The physical examples used in the book are clear and well chosen but somewhat out of date. There has been an outstanding development in quantum statistical techniques with the advent of quantum information theory and the possibility offered by quantum computation [M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*, Cambridge Univ. Press, Cambridge, 2000; MR1796805 (2003j:81038)]. In this way, new quantum statistical techniques were developed [A. S. Kholevo, *Probabilistic and statistical aspects of quantum theory*, second edition, with a foreword from the second Russian edition by K. A. Valiev, Quad./Monogr., 1, Ed. Norm., Pisa, 2011; MR2797301 (2012e:81002)], but they are not discussed in detail in the book. Furthermore, it is known that probabilities appearing in quantum theory present some conceptual difficulties which go beyond the ones lurking in the classical theory, and that the formalism involved presents important differences from that of Kolmogorov (see, for example, [F. Holik, M. Sáenz and A. Plastino, Ann. Physics **340** (2014), 293–310; MR3144266] and references therein). While it is mentioned in the book (for example, the author mentions the works of L. Accardi [in *Quantum probability and applications to the quantum theory of irreversible processes (Villa Mondragone, 1982)*, 1–19, Lecture Notes in Math., 1055, Springer, Berlin, 1984; MR0782889 (87m:81019); Phys. Rep. **77** (1981), no. 3, 169–192; MR0639024]), the problem of quantum probabilities is not discussed in detail. Indeed, in Section 7.1 this problem is mentioned explicitly in connection with the double slit experiment and the inadequacy of the set theoretical approach to represent all possible events. But the analysis suddenly stops, and these wholly important matters—for any interpretation of probability theory which aims to be based in a solid scientific foundation—are not investigated further. This is a pity, because it could be a great contribution to the field, especially if one takes into account the proposals of Chapters 8, 9 and 10 of the book. The problem of studying the applicability of the structural models approach to quantum physics is certainly in order, and this is an interesting open question examined by the book. Indeed, one may wonder if the framework discussed in Chapters 8, 9 and 10 can be related to quantum preparations and quantum states. Another important question is if it is possible to accommodate the non-commutative setting of quantum mechanics (and other theories such as quantum field theory) in the

proposed formalism.

To summarize, I highly recommend this book for those readers interested in an accessible introduction to the problems related to the foundations of probability theory and statistics, with a very interesting compilation of references on the subject. Beside this, the ideas presented in the book open the door to interesting research inquiries in the field, especially those related to the foundations of quantum probability theory, quantum information theory and quantum statistical techniques. *Federico Holik*

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