Contents

1 Random Walk, Green Function, and Equilibrium Measure 1
1.1 Some Notation 1
1.2 Simple Random Walk 2
1.3 Equilibrium Measure and Capacity 5
1.4 Notes 8

2 Random Interlacements: First Definition and Basic Properties 11
2.1 Space of Subsets of $\mathbb{Z}^d$ and Random Interlacements 11
2.2 Correlations, Shift-Invariance, and Ergodicity 13
2.3 Increasing and Decreasing Events, Stochastic Domination 16
2.4 Notes 17

3 Random Walk on the Torus and Random Interlacements 19
3.1 Preliminaries 21
3.1.1 Lazy Random Walk 21
3.1.2 Hitting of Small Sets in Short Times 22
3.1.3 Fast Mixing of Lazy Random Walk 25
3.2 Proof of Theorem 3.1 27
3.3 Notes 28

4 Poisson Point Processes 31
4.1 Poisson Distribution 31
4.2 Poisson Point Processes 32
4.3 Notes 35

5 Random Interlacement Point Process 37
5.1 A Sigma-Finite Measure on Doubly Infinite Trajectories 37
5.1.1 Spaces 37
5.1.2 Construction of the Intensity Measure Underlying Random Interlacements 39
5.2 Random Interlacement Point Process 44
5.2.1 Canonical Space and Random Interlacements 44

5.2.2 Finite Point Measures on W_+ and Random Interlacements in Finite Sets 46
5.3 Covering of a Box by Random Interlacements 48
5.4 Notes 49

6 Percolation of the Vacant Set 51
6.1 Percolation Threshold 51
6.2 Exponential Decay and Proof of Theorem 6.2 53
6.3 Proof of Proposition 6.3 55
6.4 Proof of Proposition 6.7 59
6.5 Notes 60

7 Source of Correlations and Decorrelation via Coupling 61
7.1 A Polynomial Upper Bound on Correlations 62
7.2 Perturbing the Value of u 64
7.3 Point Measures on Random Walk Excursions 65
7.4 Decorrelation via Coupling 69
7.5 Notes 72

8 Decoupling Inequalities 75
8.1 Hierarchical Events 75
8.2 Decoupling Inequalities 78
8.3 Proof of Theorem 8.5 80
8.4 Long $*$-Paths of Unlikely Events Are Very Unlikely 82
8.5 Notes 86

9 Phase Transition of $\mathscr{V}^u$ 87
9.1 Subcritical Regime 88
9.2 Supercritical Regime 91
9.3 Notes 94

10 Coupling of Point Measures of Excursions 97
10.1 Choice of Sets 97
10.2 Coupling of Point Measures of Excursions 99
10.2.1 Comparison of Intensity Measures 99
10.2.2 Construction of Coupling 105
10.2.3 Error Term 109
10.2.4 Proof of Theorem 10.4 112
10.3 Proof of Theorem 8.3 112
10.4 Notes 113

References 115

Index 119

http://www.springer.com/978-3-319-05851-1

An Introduction to Random Interlacements

Drewitz, A.; Ráth, B.; Sapozhnikov, A.

2014, X, 120 p. 8 illus., Softcover
ISBN: 978-3-319-05851-1