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Local and Nonlocal Correlations

The central concept in this book is *nonlocal correlation*. We shall see that this idea is closely related to the idea of true randomness, that is to the idea of events that are intrinsically unpredictable. Chance is already a fascinating subject in itself, but here we shall be talking about nonlocal chance. These are completely novel notions and very surprising, even revolutionary. And neither is it easy to grasp their relevance, which means that this chapter may well be the most difficult. But then the rest of the book is there to help you. In order to convince themselves that there really are nonlocal correlations and true randomness, physicists have invented a game, called Bell's game. Physicists are really big children who never stop taking their toys apart to understand what makes them tick.

But before introducing this game, we should begin by recalling what is meant by correlation. Science is essentially an exercise in observing correlations, then inventing explanations for them. John Bell used to say that correlations cry out for explanations.[1] We first present a simple example of correlations, then ask what kind of explanation can account for them. We shall see that there are in fact very few different types of explanation. If we limit ourselves to local explanations, that is, employing some mechanism that propagates continuously from point to point through space, there are actually only two different kinds.

Bell's game can then be used to study particular correlations. It is a game for two people who must work together in collaboration to obtain a maximum number of points. The rules of the game are exceedingly simple and it is easy to play, but the goal, a kind of nonlocal calculation, is not easy to apprehend at the outset. In fact, the point is not so much the game itself, as understanding how it works. And in this way we shall strike the heart of the matter—nonlocal correlations and the conceptual revolution currently under way.

But let us begin at the beginning, with the concept of correlation.

[1] Bell, J.S.: *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press (1987), p. 152.

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Correlations

Every day we make choices which have consequences. Certain choices and their consequences are more important than others.

Certain consequences depend only on our choices, but many depend also on choices made by others. In this case, the consequences of our choices are not independent of each other: they are correlated. For example, the choice of menu for the evening meal depends among other things on the price of the produce at the local grocer's, and these prices are decided by others under various constraints. The menus of those inhabiting the same part of town will thus be correlated. If there is a special offer on fresh spinach, this vegetable is likely to appear more often on the menu. Another cause of correlation between menus is the influence of the choice made by the neighbour. If there is a long queue somewhere, we may be tempted to go and see what the attraction is, or else to avoid the queue. In both cases, there will be correlation, positive in the first case and negative in the second.

Let us push the example to the extreme. Imagine two neighbours, Alice and Bob once again. (We shall see that they play a similar role to the students in the story about the peculiar telephone.) Let us suppose that they always have the same evening meal, day after day. In other words, their evening menus are perfectly correlated. How could we explain such a correlation?

A first possibility is that Bob systematically copies Alice, and thus does not actually choose his menu, or conversely, that Alice copies Bob. Here then is a first possible type of explanation for the correlation: a first event influences a second event. This explanatory scheme can be put to the test, so let us behave like scientists and do just that. In thought at least, let us separate Alice and Bob so that they are really very far removed from one another, in two different towns on two different continents, but making sure that each has access to some local grocery store. To ensure that they cannot influence one another, we insist that Alice and Bob do their shopping at exactly the same moment. And better still, let us imagine that they are on two different galaxies. Under these conditions, it would be impossible for them to communicate, or even to influence one another unknowingly, like people who yawn.[2] But now imagine that the perfect correlation between their evening menus perdures. Now we cannot explain such a correlation by an influence, so we must find some other explanatory scheme.

[2] We know that, in a group of people, if one yawns, that will trigger the same in others, whether they are aware of it or not. This is an example of an unconscious influence between people. However, the second person must necessarily see the first one yawning, so this kind of influence cannot propagate faster than light.

A second possible explanation is that the grocer's nearest to Alice and the one nearest to Bob offer one and the same product, whence there is in fact no choice whatever. Some time long ago, the two stores might have established a list of evening menus for the years to come. These menus might be different from one evening to the next, but every evening the two grocers respect the instructions featuring on their lists. This list might have been prepared by the manager of a chain of grocer's stores and communicated by email to all members of the galactic consortium. In this way, Alice and Bob necessarily end up with the same menu, day after day. According to this explanation, Alice and Bob's menus are determined by the same cause which occurred sufficiently long ago to influence both Alice and Bob, despite the huge distance that separates them. This common cause would have propagated continuously from point to point through space, without leaps or breaks. One speaks then of a common local cause, common because it arises from a shared past, and local because everything always happens locally and continuously, from one point in space to the next.

So here we have a possible logical explanation. Now think about it: is there any other possible explanation? Try your best to find a third kind of explanatory scheme that would account for the fact that Alice and Bob end up eating exactly the same meal every evening. That is, an explanation other than an influence of Alice on Bob or of Bob on Alice, and other than some common local cause. Is there really no other possible explanation? Surprising though that may seem, scientists have never found any third kind of explanation. All correlations observed in science, outside quantum physics that is, can be accounted for either by an influence of one event on another (explanation of the first type), or by common local causes, like the manager of the two grocery stores (explanation of the second type). In both types of explanation, the said influence or common cause propagates continuously from point to point through space, and in this precise sense, all such explanations are local. By extension, we speak of local correlations when we want to say that these correlations have some local explanation. In fact, we shall find that quantum physics provides us with a third possible explanation, and it is precisely the subject of this book. But outside quantum physics, there are only two types of explanation for all observed correlations, be they in geology or in medicine, in sociology or in biology. And these two types of explanation are local because they appeal to a chain of mechanisms that propagates continuously from one point to the next in space.

It is the quest for local explanations that has brought so much success to science. Indeed, science can be characterised by its incessant search for good explanations. And an explanation is considered good if it satisfies three criteria. The best known of these is accuracy. This is formalised in mathematical equations which allow one to make predictions that can then be compared with observation and experience. However, it is my opinion that this criterion, albeit essential, is nevertheless not the most important. A second characteristic of a good explanation is that it tells a story. Every science lesson begins with a story. How else could one introduce new concepts, like energy, molecule, geological layer, or correlation? Until the advent of quantum physics, all these stories took place in a perfectly continuous manner in space and through time, and hence were local stories. The third criterion for a good explanation is that it cannot easily be modified. So a good explanation can be tested by experiments because it cannot easily be adapted to fit new experimental data that would otherwise contradict it. In Popper's words, it can be falsified.

But let us return to Alice and Bob and the perfect correlation between their evening meals. The wide separation between them rules out any attempt to explain by direct influence (type 1). How could we test an explanation by common local cause (type 2)? In our example, Alice has no choice in the matter. There is only one grocer's near where she lives and this store offers just one possible menu each evening. Such a situation, with no choice whatever, is too simple to be tested, so we must make our example a little more elaborate.

Imagine now that there are two grocery stores near Alice's home, one on the left when she goes out and one on the right. Likewise, there are two grocery stores near Bob's home, one on the left and one on the right. Alice and Bob still live on two different galaxies and cannot therefore influence one another. But let us imagine that, each time they both choose, quite by chance, to do their shopping in the store on the left, they always end up with the same menu. The only local explanation for this correlation is that the left-hand grocery stores share a list which evening after evening determines the sole available evening menu. For the grocery stores on the left, the situation is precisely the same as before. But the fact that there are several stores close to both Alice and Bob means that one can imagine a range of different correlations. For example, if Alice chooses the left-hand store and Bob the one on the right, we can once again imagine that they always end up with the same menu. Likewise if Alice goes to the right-hand store and Bob to the one on the left. We then conclude that the only local explanation for these three correlations, left–left, left–right, and right–left, is once again that these four stores share the same list of menus. But now imagine that, when Alice and Bob both go to the store on the right, they *never* have the same menu. Is that possible? Well, it does sound as though it would be difficult to arrange.

At this point we come very close to the spirit of Bell's game. So let us leave our grocery stores here. We must adopt a scientific approach and simplify the situation as far as possible. Instead of an evening menu, we shall speak of results, and since it suffices to consider just two possible results, we shall not require more than that.

Bell's Game

The manufacturer of this game supplies two apparently identical boxes as shown in Fig. 2.1. Each is equipped with a joystick and a screen. At rest, the joystick is in the vertical position. A second after the joystick is pushed to the left or the right, a result appears on the screen. The results are binary, that is, there are only two possible values, either 0 (zero) or 1 (one). Computer scientists say that the results are bits of information. For each box, the results seem to be random.

To play the game, Alice and Bob each take a box, synchronise their watches, then move some distance apart. At 9 a.m. exactly, and then every minute, they each push their joystick one way or the other, then carefully note the results displayed by their boxes along with the time and the direction they chose. It is important here that they each choose left or right every minute completely freely and independently. In particular, they are not allowed to keep making the same choice, nor to come to any prior agreement about the choices they will make. It is important that Alice should not know the choice made by Bob, and that Bob should not know the choice made by Alice. Note that they do

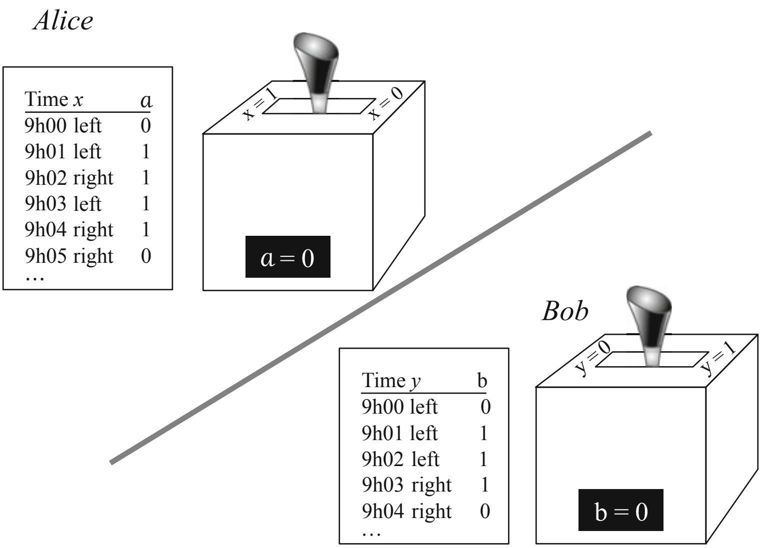

Fig. 2.1 Alice and Bob play Bell's game. Each has a box with a joystick. Every minute, they choose to push their joystick either toward the left or toward the right, whereupon each box displays a result. Alice and Bob carefully note the time, their choices, and the results produced by their boxes. At the end of the day, they compare their results and determine whether they have won or lost the game. Their objective is to understand how their boxes work, just as children learn by trying to understand their toys

not try to cheat, since their aim is to work out how the boxes supplied with the Bell game work.

They go on playing until 7 p.m., thus building up 600 pieces of data, with roughly 150 cases of left–left choices, and the same of left–right, right–left, and right–right. Then at the end of the day, they meet to calculate the points and their score according to the following rules:

1. Every time Alice pushes her joystick to the left or Bob pushes his to the left, or when both push their joysticks to the left, they get one point *if their results are the same*.
2. Every time Alice and Bob both choose to push their joysticks to the right, they get a point *if their results are different*.

The score is then calculated as follows:

- For each of the four combinations of choices left–left, left–right, right–left, and right–right, they first calculate the success rate, i.e., the number of points divided by the total number of trials, then add up these four success rates. The maximum score is thus 4, since there are four combinations of choices and each success rate can be at most equal to 1. For a score S, we shall say that Alice and Bob have won Bell's game S out of 4 times. Note that the score is an average and so can take any value between 0 and 4. For example, a score of 3.41 means that Alice and Bob have won on average 3.41 times out of 4, or 341 times out of 400.
- We shall see that it is easy to construct boxes allowing Alice and Bob to obtain a score of 3. So sometimes when we say that they win Bell's game, we shall mean that they win more often than 3 times out of 4.

To gain some familiarity with this peculiar game, let us think about the case where Alice and Bob do not in fact note down the results displayed by their boxes, but instead just write down whatever comes to mind. In short, they produce results by pure chance, quite independently of one another.[3] In this case, the success rates will all be 1/2. Indeed, half the time Alice and Bob write down the same result, and half the time opposite results, whatever their choices for the joystick. This means that the score for the Bell game will be $4 \times 1/2 = 2$. So to exceed a score of 2, Alice and Bob's boxes cannot be totally independent of one another, but must be coordinated in such a way as to produce correlated results.

[3] Note that the same reasoning applies also to the case where one of the two plays conscientiously while the other completely disregards the rules. In this case, the success rates will all be 1/2 once again and the total score 2.

Pursuing a little further, consider another example in which the two boxes always produce the same result 0, whatever the position of the joystick. In this case, Alice and Bob's choices have no influence on the result. It is easy to see that the success rates will be 1 for each of the three combinations left–left, left–right, and right–left, and 0 for the combination of choices right–right. The score will in this case be 3.

Before going on to analyse how the boxes work, let us introduce a modicum of abstraction. This will take us to the very heart of the notion of nonlocality.

Nonlocal Calculation: $a + b = x \times y$

Scientists love to encode the objects they are analysing using numbers, as we have done here with the results displayed by Bell's boxes. This helps them to focus on the essential without being confused by long sentences like "Alice pushed her joystick to the left and observed the result 0". It also allows them to carry out additions and multiplications, and we shall see that this can be used to encapsulate the notion of nonlocality in an extremely simple equation.

Let us focus first on Alice. Let x denote her choice and a the result. For example, $x = 0$ will mean that Alice chose to push her joystick to the left, and $x = 1$ will mean that she pushed it to the right. Likewise for Bob, let y denote his choice and b the result. With this notation, the little table below summarises the cases where, applying the rules of the Bell game, Alice and Bob get a point.

	$x = 0$	$x = 1$
$y = 0$	$a = b$	$a = b$
$y = 1$	$a = b$	$a \neq b$

So let us do a little elementary arithmetic, just for fun. It turns out that we can sum up the Bell game, with Alice and Bob each holding their box, widely separated from one another to avoid any possibility of copying, each making free choices and noting their results, in the form of a rather elegant equation:

$$a + b = x \times y,$$

that is, a plus b equals x times y. Indeed, the product $x \times y$ is always equal to 0 unless $x = y = 1$. The equation thus tells us that $a + b = 0$ unless $x = y = 1$.

Consider first the case where $x = y = 1$. In this case, $a + b = 1$, and since a and b are equal to 0 or 1, the equation $a + b = 1$ has only 2 solutions: either

$a = 0$ and $b = 1$, or $a = 1$ and $b = 0$. Hence, if $a + b = 1$, we certainly have $a \neq b$ and, according to the rules of the Bell game, Alice and Bob get a point.

Consider now the three other cases: $(x, y) = (0, 0)$, $(0, 1)$, or $(1, 0)$. We always have $x \times y = 0$ and the equation simplifies to $a + b = 0$. A first possible solution is $a = b = 0$. The second solution is $a = b = 1$. The second solution may look strange at first sight, because $1 + 1$ is normally equal to 2! But when we calculate with 0s and 1s (bits), the result must also be a 0 or a 1. Here, $2 = 0$ (mathematicians talk about calculating modulo 2). So the equation $a + b = 0$ is equivalent to $a = b$.

In conclusion, the beautiful equation $a + b = x \times y$ perfectly summarises the Bell game. Every time it is satisfied, Alice and Bob receive one point. And so you see that the quantum revolution can be exposed with very simple mathematics.[4]

This equation expresses the phenomenon of nonlocality. In order to win the Bell game systematically, the boxes must calculate the product $x \times y$. But if the choice x is only available to Alice's box and the choice of y is only available to Bob's box, this calculation cannot be done locally. At best, they may bet on $x \times y = 0$ and they will be right 3 times out of 4, whence they may obtain a score of 3. But any score greater than 3 requires a 'nonlocal' calculation of $x \times y$, because the two factors in the product exist only in places that are very far removed from one another.

Local Strategies for the Bell Game

Alice and Bob are in front of their respective boxes and each minute they make their choice freely and independently, carefully noting down their choices and the results displayed on the boxes. What could these boxes do to ensure that Alice and Bob get a good score?

Let us imagine that they are too far apart to influence one another. To achieve this, move Alice and Bob apart in our minds, so far from each other that no communication is possible. For example, separate them by a distance such that even light would take more than one second to go from one to the other, i.e., more than 300,000 km, roughly the distance from the Earth to the Moon. In this extreme case, it would be impossible for Alice, or rather for her box, to communicate her choice to Bob, or Bob's box. No account in terms

[4] As a good but somewhat unruly student, how many times during my studies did I ask my quantum physics teachers for explanations, only to hear that quantum physics cannot be understood because it requires such complex mathematics?

of communication or influence is thus possible and we must therefore find another explanation.

Let us begin by analysing the case where, by coincidence, the two joysticks are pushed toward the left. In this case, Alice and Bob only obtain a point if their results are the same. This is the same situation as for the customers at the grocery stores who always end up with the same menu if they choose the store on the left. We have already seen that, if we exclude all direct influence, this is only possible if the two stores leave absolutely no choice and simply impose the same menu. For the boxes in Bell's game, this means that, if the joysticks are pushed to the left, they both produce the same result. This result is predetermined at each minute, but can change from one minute to the next, just as the single available menus can vary from one evening to the next. Here we have an explanation for a maximal correlation in the case where the two joysticks are pushed toward the left. This is an explanation of the second type, that is, in terms of a common local cause. Indeed, the results predetermined each minute must be recorded in each box, that is, locally.

Let us pursue the analysis of this case a little further. The results originally recorded in the boxes may have been produced by a long series of coin tosses. From Alice's point of view, they thus look perfectly random. And the same goes for Bob. However, when they meet together and discover that they have always obtained the same result, they no will longer believe that this happened by chance. Unless perhaps this is nonlocal randomness? We shall come back to that.

Box 2. Chance. A result that occurs by chance is one that is unexpected. But unexpected for whom? Many things are unexpected, either because they result from processes that are too complex to be apprehended, or because we did not pay attention to all kinds of details that have influenced the result. However, a truly random result that occurs by 'true' chance is unexpected because it is *intrinsically* unpredictable. Such a result is not determined by one or other causal chains, however complex they may be. A truly random result is not predictable because, before it came into being, it just did not exist, it was not necessary, and its realisation is in fact an act of pure creation.

To illustrate this idea, imagine that Alice and Bob meet up by chance in the street. This might happen, for example, because Alice was going to the restaurant further down the same street and Bob to see a friend who lives in the next street. From the moment they decide to go on foot, by the shortest possible path, to the restaurant for Alice and to see his friend for Bob, their meeting was predictable. This is an example of two causal chains of events, the paths followed by Alice and Bob, which cross one another and thus produce what looks like a chance encounter to each of them. But that encounter was predictable for someone with a sufficiently global view. The apparently chance-like nature of the meeting was thus only due to ignorance: Bob did not know where Alice was going, and conversely. But what was the situation before Alice decided to go to the restaurant? If we agree that she enjoys the benefits of free will, then before she made this decision, the meeting was truly unpredictable. True chance is like this.

True chance does not therefore have a cause in the same sense as in classical physics. A result subject to true chance is not predetermined in any way. But we need to qualify this assertion, because a truly chance-like event may have a cause. It is just that this cause does not determine the result, only the probabilities of a range of different possible results. It is only the propensity of a certain result to be realised that is actually predetermined.

According to the explanatory scheme for common local causes, every minute each box produces a predetermined result. For this kind of explanation, the list of results is pre-established and memorised by each box. We may think of each box as containing some kind of little computer with a large memory, a clock, and a program which reads off the next piece of data in the memory at one minute intervals.

Depending on the program, the result can either be independent of the position of the joystick or it can depend on it. But what programs are running in Alice and Bob's boxes? Are there not infinitely many, or at least a very large number of possible programs? In fact, there are not, because the simplification we have made in our scientific approach by sticking to binary choices and results limits the number of possible programs to 4 per box. Indeed, the program need only supply one result among 2 possible for each of the 2 possible choices. In Alice's box, these 4 programs are as follows:[5]

1. The result is always $a = 0$, whatever the choice of x.
2. The result is always $a = 1$, whatever the choice of x.
3. The result is identical to the choice, i.e., $a = x$.
4. The result always differs from the choice, i.e., $a = 1 - x$.

Likewise, there will be 4 possible programs for Bob's box. This means a total of $4 \times 4 = 16$ combinations of programs for both Alice and Bob. Naturally, the programs can change from one minute to the next, both in Alice's box and in Bob's, but at each minute, one of the 4 programs in Alice's box determines its result a and one of the 4 programs in Bob's box determines b.

Let us investigate these 16 possible combinations of programs and calculate the corresponding scores. Remember that the aim is to find the maximal possible score with a local explanation. We shall see that it is impossible to build boxes using local strategies to obtain a score greater than 3. At this point, you have the choice of simply taking my word for it and moving directly to

[5] Here the notion of program is to be taken in the abstract sense of saying *what results are produced by what data*. An abstract program can clearly be written in many ways, in various programming languages, and possibly with many unnecessary lines, in such a way that it may be difficult to see that two programs written differently do in fact correspond to the same abstract program.

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