

Contents

Part I Characterization, Combination and Propagation of Errors

1	Basic Ideas of Measurement	3
1.1	Prologue	3
1.2	True Values	3
1.3	Measurement Uncertainties	4
1.4	Breakdown of the Gaussian Approach	5
1.5	Non-Gaussian Prospect	6
1.6	Appraising Measurement Uncertainties	11
1.7	Quotation of Numerical Values	14
2	Formalization of Measuring Processes	17
2.1	Random Variables	17
2.2	Probability Densities	18
2.3	Generalized Gaussian Error Calculus	23
2.4	Elements of Evaluation	25
2.5	Consistency	26
2.6	Traceability	27
3	Normal Parent Distributions	29
3.1	One-Dimensional Normal Density	29
3.2	Multidimensional Normal Density	33
3.3	Chi-Square and F Density	37
3.4	Student's (Gosset's) Density	40
3.5	Fisher's Density	44
3.6	Hotelling's Density	47
4	Estimators and Expectations	53
4.1	Expectations	53
4.2	Statistical Ensemble	54
4.3	Estimators	56
4.4	Behrens–Fisher Problem	61
4.5	Downfall of the Analysis of Variance	64

5	Bias and Randomness	67
5.1	Localization of the True Value x_0	67
5.2	Localization of the Parameter μ_x	67
5.3	Uncertainty of the Arithmetic Mean	68
5.4	Second Thoughts About Systematic Errors	73
5.5	Function of a Measurand	74
5.6	Systematic Error of the Measuring Object	80
6	Error Propagation, Two Variables	81
6.1	Series Expansions	81
6.2	Bias	82
6.3	Random Errors	83
6.4	Overall Uncertainty	86
7	Error Propagation, m Variables	97
7.1	Series Expansions	97
7.2	Bias	98
7.3	Random Errors	98
7.4	Overall Uncertainties	99
7.5	Traceability	101
8	Concatenated Functions	105
8.1	Series Expansions	105
8.2	Biases	107
8.3	Random Errors	108
8.4	Overall Uncertainty	108
8.5	Composed Covariance	109
Part II Least Squares Adjustment		
9	Least Squares Formalism	113
9.1	Geometry of Adjustment	113
9.2	Unconstrained Adjustment	116
9.3	Constrained Adjustments	120
9.4	Quintessence of the Method of Least Squares	121
10	Consequences of Systematic Errors	123
10.1	Structure of the Solution Vector	123
10.2	Minimized Sum of Squared Residuals	124
10.3	Gauss–Markoff Theorem	128
10.4	Choice of Weights	130
10.5	Consistency of the Input Data	132
11	Uncertainties of Least Squares Estimators	135
11.1	Non-weighted Adjustment	135
11.2	Weighted Adjustment	140
11.3	Weighted and Non-weighted Mean of Means	144
11.4	Function of Least Squares Estimators	150

12	Uncertainty Spaces	153
12.1	The New Solids	153
12.2	Confidence Ellipsoids	154
12.3	Security Polytopes	155
12.4	Composite Bounding Surfaces	164

Part III Linear and Linearized Systems

13	Straight Lines	173
13.1	Fitting Conditions	173
13.2	Straight Line (i)	175
13.3	Straight Line (ii)	182
13.4	Straight Line (iii)	191
14	Exponentials	203
14.1	Fitting Conditions	203
14.2	Series Expansion	203
14.3	Transformation	209
15	Planes	217
15.1	Fitting Conditions	217
15.2	Plane (i)	219
15.3	Plane (ii)	225
15.4	Plane (iii)	234
16	Circles	245
16.1	Fitting Conditions	245
16.2	Series Expansion	246
16.3	Stereographic Projection	252
17	Parabolas	265
17.1	Fitting Conditions	265
17.2	Parabola (i)	267
17.3	Parabola (ii)	272
17.4	Parabola (iii)	287
18	Least Squares Trigonometric Polynomials	299
18.1	Representation	299
18.2	Fitting Conditions	300
18.3	Uncertainty Band	304

Part IV Generic Metrological Issues

19	Dissemination of Units	311
19.1	Transfer of True Values	311
19.2	Working Standards	312
19.3	Key Comparisons	314

20	Calibration Chains	323
20.1	Mass Decades	323
20.2	Weighing Scheme	324
20.3	Least Squares Approach	326
20.4	Uncertainties	327
21	Pairwise Comparisons	329
21.1	Singular Design Matrix	329
21.2	On Constraints	330
21.3	Measuring Conditions	331
21.4	Least Squares Approach	332
21.5	Uncertainties	332
22	Fundamental Constants of Physics	337
22.1	The Constants at Present	337
22.2	Concatenations	339
22.3	Input Data	340
22.4	Linearization	342
22.5	Least Squares Estimators	343
22.6	Uncertainties	345
23	Essence of Metrology	349
23.1	Consistency	349
23.2	Traceability	349
Appendix A	Rank of Matrices	351
Appendix B	Variance–Covariance Matrices	353
Appendix C	Linear Functions of Normal Variables	357
Appendix D	Orthogonal Projections	361
Appendix E	Least Squares Adjustments	365
Appendix F	Expansion of Solution Vectors	369
Appendix G	Student’s Density	383
Appendix H	Quantiles of Hotelling’s Density	387
Appendix I	Graphical Scale Transformations	389
	References	395
	Index	399

Measurement Uncertainties in Science and Technology

Grabe, M.

2014, XIV, 401 p. 80 illus., 41 illus. in color., Hardcover

ISBN: 978-3-319-04887-1