Preface

I have a principled argument for why this book should exist. I have no such argument for why it contains just what it contains. The principles are these:

- You cannot understand philosophy of mathematics without understanding mathematics.
- You cannot understand mathematics without doing mathematics.

The main point of this book, with its 298 exercises, is to give students opportunities to recreate some mathematics that will illuminate important readings in philosophy of mathematics. As for the particular mathematical materials I have chosen: they are the unforced fruits of lengthy experience. I have taught undergraduates for three decades. In the 14 times I taught philosophy of mathematics, I discovered again and again that some important text was opaque to my students largely because they lacked some particular bit of mathematical background. Most of the missing bits came from a handful of subject areas: Primitive Recursive Arithmetic, Peano Arithmetic, Gödel's theorems (completeness, compactness, and incompleteness), interpretability, the hierarchy of sets (especially the landscape of $V(\omega + \omega)$), Frege Arithmetic, and intuitionist sentential logic. This book offers exercises in these areas supported by explanatory materials and just a dash of philosophy. I have made no attempt to impose a grand unifying narrative. There is no central thesis. Some students of philosophy of mathematics have found it helpful to work their way into these subject areas. Other readers may enjoy the same experience.

I cannot think of any undergraduate course in which this book would be an appropriate stand-alone text. I offer it as a supplement to primary texts chosen by instructors or automaths. For the benefit of the latter, the book offers some guidance about what those primary texts might be. Professors will, of course, make their own choices, both about primary texts and about assignments within this book. I would be amazed if any instructor assigned every section and every exercise. I do not see how there could be time for it. I would expect instructors to let their own interests guide them as they pick and choose. Not every choice will make sense. Chapter 2, for example, would be inscrutable without Chap. 1. Parts of Chap. 6 presuppose parts of Chap. 2. Chapters 4 and 5 build on Chap. 3. On the other hand, a leap-frog journey consisting of Chaps. 1, 3, and 7 would make

sense—as would many other paths over and around sections instructors might choose to skip. That is one respect in which the book's somewhat motley character is a virtue.

The readers most likely to benefit from this book are exactly those most likely to benefit from a philosophy of mathematics course. Those readers will have some background in formal logic, they will find mathematics engaging and non-threatening, they will understand basic properties of the natural and real numbers, and they will see the point of asking "why" and "how" questions that emerge from mathematical experience, but cannot be answered by just producing more mathematics. I would describe this last trait as "philosophical inclination"—and it is the inclination, rather than any particular philosophical training, that is likely to be most important here. (Once, in a philosophy of social science course, a student of mine interrupted a discussion of deductive-nomological explanation by asking, "Why would anyone want to understand what explanation is?" There, I suspect, was a student who had never felt a philosophical impulse in his life. I would not recommend this book to him.)

I shared the joys and frustrations of a work-in-progress with my students. Some responded by dropping my course. Others stuck it out and helped make the book better. I like to think that, in return, they learned what it is like to *do* mathematics and philosophy. In any case, they have my thanks. Thanks, too, to Florence Emily Pollard for her careful reading of the penultimate draft.

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