

Chapter 2

Signal Processing at Receivers: Detection Theory

As an application of the statistical hypothesis testing, signal detection plays a key role in signal processing at receivers of wireless communication systems. To accept or reject a hypothesis based on observations, the hypotheses are possible statistical descriptions of observations using statistical hypothesis testing tools. As realizations of a certain random variable, observations can be characterized by a set of candidate probability distributions of the random variable.

In this chapter, based on the statistical hypothesis testing, we introduce the theory of signal detection and key techniques for performance analysis. We focus on the fundamentals of signal detection in this chapter, while the signal detection over multiple-antenna systems will be considered in the following parts of the book.

2.1 Principles of Hypothesis Testing

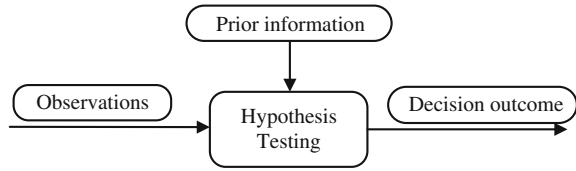
Three key elements are carried out in the statistical hypothesis testing, including

- (1) Observations.
- (2) Set of hypotheses.
- (3) Prior information.

The decision process or hypothesis testing is illustrated in Fig. 2.1. In Fig. 2.1, is shown that observations and prior information are taken into account to obtain the final decision. However, considering the cases that no prior information is available or prior information could be useless, the hypothesis test can also be developed with observations only.

Under the assumption that there exist $M(\geq 2)$ hypotheses, we can have an M -ary hypothesis testing in which we need to choose one of the M hypotheses that explains observations and prior information best. In order to choose a hypothesis, different criteria can be considered. According to these criteria, different hypothesis tests are

Fig. 2.1 Block diagram for hypothesis testing



available. Based on the likelihood ratio (LR)¹ hypothesis test; three well-known hypothesis tests are given as follows:

- (1) Maximum a posteriori probability (MAP) hypothesis test.
- (2) Bayesian hypothesis test.
- (3) Maximum likelihood (ML) hypothesis test.

In the following section, the hypothesis tests in the above are illustrated respectively.

2.2 Maximum a Posteriori Probability Hypothesis Test

Let us first introduce the MAP hypothesis test or MAP decision rule. Consider that there are different balls contained in two boxes (A and B), where a certain number is marked on each ball. Under the assumption that the distribution of the numbers on balls is different for each box, as a ball is drawn from one of the boxes, we want to determine the box where the ball is drawn from based on the number of the ball. Accordingly, the following two hypotheses can be founded:

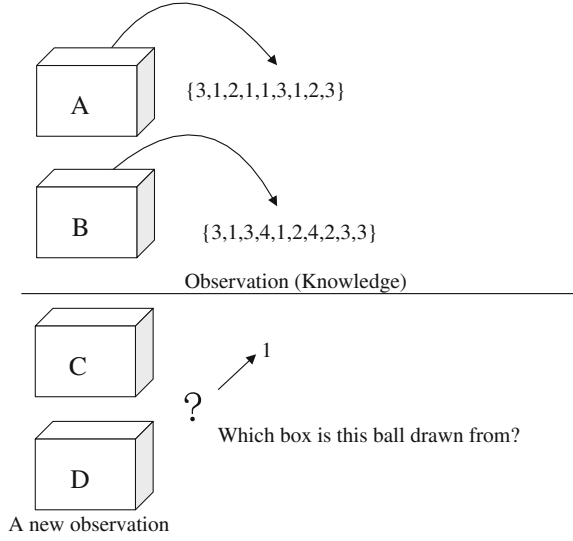
$$\begin{cases} \mathcal{H}_0 : \text{the ball is drawn from box A;} \\ \mathcal{H}_1 : \text{the ball is drawn from box B.} \end{cases}$$

For example, suppose that 10 balls are drawn from each box as shown in Fig. 2.2. Based on the empirical distribution results in Fig. 2.2, conditional distributions of the number on balls are given by

$$\begin{aligned} \Pr(1|\mathcal{H}_0) &= \frac{4}{10}; \\ \Pr(2|\mathcal{H}_0) &= \frac{3}{10}; \\ \Pr(3|\mathcal{H}_0) &= \frac{3}{10}; \end{aligned}$$

and

¹ Note that in Chaps. 2 and 3, we use LR to denote the term “likelihood ratio,” while in the later chapters of the book, the LR is used to represent “lattice reduction.”

Fig. 2.2 Balls drawn from two boxes

$$\Pr (1|\mathcal{H}_1) = \frac{2}{10};$$

$$\Pr (2|\mathcal{H}_1) = \frac{2}{10};$$

$$\Pr (3|\mathcal{H}_1) = \frac{4}{10};$$

$$\Pr (4|\mathcal{H}_1) = \frac{2}{10}.$$

In addition, the probability that A ($\mathcal{H}_0$) or B ($\mathcal{H}_1$) box is chosen is assumed to be the same, i.e.,

$$\Pr (\mathcal{H}_0) = \Pr (\mathcal{H}_1) = \frac{1}{2}. \quad (2.1)$$

Then, we can easily have

$$\Pr (1) = \Pr (\mathcal{H}_0) \Pr (1|\mathcal{H}_0) + \Pr (\mathcal{H}_1) \Pr (1|\mathcal{H}_1) = \frac{6}{20};$$

$$\Pr (2) = \Pr (\mathcal{H}_0) \Pr (2|\mathcal{H}_0) + \Pr (\mathcal{H}_1) \Pr (2|\mathcal{H}_1) = \frac{5}{20};$$

$$\Pr (3) = \Pr (\mathcal{H}_0) \Pr (3|\mathcal{H}_0) + \Pr (\mathcal{H}_1) \Pr (3|\mathcal{H}_1) = \frac{7}{20};$$

$$\Pr (4) = \Pr (\mathcal{H}_0) \Pr (4|\mathcal{H}_0) + \Pr (\mathcal{H}_1) \Pr (4|\mathcal{H}_1) = \frac{2}{20},$$

where $\Pr(n)$ denotes the probability that the ball with number n is drawn. Taking $\Pr(\mathcal{H}_k)$ as the a priori probability (APRP) of $\mathcal{H}_k$, the a posteriori probability (APP) of $\mathcal{H}_k$ is shown as follows:

$$\begin{aligned}\Pr(\mathcal{H}_0|1) &= \frac{2}{3}; \\ \Pr(\mathcal{H}_0|2) &= \frac{3}{5}; \\ \Pr(\mathcal{H}_0|3) &= \frac{3}{7}; \\ \Pr(\mathcal{H}_0|4) &= 0,\end{aligned}$$

and

$$\begin{aligned}\Pr(\mathcal{H}_1|1) &= \frac{1}{3}; \\ \Pr(\mathcal{H}_1|2) &= \frac{2}{5}; \\ \Pr(\mathcal{H}_1|3) &= \frac{4}{7}; \\ \Pr(\mathcal{H}_1|4) &= 1.\end{aligned}$$

Here, $\Pr(\mathcal{H}_k|n)$ is formed as the conditional probability that the hypothesis $\mathcal{H}_k$ is true under the condition that the number on the drawn ball is n . For example, if the number of the ball is $n = 1$, since $\Pr(\mathcal{H}_0|1) = \frac{2}{3}$ is greater than $\Pr(\mathcal{H}_1|1) = \frac{1}{3}$, we can decide that the ball is drawn from box A, where the hypothesis $\mathcal{H}_0$ is accepted. The corresponding decision rule is named as the MAP hypothesis testing, since we choose the hypothesis that maximizes the APP.

Generally, in the binary hypothesis testing, $\mathcal{H}_0$ and $\mathcal{H}_1$ are referred to as the null hypothesis and the alternative hypothesis, respectively. Under the assumption that the APRPs $\Pr(\mathcal{H}_0)$ and $\Pr(\mathcal{H}_1)$ are known and the conditional probability, $\Pr(Y|\mathcal{H}_k)$, is given, where Y denotes the random variable for an observation, the MAP decision rule for binary hypothesis testing is given by

$$\begin{cases} \mathcal{H}_0 : \Pr(\mathcal{H}_0|Y = y) > \Pr(\mathcal{H}_1|Y = y); \\ \mathcal{H}_1 : \Pr(\mathcal{H}_0|Y = y) < \Pr(\mathcal{H}_1|Y = y), \end{cases} \quad (2.2)$$

where y denotes the realization of Y . Note that $\mathcal{H}_0$ is chosen if $\Pr(\mathcal{H}_0|Y = y) > \Pr(\mathcal{H}_1|Y = y)$ and vice versa. Here, we do not consider the case of $\Pr(\mathcal{H}_0|Y = y) = \Pr(\mathcal{H}_1|Y = y)$ in (2.2), where a decision can be made arbitrarily. Thus, the decision outcome in (2.3) can be considered as a function of y . Using Bayes rule, we can also show that

$$\begin{cases} \mathcal{H}_0 : \frac{\Pr(Y = y|\mathcal{H}_0)}{\Pr(Y = y|\mathcal{H}_1)} > \frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_0)}; \\ \mathcal{H}_1 : \frac{\Pr(Y = y|\mathcal{H}_0)}{\Pr(Y = y|\mathcal{H}_1)} < \frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_0)}. \end{cases} \quad (2.3)$$

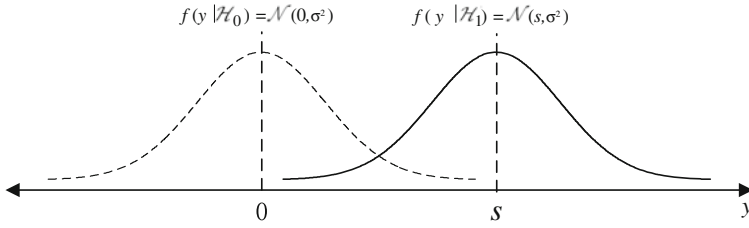


Fig. 2.3 The pdf of the hypothesis pair

Notice that as Y is a continuous random variable, $\Pr(Y = y | \mathcal{H}_k)$ is replaced by $f(Y = y | \mathcal{H}_k)$, where $f(Y = y | \mathcal{H}_k)$ represents the conditional probability density function (pdf) of Y given $\mathcal{H}_k$.

Example 2.1. Define by $\mathcal{N}(\mu, \sigma^2)$ the pdf of a Gaussian random variable (i.e., x) with mean μ and variance σ^2 , where

$$\mathcal{N}(\mu, \sigma^2) = \frac{\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}{\sqrt{2\pi\sigma^2}}. \quad (2.4)$$

Let the noise n be a Gaussian random variable with mean zero and variance σ , while s be a positive constant. Consider the case that a constant signal, s , is transmitted, while a received signal, y , may be corrupted by the noise, n , as shown in Fig. 2.3. Then, we can have the following hypothesis pair to decide whether or not s is present when y is corrupted by n :

$$\begin{cases} \mathcal{H}_0 : y = n; \\ \mathcal{H}_1 : y = s + n. \end{cases} \quad (2.5)$$

Then, as shown in Fig. 2.3, we have

$$\begin{cases} f(y | \mathcal{H}_0) = \mathcal{N}(0, \sigma^2); \\ f(y | \mathcal{H}_1) = \mathcal{N}(s, \sigma^2), \end{cases} \quad (2.6)$$

and

$$\frac{f(Y = y | \mathcal{H}_0)}{f(Y = y | \mathcal{H}_1)} = \exp\left(-\frac{s(2y - s)}{2\sigma^2}\right), \quad (2.7)$$

when $s > 0$. Letting

$$\rho = \frac{\Pr(\mathcal{H}_0)}{\Pr(\mathcal{H}_1)},$$

the MAP decision rule is simplified as follows:

Table 2.1 MAP decision rule

Accept	$\mathcal{H}_0$	$\mathcal{H}_1$
$\mathcal{H}_0$ is true	Correct	Type I (false alarm)
$\mathcal{H}_1$ is true	Type II (miss)	Correct (detection)

Table 2.2 The probabilities of type I and II errors

Error type	Case	Error probability
Type I	Accept $\mathcal{H}_1$ when $\mathcal{H}_0$ is true	P_A
Type II	Accept $\mathcal{H}_0$ when $\mathcal{H}_1$ is true	P_B

$$\begin{cases} \mathcal{H}_0 : y < \frac{s}{2} + \frac{\sigma^2 \ln \rho}{s}; \\ \mathcal{H}_1 : y > \frac{s}{2} + \frac{\sigma^2 \ln \rho}{s}. \end{cases} \quad (2.8)$$

Since the decision rule is a function of y , we can express the decision rule as follows:

$$\begin{cases} r(y) = 0 : y \in \mathcal{A}_0; \\ r(y) = 1 : y \in \mathcal{A}_1, \end{cases} \quad (2.9)$$

where $\mathcal{A}_0$ and $\mathcal{A}_1$ represent the decision regions of $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Therefore, for the MAP decision rule in (2.8), the corresponding decision regions are given by

$$\begin{cases} \mathcal{A}_0 = \left\{ y \mid y < \frac{s}{2} - \frac{\sigma^2 \ln \rho}{s} \right\}; \\ \mathcal{A}_1 = \left\{ y \mid y > \frac{s}{2} - \frac{\sigma^2 \ln \rho}{s} \right\}, \end{cases} \quad (2.10)$$

where $\mathcal{A}_0$ and $\mathcal{A}_1$ are regarded as the acceptance region and the rejection/critical region, respectively, in the binary hypothesis testing.

Table 2.1 shows four possible cases of decision, where type I and II errors are usually carried out to analyze the performance. Note that since the null hypothesis, $\mathcal{H}_0$, normally represents the case that no signal is present, while the other hypothesis, $\mathcal{H}_1$, represents the case that a signal is present, the probabilities of type I and II errors are regarded as the false alarm and miss probabilities, respectively. The two types of decision errors are summarized in Table 2.2. Using the decision rule $r(y)$, P_A and P_B are given by

$$P_A = \Pr(Y \in \mathcal{A}_1 | \mathcal{H}_0) \quad (2.11)$$

$$= \int r(y) f(y | \mathcal{H}_0) dy \quad (2.12)$$

$$= E[r(Y) | \mathcal{H}_0] \quad (2.13)$$

and

$$P_B = \Pr(Y \in \mathcal{A}_0 | \mathcal{H}_1) \quad (2.14)$$

$$= \int (1 - r(y)) f(y | \mathcal{H}_1) dy \quad (2.15)$$

$$= E[(1 - r(Y)) | \mathcal{H}_1], \quad (2.16)$$

respectively. Then, the probability of detection becomes

$$P_D = 1 - P_B \quad (2.17)$$

$$= E[r(Y) | \mathcal{H}_1]. \quad (2.18)$$

2.3 Bayesian Hypothesis Test

In order to minimize the cost associated with the decision, the Bayesian decision rule is carried out. Denote by D_k the decision that accepts $\mathcal{H}_k$, while by G_{ik} the associated cost of D_i when the hypothesis $\mathcal{H}_k$ is true. Assuming the cost of erroneous decision to be higher than that of correct decision, we have $G_{10} > G_{00}$ and $G_{01} > G_{11}$. The average cost $E[G_{ik}]$ is given by

$$\bar{G} = E[G_{ik}] \quad (2.19)$$

$$= \sum_i \sum_k G_{ik} \Pr(D_i, \mathcal{H}_k) \quad (2.20)$$

$$= \sum_i \sum_k G_{ik} P(D_i | \mathcal{H}_k) \Pr(\mathcal{H}_k). \quad (2.21)$$

Let $\mathcal{A}_0^c$ denote the complementary set of the decision region, $\mathcal{A}_0$, and assume that $\mathcal{A}_1 = \mathcal{A}_0^c$ for convenience. Since

$$\Pr(D_1 | \mathcal{H}_k) = 1 - \Pr(D_0 | \mathcal{H}_k), \quad (2.22)$$

the average cost in (2.21) is rewritten as

$$\bar{G} = G_{10} \Pr(\mathcal{H}_0) + G_{11} \Pr(\mathcal{H}_1) + \int_{\mathcal{A}_0} g_1(y) - g_0(y) dy, \quad (2.23)$$

where

$$\begin{cases} g_0(y) = \Pr(\mathcal{H}_0)(G_{10} - G_{00})f(y | \mathcal{H}_0); \\ g_1(y) = \Pr(\mathcal{H}_1)(G_{01} - G_{11})f(y | \mathcal{H}_1). \end{cases} \quad (2.24)$$

Then, it is possible to minimize the average cost $\bar{G}$ by properly defining the acceptance regions, while the problem is formulated as

$$\min_{\mathcal{A}_0, \mathcal{A}_1} \bar{G}. \quad (2.25)$$

Since (2.23) follows

$$\bar{G} = \text{Constant} + \int_{\mathcal{A}_0} g_1(y) - g_0(y) dy, \quad (2.26)$$

we can show that

$$\min_{\mathcal{A}_0} \bar{G} \Leftrightarrow \min_{\mathcal{A}_0} \left\{ \int_{\mathcal{A}_0} g_1(y) - g_0(y) dy \right\}, \quad (2.27)$$

while the optimal regions that minimize the cost are given by

$$\begin{cases} \mathcal{A}_0 = \{y | g_1(y) \leq g_0(y)\}; \\ \mathcal{A}_1 = \{y | g_1(y) > g_0(y)\}. \end{cases} \quad (2.28)$$

Hence, we can conclude the Bayesian decision rule that minimizes the cost as follows:

$$\begin{cases} \mathcal{H}_0 : g_0(y) > g_1(y); \\ \mathcal{H}_1 : g_0(y) < g_1(y), \end{cases} \quad (2.29)$$

or

$$\begin{cases} \mathcal{H}_0 : \frac{f(y|\mathcal{H}_0)}{f(y|\mathcal{H}_1)} > \frac{\Pr(\mathcal{H}_0) G_{01} - G_{11}}{\Pr(\mathcal{H}_1) G_{10} - G_{00}}; \\ \mathcal{H}_1 : \frac{f(y|\mathcal{H}_0)}{f(y|\mathcal{H}_1)} < \frac{\Pr(\mathcal{H}_0) G_{01} - G_{11}}{\Pr(\mathcal{H}_1) G_{10} - G_{00}}, \end{cases} \quad (2.30)$$

where $(G_{10} - G_{00})$ and $(C_{01} - C_{11})$ are positive. More importantly, for binary hypothesis testing, we can find out that the ratio of the cost differences, $\frac{G_{01}-G_{11}}{G_{10}-G_{00}}$, is able to characterize the Bayesian decision rule rather than the values of individual costs, G_{ik} 's. Specifically, as $\frac{G_{01}-G_{11}}{G_{10}-G_{00}} = 1$, the Bayesian hypothesis test becomes the MAP hypothesis test.

2.4 Maximum Likelihood Hypothesis Test

The MAP decision rule can be employed under the condition that the APRP is available. Considering the case that the APRP is not available, another decision rule based on likelihood functions can be developed. For a given value of observation, y , the likelihood function is defined by

$$\begin{cases} f_0(y) = f(y|\mathcal{H}_0); \\ f_1(y) = f(y|\mathcal{H}_1). \end{cases} \quad (2.31)$$

Notice that the likelihood function is not a function of y since y is given, but a function of the hypothesis. With respect to the ML function, the ML decision rule is to choose the hypothesis as follows:

$$\begin{cases} \mathcal{H}_0 : f_0(y) > f_1(y); \\ \mathcal{H}_1 : f_0(y) < f_1(y), \end{cases} \Leftrightarrow \begin{cases} \mathcal{H}_0 : \frac{f_0(y)}{f_1(y)} > 1; \\ \mathcal{H}_1 : \frac{f_0(y)}{f_1(y)} < 1, \end{cases} \quad (2.32)$$

where the ratio, $\frac{f_0(y)}{f_1(y)}$, is regarded as the LR. For convenience, given by

$$\text{LLR}(y) = \log \frac{f_0(y)}{f_1(y)} \quad (2.33)$$

the e-based log-likelihood ratio (LLR), the ML decision rule can be rewritten as

$$\begin{cases} \mathcal{H}_0 : \text{LLR}(y) > 0; \\ \mathcal{H}_1 : \text{LLR}(y) < 0. \end{cases} \quad (2.34)$$

Note that the ML decision rule can be considered as a special case of the MAP decision rule when the APRPs are the same, i.e., $\Pr(\mathcal{H}_0) = \Pr(\mathcal{H}_1)$. In this case, the MAP decision rule is reduced to the ML decision rule.

2.5 Likelihood Ratio-Based Hypothesis Test

Let the LR-based decision rule be

$$\begin{cases} \mathcal{H}_0 : \frac{f_0(y)}{f_1(y)} > \rho; \\ \mathcal{H}_1 : \frac{f_0(y)}{f_1(y)} < \rho, \end{cases} \quad (2.35)$$

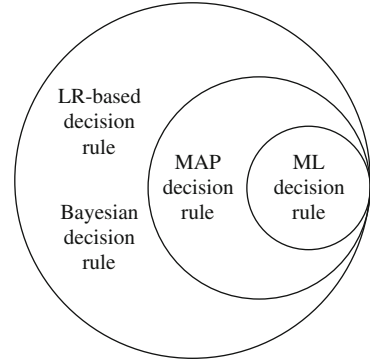
where ρ denotes a predetermined threshold. Consider the following hypothesis pair of received signals:

$$\begin{cases} \mathcal{H}_0 : y = \mu_0 + n; \\ \mathcal{H}_1 : y = \mu_1 + n, \end{cases} \quad (2.36)$$

where $\mu_1 > \mu_0$ and $n \sim \mathcal{N}(0, \sigma^2)$. Then, it follows that

$$\begin{cases} f_0(y) = \mathcal{N}(\mu_0, \sigma^2); \\ f_1(y) = \mathcal{N}(\mu_1, \sigma^2), \end{cases} \quad (2.37)$$

Fig. 2.4 The relationship of different decision rules



while the LLR becomes

$$\text{LLR}(x) = -\frac{(\mu_1 - \mu_0) \left(y - \frac{\mu_0 + \mu_1}{2} \right)}{\sigma^2}. \quad (2.38)$$

The corresponding LR-based decision rule is given by

$$\begin{cases} \mathcal{H}_0 : \left(y - \frac{\mu_0 + \mu_1}{2} \right) < \frac{\sigma^2 \ln \rho}{\mu_1 - \mu_0}; \\ \mathcal{H}_1 : \left(y - \frac{\mu_0 + \mu_1}{2} \right) > \frac{\sigma^2 \ln \rho}{\mu_1 - \mu_0}. \end{cases} \quad (2.39)$$

Letting

$$\bar{\rho} = \frac{\sigma^2 \ln \rho}{\mu_1 - \mu_0} + \frac{\mu_0 + \mu_1}{2}, \quad (2.40)$$

(2.39) can be rewritten as

$$\begin{cases} \mathcal{H}_0 : y < \bar{\rho}; \\ \mathcal{H}_1 : y > \bar{\rho}. \end{cases} \quad (2.41)$$

Specifically, if we let

$$\rho = \frac{\Pr(\mathcal{H}_0)(G_{10} - G_{00})}{\Pr(\mathcal{H}_1)(G_{01} - G_{11})}, \quad (2.42)$$

then the LR-based decision rule becomes the Bayesian decision rule. In summary, The LR-based decision rule can be regarded as a generalization of the MAP, Bayesian, and ML decision rules, where the relationship of various decision rules for binary hypothesis testing is shown in Fig. 2.4.

2.6 Neyman–Pearson Lemma

It is possible to define the Correct (detection) and type I error (false alarm) probabilities for each decision rule. On the contrary, for a given target detection probability or error probability, we may be able to derive an optimal decision rule. Let us consider the following optimization problem:

$$\max_d P_D(d) \quad \text{subject to} \quad P_A(d) \leq \sigma, \quad (2.43)$$

where d and σ represent a decision rule and the maximum false alarm probabilities, respectively. In order to find the decision rule, d , with the maximum false alarm probability constraint, the Neyman–Pearson Lemma is presented as follows.

Lemma 2.1. *Let the decision rule of $d'(s)$ be*

$$d'(s) = \begin{cases} 1, & \text{if } f_1(s) > \eta f_0(s); \\ \gamma(s) = \begin{cases} 1, & \text{with probability } p; \\ 0, & \text{with probability } 1 - p, \end{cases} & \text{if } f_1(s) = \eta f_0(s); \\ 0, & \text{if } f_1(s) < \eta f_0(s), \end{cases} \quad (2.44)$$

where $\eta > 0$ and p is decided such that $P_A = \sigma$. The decision rule in (2.44) is named as the Neyman–Pearson (NP) rule and becomes the solution of the problem in (2.43).

Proof. Under the condition of $P_A \leq \sigma$, we can assume that the decision rule becomes $\hat{d}$. In order to show the optimality of the problem in (2.43), we need to verify that for any $\hat{d}$, we have $P_D(t') \geq P_D(\hat{t})$. Denote by $\mathcal{S}$ the observation set, where $s \in \mathcal{S}$. Using the definition of d' , for any $s \in \mathcal{S}$, it shows that

$$(d'(s) - \hat{d}(s)) (f_1(s) - \eta f_0(s)) \geq 0. \quad (2.45)$$

Then, it is derived that

$$\int_{s \in \mathcal{S}} (d'(s) - \hat{d}(s)) (f_1(s) - \eta f_0(s)) ds \geq 0 \quad (2.46)$$

and

$$\int_{s \in \mathcal{S}} d'(s) f_1(s) ds - \int_{s \in \mathcal{S}} \hat{d}(s) f_1(s) ds \geq \eta \left(\int_{s \in \mathcal{S}} d'(s) f_0(s) ds - \int_{s \in \mathcal{S}} \hat{d}(s) f_0(s) ds \right), \quad (2.47)$$

which can further show that

$$P_D(d') - P_D(\hat{d}) \geq \eta (P_A(d') - P_A(\hat{d})) \geq 0. \quad (2.48)$$

Since (2.48) shows that $P_D(d') \geq P_D(\hat{d})$, the proof is completed. $\square$

From (2.43), we can show that the NP decision rule is the same as the LR-based decision rule with the threshold $\rho = \frac{1}{\eta}$ except the randomized rule, i.e., $f_1(s) = \eta f_0(s)$.

Example 2.2. Let us consider the following hypothesis pair:

$$\begin{cases} \mathcal{H}_0 : & y = n; \\ \mathcal{H}_1 : & y = s + n, \end{cases} \quad (2.49)$$

where $s > 0$ and $n \sim \mathcal{N}(0, \sigma^2)$. Using (2.41), the type I error probability is shown as

$$\begin{aligned} \Delta &= \int_{-\infty}^{\infty} d(y) f_0(y) dy \\ &= \int_{\bar{\rho}}^{\infty} f_0(y) dy \\ &= \int_{\bar{\rho}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{y^2}{2\sigma^2}\right) dy \\ &= \mathcal{Q}\left(\frac{\bar{\rho}}{\sigma}\right), \end{aligned}$$

where $\mathcal{Q}(x)$ denotes the Q-function and is defined by

$$\mathcal{Q}(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt. \quad (2.50)$$

Note that the function $\mathcal{Q}(x)$, $x \geq 0$, is the tail of the normalized Gaussian pdf (i.e., $\mathcal{N}(0, 1)$) from x to ∞ . Then, it follows that

$$\Delta = \mathcal{Q}\left(\frac{\bar{\rho}}{\sigma}\right) \quad (2.51)$$

or

$$\bar{\rho} = \sigma \mathcal{Q}^{-1}(\Delta). \quad (2.52)$$

Thus, the detection probability becomes

$$\begin{aligned} P_D &= \int_{\bar{\rho}}^{\infty} f_1(y) dy \\ &= \int_{\bar{\rho}}^{\infty} \frac{1}{\sqrt{2\pi N_0^2}} \exp\left(-\frac{(y-s)^2}{2N_0^2}\right) dy \end{aligned}$$

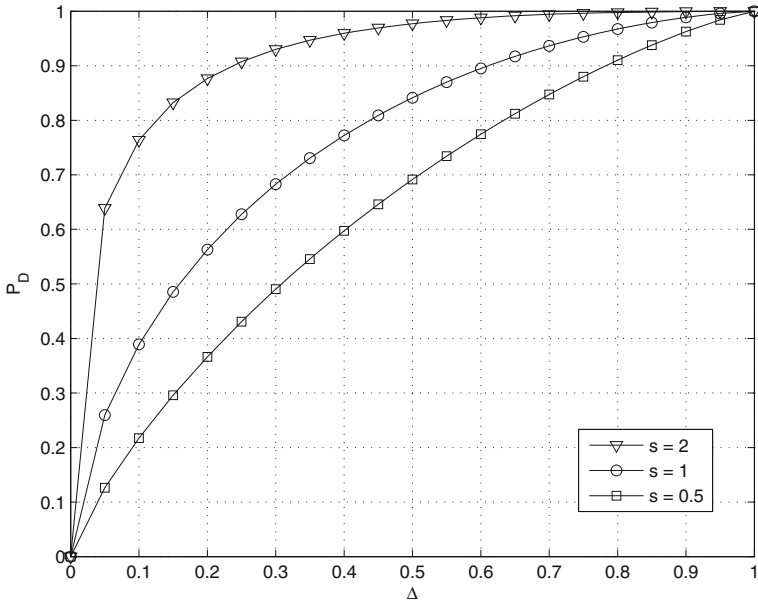


Fig. 2.5 Receiver operating characteristics (ROCs) of the NP decision rule.

$$\begin{aligned}
 &= Q\left(\frac{\bar{\rho} - s}{\sigma}\right) \\
 &= Q\left(Q^{-1}(\Delta) - \frac{s}{\sigma}\right), \tag{2.53}
 \end{aligned}$$

which shows that the detection probability is a function of the type I error probability.

In Fig. 2.5, the receiver operating characteristics (ROCs) of the NP decision rule are shown for different values of s . Note that the ROCs are regarded as the relationship between the detection and type I error probabilities.

2.7 Detection of Symmetric Signals

Symmetric signals are widely considered in digital communications. In this section, we focus on the detection problem of symmetric signals, namely the symmetric signal detection.

Again, let us consider the hypotheses of interest as follows:

$$\begin{cases} \mathcal{H}_0 : y = s + n; \\ \mathcal{H}_1 : y = -s + n, \end{cases} \tag{2.54}$$

where y is the received signal, $s > 0$, and $n \sim \mathcal{N}(0, \sigma^2)$. According to the symmetry of transmitted signals s and $-s$, the probabilities of type I and II errors become the same and given by

$$P_{\text{Error}} = \Pr(\text{Accept } \mathcal{H}_0 \mid \mathcal{H}_1) \quad (2.55)$$

$$= \Pr(\text{Accept } \mathcal{H}_1 \mid \mathcal{H}_0). \quad (2.56)$$

For a given $Y = y$, the LLR function is written as

$$\begin{aligned} \text{LLR}(Y) &= \log \left(\frac{f_0(Y)}{f_1(Y)} \right) \\ &= -\frac{1}{2\sigma^2} \left((Y - s)^2 - (Y + s)^2 \right) \\ &= \frac{2s}{\sigma^2} Y. \end{aligned} \quad (2.57)$$

In addition, based on the ML decision rule, we have

$$\begin{cases} \mathcal{H}_0 : \text{LLR}(Y) > 0; \\ \mathcal{H}_1 : \text{LLR}(Y) < 0, \end{cases}$$

which can be further simplified as

$$\begin{cases} \mathcal{H}_0 : Y > 0; \\ \mathcal{H}_1 : Y < 0. \end{cases} \quad (2.58)$$

From this, we can show that the ML detection is simply a hard-decision of the observation $Y = y$.

2.7.1 Error Probability

With the symmetry of transmitted signals s and $-s$ in (2.54), the error probability is found as

$$P_{\text{Error}} = \Pr(\text{LLR}(y) > 0 \mid \mathcal{H}_1) \quad (2.59)$$

$$= \Pr(\text{LLR}(y) < 0 \mid \mathcal{H}_0),$$

which can be derived as

$$\begin{aligned}
P_{\text{Error}} &= \Pr(\text{LLR}(y) > 0 | \mathcal{H}_1) \\
&= \Pr(y > 0 | \mathcal{H}_1) \\
&= \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(Y+s)^2\right) dY
\end{aligned} \tag{2.60}$$

and

$$\begin{aligned}
P_{\text{Error}} &= \int_{\frac{s}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{Y^2}{2}\right) dY \\
&= \mathcal{Q}\left(\frac{s}{\sigma}\right).
\end{aligned} \tag{2.61}$$

Letting the signal-to-noise ratio (SNR) be $\text{SNR} = \frac{s^2}{\sigma^2}$, then we can have $P_{\text{Error}} = \mathcal{Q}(\sqrt{\text{SNR}})$. Note that the error probability decreases with the SNR, since $\mathcal{Q}$ is a decreasing function.

2.7.2 Bound Analysis

In order to show the characteristics of the error probability, let us derive the bounds on the error probability. Define the error function as

$$\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp(-y^2) dy \tag{2.62}$$

and the complementary error function as

$$\begin{aligned}
\text{erfc}(y) &= 1 - \text{erf}(y) \\
&= \frac{2}{\sqrt{\pi}} \int_y^\infty \exp(-t^2) dt, \text{ for } y > 0,
\end{aligned} \tag{2.63}$$

where the relationship between the Q-function and the complementary error function is given by

$$\begin{aligned}
\text{erfc}(y) &= 2\mathcal{Q}(\sqrt{2}y); \\
\mathcal{Q}(y) &= \frac{1}{2}\text{erfc}\left(\frac{y}{\sqrt{2}}\right).
\end{aligned} \tag{2.64}$$

Then, for a given $Y = y$, we can show that the complementary error function has the following bounds:

$$\left(1 - \frac{1}{2Y^2}\right) \frac{\exp(-Y^2)}{\sqrt{\pi}Y} < \operatorname{erfc}(Y) < \frac{\exp(-Y^2)}{\sqrt{\pi}Y}, \quad (2.65)$$

where the lower bound is valid if $Y > 1/\sqrt{2}$. Accordingly, the Q-function $Q(Y)$ is bounded as follows:

$$\left(1 - \frac{1}{Y^2}\right) \frac{\exp(-Y^2/2)}{\sqrt{2\pi}Y} < Q(Y) < \frac{\exp(-Y^2/2)}{\sqrt{2\pi}Y}, \quad (2.66)$$

where the lower bound is valid if $x > 1$. Thus, the upper bound of error probability is given by

$$P_{\text{Error}} = Q(\sqrt{\text{SNR}}) < \frac{\exp(-\text{SNR}/2)}{\sqrt{2\pi}\sqrt{\text{SNR}}}. \quad (2.67)$$

In order to obtain an upper bound on the probability of an event that happens rarely, the Chernoff bound is widely considered, which can be used for any background noise.

Let $p(y)$ denote the step function, where $p(y) = 1$, if $y \geq 0$, and $p(y) = 0$, if $y < 0$. Using the step function, the probability for the event that $Y \geq y$, which can be also regarded as the tail probability, is given by

$$\begin{aligned} \Pr(Y \geq y) &= \int_y^\infty f_Y(\rho) d\rho \\ &= \int_{-\infty}^\infty p(\rho - y) f_Y(\rho) d\rho \\ &= E[p(Y - y)], \end{aligned} \quad (2.68)$$

where $f_Y(\rho)$ represents the pdf of Y . From Fig. 2.6, we can show that

$$p(y) \leq \exp(y). \quad (2.69)$$

Thus, we can have

$$\Pr(Y \geq y) \leq E[\exp(Y - y)] \quad (2.70)$$

or

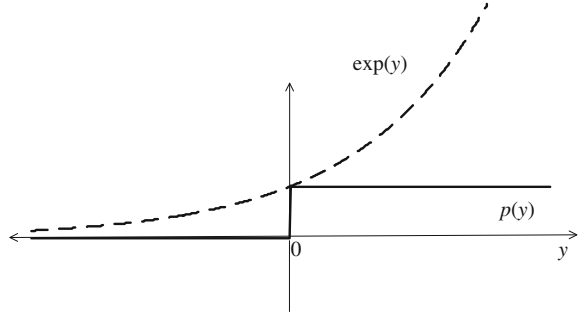
$$\Pr(Y \geq y) \leq E[\exp(\lambda(Y - y))] \quad (2.71)$$

for $\lambda \geq 0$. By minimizing the right-hand side with respect to λ in (2.71), the tightest upper bound can be obtained which is regarded as the Chernoff bound and given by

$$\Pr(Y \geq y) \leq \min_{\lambda \geq 0} \exp(-\lambda y) E[\exp(\lambda Y)]. \quad (2.72)$$

In (2.72), $E[\exp(\lambda Y)]$ is called the moment generating function (MGF).

Fig. 2.6 An illustration of step function



Let Y be a Gaussian random variable with mean μ and variance σ^2 . The MGF of Y is shown as

$$E[\exp(\lambda Y)] = \exp\left(\lambda\mu + \frac{1}{2}\lambda^2\sigma^2\right), \quad (2.73)$$

while the corresponding Chernoff bound is given by

$$\begin{aligned} \Pr(Y \geq y) &\leq \min_{\lambda \geq 0} \exp(-\lambda y) \exp\left(\lambda\mu + \frac{1}{2}\lambda^2\sigma^2\right) \\ &= \min_{\lambda \geq 0} \exp\left(\lambda(\mu - y) + \frac{1}{2}\lambda^2\sigma^2\right). \end{aligned} \quad (2.74)$$

The solution of the minimization is found as

$$\begin{aligned} \lambda^* &= \arg \min_{\lambda \geq 0} \exp\left(\lambda(\mu - y) + \frac{1}{2}\lambda^2\sigma^2\right) \\ &= \max\left\{0, \frac{y - \mu}{\sigma^2}\right\}. \end{aligned} \quad (2.75)$$

Under the condition that $\lambda^* > 0$, the Chernoff bound is derived as

$$\Pr(Y \geq y) \leq \exp\left(-\frac{(y - \mu)^2}{2\sigma^2}\right). \quad (2.76)$$

With respect to the error probability in (2.60), the Chernoff bound is given by

$$P_{\text{Error}} \leq \exp\left(-\frac{s^2}{2\sigma^2}\right) = \exp\left(-\frac{\text{SNR}}{2}\right). \quad (2.77)$$

In summary, the Chernoff bound can be given by

$$\mathcal{Q}(y) \leq \exp\left(-\frac{y^2}{2}\right), \quad (2.78)$$

which is actually a special case of the Chernoff bound in (2.72).

2.8 Binary Signal Detection

In general, signals are transmitted by waveforms rather than discrete signals over wireless channels. Using binary signaling, for $0 \leq t < T$, the received signal can be written as

$$Y(t) = S(t) + N(t), \quad 0 \leq t < T, \quad (2.79)$$

where T and $N(t)$ denote the signal duration and a white Gaussian random process, respectively. Note that we have $E[N(t)] = 0$ and $E[N(t)N(\rho)] = \frac{N_0}{2}\delta(t - \rho)$, where $\delta(t)$ represents the Dirac delta function. The channel in (2.79) is called the additive white Gaussian noise (AWGN) channel, while $S(t)$ is a binary waveform that is given by

$$\begin{cases} \text{under hypothesis } \mathcal{H}_0 : S(t) = s_0(t); \\ \text{under hypothesis } \mathcal{H}_1 : S(t) = s_1(t). \end{cases} \quad (2.80)$$

Note that the transmission rate is $\frac{1}{T}$ bits per second for the signaling in (2.79).

Let a heuristic approach be carried out to deal with waveform signal detection problem. At the receiver, the decision is made with $Y(t)$, $0 \leq t < T$. Denote by $y(t)$ a realization or observation of $Y(t)$, where L samples are taken from $y(t)$. Letting $s(t)$ and $n(t)$ be a realization of $S(t)$ and $N(t)$, respectively, we can have

$$\begin{aligned} y_l &= \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} y(t)dt, \\ s_{m,l} &= \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} s_m(t)dt, \\ n_l &= \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} n(t)dt, \end{aligned} \quad (2.81)$$

where

$$\begin{cases} \mathcal{H}_0 : y_l = s_{0,l} + n_l; \\ \mathcal{H}_1 : y_l = s_{1,l} + n_l. \end{cases} \quad (2.82)$$

In addition, since $N(t)$ is a white process, the n_l 's are independent, while the mean of n_l is zero and the variance can be derived as

$$\begin{aligned}
\sigma^2 &= E[n_l^2] \\
&= E \left[\left(\int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} N(t) dt \right)^2 \right] \\
&= \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} E[N(t)N(\rho)] dt d\rho \\
&= \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} \int_{\frac{(l-1)T}{L}}^{\frac{lT}{L}} \frac{N_0}{2} \delta(t - \rho) dt d\rho \\
&= \frac{N_0 T}{2L}.
\end{aligned} \tag{2.83}$$

Letting $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_L]^T$, the LLR becomes

$$\begin{aligned}
\text{LLR}(\mathbf{y}) &= \frac{\prod_{l=1}^L f(y_l | \mathcal{H}_0)}{\prod_{l=1}^L f(y_l | \mathcal{H}_1)} \\
&= \log \frac{f_0(\mathbf{y})}{f_1(\mathbf{y})},
\end{aligned} \tag{2.84}$$

which follows that

$$\begin{aligned}
\text{LLR}(\mathbf{y}) &= \sum_{l=1}^L \log \frac{f_0(y_l)}{f_1(y_l)} \\
&= \sum_{l=1}^L \log \left[\exp \left(-\frac{1}{N_0} \left((y_l - s_{0,l})^2 - (y_l - s_{1,l})^2 \right) \right) \right] \\
&= \frac{1}{N_0} \sum_{l=1}^L \left((y_l - s_{1,l})^2 - (y_l - s_{0,l})^2 \right) \\
&= \frac{1}{N_0} \left(\sum_{l=1}^L 2y_l(s_{0,l} - s_{1,l}) + \sum_{l=1}^L (s_{1,l}^2 - s_{0,l}^2) \right).
\end{aligned} \tag{2.85}$$

In addition, letting $\mathbf{s}_m = [s_{m,1} \ s_{m,2} \ \dots \ s_{m,L}]^T$, (2.85) can be rewritten as

$$\text{LLR}(\mathbf{y}) = \frac{1}{N_0} \left(2\mathbf{y}^T(\mathbf{s}_0 - \mathbf{s}_1) - (\mathbf{s}_0^T \mathbf{s}_0 - \mathbf{s}_1^T \mathbf{s}_1) \right). \tag{2.86}$$

From the LLR in (2.86), the MAP decision rule is given by

$$\begin{cases} \mathcal{H}_0 : \mathbf{y}^T(\mathbf{s}_0 - \mathbf{s}_1) > \sigma^2 \log \left(\frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_2)} \right) + \frac{1}{2}(\mathbf{s}_0^T \mathbf{s}_0 - \mathbf{s}_1^T \mathbf{s}_1); \\ \mathcal{H}_1 : \mathbf{y}^T(\mathbf{s}_0 - \mathbf{s}_1) < \sigma^2 \log \left(\frac{\Pr(\mathcal{H}_1)}{\Pr(\mathcal{H}_2)} \right) + \frac{1}{2}(\mathbf{s}_0^T \mathbf{s}_0 - \mathbf{s}_1^T \mathbf{s}_1). \end{cases} \quad (2.87)$$

For the LR-based decision rule, we can replace $\frac{\Pr(\mathcal{H}_0)}{\Pr(\mathcal{H}_1)}$ by the threshold ρ , which is

$$\begin{cases} \mathcal{H}_0 : \mathbf{y}^T(\mathbf{s}_0 - \mathbf{s}_1) > \sigma^2 \log \rho + \frac{1}{2}(\mathbf{s}_0^T \mathbf{s}_0 - \mathbf{s}_1^T \mathbf{s}_1); \\ \mathcal{H}_1 : \mathbf{y}^T(\mathbf{s}_0 - \mathbf{s}_1) < \sigma^2 \log \rho + \frac{1}{2}(\mathbf{s}_0^T \mathbf{s}_0 - \mathbf{s}_1^T \mathbf{s}_1). \end{cases} \quad (2.88)$$

As the number of samples during T seconds is small, there could be signal information loss due to sampling operation using the approach in (2.81). To avoid any signal information loss, suppose that L is sufficiently large to approximate as

$$\mathbf{y}^T \mathbf{s}_i \approx \frac{1}{T} \int_0^T y(t) s_m(t) dt. \quad (2.89)$$

Accordingly, the LR-based decision rule can be written as

$$\begin{cases} \mathcal{H}_0 : \int_0^T y(t)(s_0(t) - s_1(t)) dt > \sigma^2 \log \rho + \frac{1}{2} \int_0^T (s_0^2(t) - s_1^2(t)) dt; \\ \mathcal{H}_1 : \int_0^T y(t)(s_0(t) - s_1(t)) dt < \sigma^2 \log \rho + \frac{1}{2} \int_0^T (s_0^2(t) - s_1^2(t)) dt, \end{cases} \quad (2.90)$$

or

$$\begin{cases} \mathcal{H}_0 : \int_0^T y(t)(s_0(t) - s_1(t)) dt > W_T; \\ \mathcal{H}_1 : \int_0^T y(t)(s_0(t) - s_1(t)) dt < W_T, \end{cases} \quad (2.91)$$

if we let

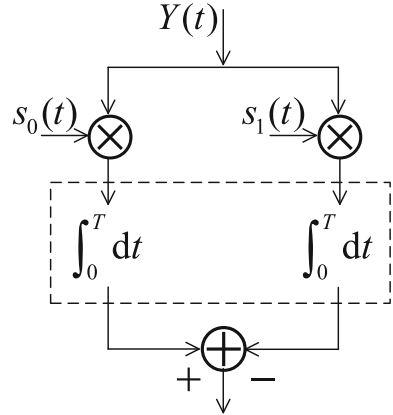
$$W_T = \sigma^2 \log \rho + \frac{1}{2} \int_0^T (s_0^2(t) - s_1^2(t)) dt. \quad (2.92)$$

Note that the decision rule is named as the correlator detector, which can be implemented as in Fig. 2.7.

Let us then analyze its performance. For the ML decision rule, we can have $\rho = 1$ in the LR-based decision rule, which leads to $W_T = \frac{1}{2} \int_0^T (s_0^2(t) - s_1^2(t)) dt$. Letting $X = \int_0^T y(t)(s_0(t) - s_1(t)) dt - W_T$, we can show that

$$\begin{cases} \Pr(D_0|\mathcal{H}_1) = \Pr(X > 0|\mathcal{H}_1); \\ \Pr(D_1|\mathcal{H}_0) = \Pr(X < 0|\mathcal{H}_0). \end{cases} \quad (2.93)$$

Fig. 2.7 Correlator detector using binary signalling



In order to derive the error probabilities, the random variable X has to be characterized. Note that X is a Gaussian random variable, since $N(t)$ is assumed to be a Gaussian process. On the other hand, if $\mathcal{H}_m$ is true, the statistical properties of G depend on $\mathcal{H}_m$ as $Y(t) = s_m(t) + N(t)$. Then, letting $\mathcal{H}_m$ be true, to fully characterize X , the Gaussian mean and variance of X are given by

$$\begin{aligned} E[X|\mathcal{H}_m] &= \int_0^T E[Y(t)|\mathcal{H}_m] (s_0(t) - s_1(t)) dt - W_T \\ &= \int_0^T s_m(t) (s_0(t) - s_1(t)) dt - W_T \end{aligned} \quad (2.94)$$

and

$$\sigma_m^2 = E[(X - E[X|\mathcal{H}_m])^2|\mathcal{H}_m], \quad (2.95)$$

respectively. Denote by E_s the average energy of the signals, $s_m(t)$, $m = 1, 2$, which are equally likely transmitted, we can show that $E_s = \frac{1}{2} \int_0^T (s_0^2(t) + s_1^2(t)) dt$. In the meanwhile, letting $\tau = \frac{1}{E_s} \int_0^T s_0(t)s_1(t)dt$, we have $\sigma^2 = \sigma_0^2 = \sigma_1^2 = N_0 E_s(1 - \tau)$ and

$$\begin{cases} E[X|\mathcal{H}_0] = E_s(1 - \tau); \\ E[X|\mathcal{H}_1] = -E_s(1 - \tau). \end{cases} \quad (2.96)$$

Up to this point, the pdfs of X under $\mathcal{H}_0$ and $\mathcal{H}_1$ become

$$\begin{cases} f_0(g) = \frac{\exp\left(-\frac{(g - E_s(1 - \tau))^2}{2N_0 E_s(1 - \tau)}\right)}{\sqrt{2\pi N_0 E_s(1 - \tau)}}; \\ f_1(g) = \frac{\exp\left(-\frac{(g + E_s(1 - \tau))^2}{2N_0 E_s(1 - \tau)}\right)}{\sqrt{2\pi N_0 E_s(1 - \tau)}}. \end{cases} \quad (2.97)$$

By following the same approach in Sect. 2.7.1, the error probability can be derived as

$$P_{\text{Error}} = Q\left(\sqrt{\frac{E_s(1-\tau)}{N_0}}\right). \quad (2.98)$$

For a fixed signal energy, E_s , since the error probability can be minimized when $\tau = -1$, the corresponding minimum error probability can be written as

$$P_{\text{Error}} = Q\left(\sqrt{\frac{2E_s}{N_0}}\right). \quad (2.99)$$

Note that the resulting signals that minimize the error probability are regarded as antipodal signals, which can be easily shown as $s_0(t) = -s_1(t)$. In addition, for an orthogonal signal set where $\rho = 0$, we have the corresponding error probability to be

$$P_{\text{Error}} = Q\left(\sqrt{\frac{E_s}{N_0}}\right). \quad (2.100)$$

Therefore, it can be shown that there is a 3 dB gap (in SNR) between the antipodal signal set in (2.99) and orthogonal signal set in (2.100).

2.9 Detection of M -ary Signals

After introducing the binary signal detection for $M = 2$, let us consider a set of M waveforms, $\{s_1(t), s_2(t), \dots, s_M(t)\}$, $0 \leq t < T$, for M -ary communications, where the transmission rate is given by

$$R = \frac{\log_2 M}{T} \quad (2.101)$$

bits per seconds. In (2.101), it can be observed that the transmission rate increases with M , while a large M would be preferable. However, the detection performance becomes worse as M increases in general.

Let the received signal be

$$y(t) = s_m(t) + n(t) \quad (2.102)$$

for $0 \leq t < T$ under the m th hypothesis. The likelihood with L samples is shown as

$$\begin{aligned}
 f_m(\mathbf{y}) &= \prod_{l=1}^L f_m(y_l) \\
 &= \frac{\prod_{l=1}^L \exp\left(-\frac{(y_l - s_{m,l})^2}{N_0}\right)}{(\pi N_0)^{\frac{L}{2}}}.
 \end{aligned} \tag{2.103}$$

Taking the logarithm on (2.103), we can rewrite the log-likelihood function as follows:

$$\begin{aligned}
 \log f_m(\mathbf{y}) &= \log \frac{1}{(\pi N_0)^{\frac{L}{2}}} + \sum_{l=1}^L \log \left(\exp \left(-\frac{(y_l - s_{m,l})^2}{N_0} \right) \right) \\
 &= \log \frac{1}{(\pi N_0)^{\frac{L}{2}}} - \frac{(y_l - s_{m,l})^2}{N_0},
 \end{aligned} \tag{2.104}$$

which can be further simplified as

$$\log f_m(\mathbf{y}) = \frac{1}{N_0} \sum_{l=1}^L \left(y_l s_{m,l} - \frac{1}{2} |s_{m,l}|^2 \right) \tag{2.105}$$

by canceling the common terms for all the hypotheses. In addition, (2.105) can be rewritten as

$$\log f_m(y(t)) = \frac{1}{N_0} \left(\int_0^T y(t) s_m(t) dt - \frac{1}{2} \int_0^T s_m^2(t) dt \right), \tag{2.106}$$

when L goes infinity. Note that $E_m = \int_0^T s_m^2(t) dt$ in (2.106) represents the signal energy, while $\int_0^T y(t) s_m(t) dt$ denotes the correlation between $y(t)$ and $s_m(t)$.

Based on the log-likelihood functions, the ML decision rule can be found by

$$\text{If } \log f_m(y(t)) \geq \log f_{m'}(y(t)) \text{ accept } \mathcal{H}_m \tag{2.107}$$

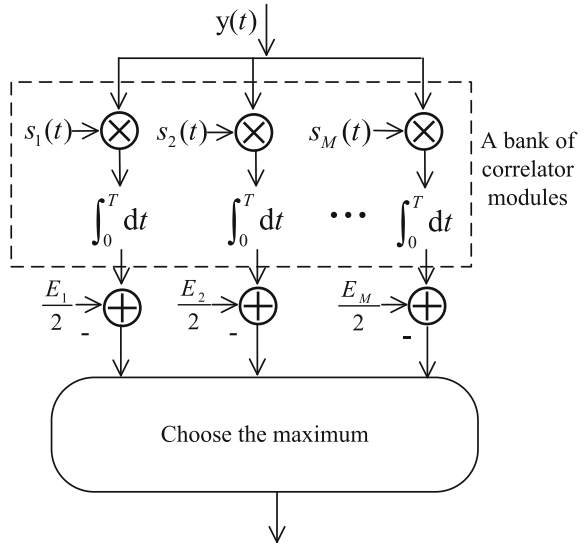
or

$$\text{If } \log \frac{f_m(y(t))}{f_{m'}(y(t))} \geq 0 \text{ accept } \mathcal{H}_m, \tag{2.108}$$

for $m' \in \{1, 2, \dots, M\} \setminus \{m\}$, where $\setminus$ denotes the set minus (i.e., $A \setminus B = \{x \mid x \in A, x \notin B\}$).

In Fig. 2.8, the implementation of ML decision rule using a bank of the correlator modules is carried out, while the MAP decision rule can also be derived by taking into account the APRP on the ML decision rule.

Fig. 2.8 A bank of correlator modules using M -ary signals



2.10 Concluding Remarks

In this chapter, different decision rules have been introduced with their applications to signal detection. Since multiple signals are transmitted or received via multiple channels simultaneously, in multiple-input multiple-output (MIMO) systems, it is preferable to describe signals in vector forms. In the next chapter, we will focus on signal detection in a vector space and the idea of MIMO detection.

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