

Preface

Combinatorics and Algebraic Geometry have enjoyed a fruitful interplay since the nineteenth century. Classical interactions include invariant theory, theta functions, and enumerative geometry. The aim of this volume is to introduce recent development in combinatorial algebraic geometry.

The five chapters of this book are based on the lectures delivered at the CIME-CIRM summer-school, Levico Terme, June 10–15, 2013.

We here regard algebraic geometry with a view towards applications, such as tensor calculus and algebraic statistics. A common theme is the study of algebraic varieties endowed with a rich combinatorial structure. Relevant techniques are polyhedral geometry, free resolutions, multilinear algebra, projective duality, and compactifications.

Aldo Conca offers an introduction to *Koszul Algebras and Their Syzygies*. Koszul algebras are fundamental in commutative algebra, and they have numerous applications in algebraic geometry. One application presented here is the study of Castelnuovo–Mumford regularity of projective varieties. Other results presented in this chapter concern syzygies of Koszul algebras, the Koszul property of Veronese algebras, and algebras in the theory of hyperspace arrangements.

Systems of polynomial equations in infinitely many variables arise naturally in applied algebraic geometry. Typically, these infinite-dimensional systems have a lot of symmetry, and, in favorable circumstances, one encounters *Noetherianity up to Symmetry*. Jan Draisma offers a glimpse on recent developments in this field. His chapter focuses on examples from algebraic statistics and on the combinatorics of well-quasi-ordered sets.

Maximum Likelihood Geometry studies the critical points of monomial functions over a variety inside the probability simplex. The number of complex critical points, known as its maximum likelihood degree, is a topological invariant. June Huh joined Bernd Sturmfels in writing a chapter, which introduces this theory and its statistical motivations. Many favorites from combinatorial algebraic geometry are featured: toric varieties, matroids, A-discriminants, Grassmannians, and determinantal varieties.

Sandra Di Rocco lectured on *Linear Toric Fibrations*, that is, toric varieties which are birational to projective toric bundles. On the combinatorial side, these correspond to Cayley polytopes, a class of highly structured lattice polytopes that has received much attention in the recent literature. This chapter presents geometrical phenomena, in algebraic geometry and neighboring fields, which are characterized by a Cayley structure.

Filippo Viviani takes the reader on *A Tour of Hermitian Symmetric Manifolds*. These are Hermitian manifolds which are homogeneous and such that every point has a symmetry preserving the Hermitian structure. Examples of such manifolds serve as moduli spaces in algebraic and analytic geometry. This chapter offers an introduction to several different perspectives from which Hermitian symmetric manifolds can be studied.

We thank the CIME foundation and the CIRM center for hosting the school and for their generous support. All of us had a wonderful time at Levico Terme. The beautiful scenery of Trentino made the mathematical interactions and the stimulating lectures even more enjoyable.

Stockholm, Sweden
Berkeley, CA
October 2013

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CIME activity is carried out with the collaboration and financial support of:

- INdAM (Istituto Nazionale di Alta Matematica)
- MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca)
- Ente Cassa di Risparmio di Firenze

Combinatorial Algebraic Geometry

Levico Terme, Italy 2013, Editors: Sandra Di Rocco,
Bernd Sturmfels

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B.; Viviani, F.

2014, VII, 239 p. 26 illus., 4 illus. in color., Softcover
ISBN: 978-3-319-04869-7