Chapter 2
An Overview of Multiple Criteria Decision Aid

Abstract This chapter provides an overview of the multicriteria decision aid paradigm. The discussion covers the main features and concepts in the field as well as an introduction to the main methodological approaches and techniques.

Keywords Multicriteria decision aid · Multiobjective optimization · Goal programming · Multiattribute value theory · Outranking relations · Preference disaggregation analysis

2.1 Introduction

The discussion in the previous chapter highlights the multi-dimensional nature of financial decisions. When it comes to actual decision support, the particular characteristics of the decision environment in a given instance should be considered, together with the preferences and judgment policy of the decision makers, and the domain knowledge provided by normative and descriptive financial theories. For instance, one should consider specific budgetary, regulatory, or policy constraints and conditions, qualitative expert judgments describing special aspects of the problem, and other special features which are relevant for a given decision context.

Thus, the complex and often ill-defined nature of important concepts such as risk and return, the multiple explanatory and decision factors involved, as well as the complex framework in which financial decisions are taken and implemented, calls for integrated decision aid tools. These should support the structuring of the problem, the modeling process, the identification and evaluation of alternative ways of action, as well as the implementation of the selected solutions. Operations research and management science (OR/MS) techniques address these issues and provide a wide range of modeling tools, suitable for handling financial decision problems under different schemes with regard to the decision context, the available information, and data.

M. Doumpos and C. Zopounidis, *Multicriteria Analysis in Finance*,
SpringerBriefs in Operations Research, DOI: 10.1007/978-3-319-05864-1_2,

© The Author(s) 2014

Multiple-criteria decision aid (MCDA) has evolved over the past decades becoming a major discipline in OR/MS. The field of MCDA is devoted to the development and implementation of decision support tools and methodologies for facilitating decision making in ill-structured problems involving conflicting multiple criteria, goals, objectives, and points of view.

In the context of MCDA a wide variety of decision settings can be considered, including among others static deterministic problems, decisions under uncertainty and fuzziness, dynamic problems, as well as group decision making. In all cases, the MCDA paradigm is based on the comprehensive description of a particular decision problem taking into account all the pertinent decision factors, on the basis of the decision makers' preferences. This is an appealing approach in many domains, including finance, given the high complexity that characterize the decisions that firms and organizations take and the multiple points of view which are involved (financial, regulatory, social, environmental, etc.). The following sections provide a brief overview of the MCDA field. An comprehensive introduction to the main concepts, principles and techniques in this field can be found in the book of Belton and Stewart [24], whereas the recent advances and research trends are presented in the books of Ehrgott et al. [80], and Zopounidis and Pardalos [270].

2.2 Main Concepts of Multicriteria Analysis

The main goal of MCDA is to provide decision aiding in complex and ill-structured problems, in accordance with the decision makers' preferential system and judgment policy. When multiple decision criteria are involved, there cannot be a unanimous optimal decision (in the traditional optimization sense), as different goals and objectives naturally lead to the formulation of different recommendations. However, having formal procedures and analytic techniques for problem structuring and the assessment of alternative ways of action, greatly facilitates the decision process.

MCDA intervenes in all phases of the decision process, beginning from problem structuring [24, 256] up to the implementation of the recommended solutions. An outline of the decision aiding process in the context of MCDA is illustrated in Fig. 2.1, following the approach introduced by Roy [207].

The first level of the above process, involves the specification of a set $\mathscr{A}$ of feasible alternative solutions to the problem at hand (alternatives). The objective of the decision is also determined. The set $\mathscr{A}$ can be continuous or discrete. In the former case $\mathscr{A}$ is specified through constraints imposed by the decision maker or by the decision environment. In the case where $\mathscr{A}$ is discrete, a finite set of alternatives are subject to evaluation.

The determination of the objective of the decision specifies the way that $\mathscr{A}$ should be considered to take the final decision. This involves the selection of the decision problematic that is most suitable to the problem at hand:

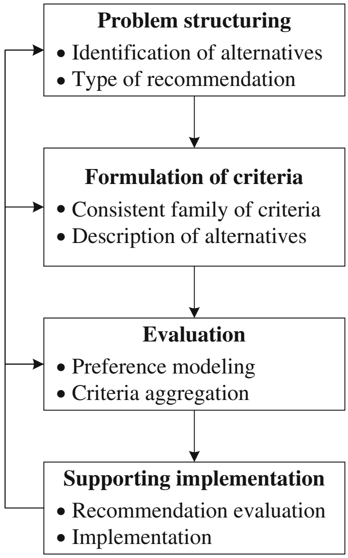

Fig. 2.1 The decision aiding process in MCDA

- Choice of the best alternative(s).
- Ranking of the alternatives from the best to the worst.
- Classification/sorting of the alternatives into pre-defined performance categories.
- Description of the alternatives.

The selection of an investment project is a typical choice example, whereas the ranking of a bank's branches on the basis of their efficiency and performance is an example of a ranking problem. Financial decisions that require a classification of the available options include credit scoring (see Chap. 4), mergers and acquisitions (e.g., identification of firms that could be takeover targets), and country risk assessment (see Chap. 6), among others. Finally, the descriptive problematic may involve the identification of alternatives with similar performance characteristics (firms, investments, countries, etc.). It must be noted, however, that often a combination of different problematics is required in order to address a given problem instance. For instance, in Sect. 3.3 we shall analyze a case study regarding the development of a bank rating methodology that uses a ranking scheme to define a classification of banks into performance categories.

The second stage involves the identification of all factors related to the decision. MCDA assumes that these factors have the form of criteria. A criterion is a real function f measuring the performance of the alternatives on each of their individual characteristics, defined such that:

$$f(\mathbf{x}) > f(\mathbf{y}) \Leftrightarrow \mathbf{x} \succ \mathbf{y} \text{ (alternative } \mathbf{x} \text{ is preferred over alternative } \mathbf{y})$$
$$f(\mathbf{x}) = f(\mathbf{y}) \Leftrightarrow \mathbf{x} \sim \mathbf{y} \text{ (alternatives } \mathbf{x} \text{ and } \mathbf{y} \text{ are indifferent)}$$

The set of the criteria $F = \{f_1, f_2, \dots f_K\}$ identified at this second stage of the decision aiding process, must form a consistent family of criteria. A consistent family of criteria is characterized by the following properties [31]:

- Monotonicity: If alternative $\mathbf{x}$ is preferred over alternative $\mathbf{y}$, the same should also hold for any alternative $\mathbf{z}$ such that $f_k(\mathbf{z}) \geq f_k(\mathbf{x})$ for all k.
- Completeness: If $f_k(\mathbf{x}) = f_k(\mathbf{y})$ for all criteria, then the decision maker should be indifferent between alternatives $\mathbf{x}$ and $\mathbf{y}$.
- Non-redundancy: The set of criteria satisfies the non-redundancy property if the elimination of any criterion results to the violation of monotonicity and/or completeness.

Once a consistent family of criteria has been specified, the next step is to proceed with the specification of the criteria aggregation model that meets the requirements of the problem. Finally, the last stage involves all the necessary supportive actions needed for the successful implementation of the results of the analysis and the justification of the model's recommendations.

2.3 Methodological Approaches

MCDA provides a wide range of methodologies for addressing decision-making problems of different types. The differences between these methodologies involve the form of the models, the model development process, and their scope of application. On the basis of these characteristics, the following four main streams in MCDA research can be distinguished [194]:

- Multiobjective optimization.
- Multiattribute utility/value theory.
- Outranking relations.
- Preference disaggregation analysis.

The following sections provide a brief overview of these methodological streams.

2.3.1 *Multiobjective Optimization*

Multiobjective optimization (MOO) extends the traditional single optimization framework to problems with multiple objectives. Formally, a MOO problem has the following form:

$$\begin{array}{ll} \max & f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_K(\mathbf{x}) \\ \text{subject to:} & \mathbf{x} \in \mathscr{A} \end{array} \tag{2.1}$$

where $\mathbf{x}$ is the vector of decision variables, $f_1, f_2, \dots, f_K$ are the objective functions (all assumed to be in maximization form) and $\mathscr{A}$ is the set of feasible solutions. The objectives are assumed to be in conflict, which implies that they are not all optimized simultaneously at a single solution.

In this context, optimality is defined on the basis of the concept of dominance. A feasible solution $\mathbf{x}^*$ dominates another solution $\mathbf{x} \in \mathscr{A}$ if and only if $f_k(\mathbf{x}^*) \geq f_k(\mathbf{x}), \forall k = 1, \ldots, K$, with at least one of the inequalities being strict. Thus, solving problem (2.1) requires the identification of Pareto optimal solutions, that is solutions not dominated by others.

The identification of the set of Pareto optimal solutions can be done with several techniques. A comprehensive discussion of various algorithmic procedures and formulations can be found in the books of Miettinen [179] and Steuer [229]. For instance, a commonly used approach is based on aggregating the objectives through a scalarization function. A typical example is the Chebyshev scalarization model, which is suitable for both convex and non-convex problem instances [229]. The model can be expressed as follows:

$$
\begin{array}{ll}
\min & \gamma + \rho \sum_{k=1}^{K} [f_k^* - f_k(\mathbf{x})] \\
\text{s.t.} & \gamma \geq w_k [f_k^* - f_k(\mathbf{x})] \\
& \mathbf{x} \in \mathscr{A}, \gamma \geq 0
\end{array}
\tag{2.2}
$$

where w_k is the non-negative trade-off constant for objective k, f_k^* is the maximum value of objective k (which can be found by performing K single objective optimizations), and ρ is a small positive constant used to exclude the possibility of obtaining weakly efficient solutions.[1] The full set of efficient (Pareto) solutions can be traced by solving the above optimization problem with different trade-offs for the objectives.

MOO problems can also be expressed in the form of goal programming (GP) formulations. In a GP context, the decision maker specifies target levels $t_1, t_2, \ldots, t_K$ for the objectives. A GP model can be expressed in the following general form:

$$
\begin{array}{ll}
\min & D(d_k^+, d_k^-) \\
\text{subject to:} & f_k(\mathbf{x}) + g_k(d_k^+, d_k^-) \leq = \geq t_k, \quad k = 1, 2, \ldots, K \\
& \mathbf{x} \in \mathscr{A} \\
& d_k^+, d_k^- \geq 0, \quad k = 1, 2, \ldots, K
\end{array}
\tag{2.3}
$$

where d_k^+, d_k^- are slack variables indicating the deviations from the pre-specified target levels, whereas D and $g_1, \ldots, g_K$ are functions of the slack variables. The first set of constraints defines the relationship between the objectives, the associated target levels, and the slack variables.

For instance, a goal of the form "objective k should be approximately equal to t_k" can be formulated as $f_k(\mathbf{x}) + d_k^+ - d_k^- = t_k$, with $d_k^+ + d_k^-$ being minimized. Similarly, a goal of the form "objective k should be at least equal to t_k, if possible" is formulated as $f_j(\mathbf{x}) + d_k^+ \geq t_k$, such that d_k^+ is minimized. Following the same

[1] A feasible solution $\mathbf{x}^*$ is called weakly efficient if there is no other feasible solution $\mathbf{x}$ such that $f_k(\mathbf{x}) > f_k(\mathbf{x}^*)$, for all $k = 1, \ldots, K$.

approach, different types of goals can be introduced in the general model (2.3). An detailed analysis of GP models and their applications can be found in the book of Jones and Tamiz [134].

2.3.2 Multiattribute Value Theory

Utility theory has played a central role in the field of decision analysis since its axiomatization by von Neumann and Morgenstern [185]. In a multicriteria context, multiattribute utility/value theory (MAUT/MAVT)[2] provides a normative approach for characterizing and analyzing rational decision making [139]. MAVT is mostly involved with the way decision makers make choices among a finite set of alternatives, but it also has important implications for MOO and GP models [78].

In particular, MAVT is involved with functional decision models (utility functions) aggregating multiple criteria into a composite indicator. A value function V aggregates a vector $\mathbf{x}$ of K decision criteria such that:

$$\begin{aligned}
&V(\mathbf{x}_i) > V(\mathbf{x}_j) \Rightarrow \text{alternative } i \text{ is preferred over alternative } j \ (\mathbf{x}_i \succ \mathbf{x}_j)\\
&V(\mathbf{x}_i) = V(\mathbf{x}_j) \Rightarrow \text{alternatives } i \text{ and } j \text{ are indifferent } (\mathbf{x}_i \sim \mathbf{x}_j)
\end{aligned}$$

Depending on the criteria independence conditions, different form of value functions can be defined. For instance, if it is assumed that the preferences of the decision maker on any subset of criteria do not depend on the other criteria (mutual preferential independence), then V is expressed in additive form:

$$V(\mathbf{x}) = \sum_{k=1}^{K} w_k v_k(x_k)$$

where $w_k \geq 0$ is the trade-off constant for criterion k and $v_k(x_k)$ is the associated marginal value function. This is a compensatory model, in the sense that the low performance in one criterion can be compensated by a high performance on others. The trade-off constants define this level of compensation. For instance, a poor performance on a criterion that has a high trade-off constant is not easily compensated by high performance by other criteria with low trade-offs. The trade-offs are by definition non-negative and they are usually normalized such that they sum up to a predefined scaling constant (e.g., $w_1 + w_2 + \cdots + w_K = 1$). On the other hand, the marginal value functions decompose the overall performance score into partial scores at the criteria level; they are non-decreasing for criteria in maximization form (e.g., profit related criteria) and non-increasing for minimization criteria (e.g.,

[2] The term "utility theory" is usually used in the context of decisions under uncertainty, whereas "value theory" is often preferred for deterministic problems. Having this distinction in mind, in order to simplify the presentation in the remainder of the book we shall use the term "value" to cover both situations.

risk criteria). Similarly to the trade-offs, the marginal values are also appropriately scaled, usually between 0 (for the worst performing alternative) and 1 (for the best performance).

Under weaker preferential independence assumptions alternative value models can be introduced. For instance, a multiplicative value function is expressed as follows:

$$1 + \lambda V(\mathbf{x}) = \prod_{k=1}^{K} [1 + \lambda w_k v_k(x_k)]$$

where $\lambda > -1$ is a scaling constant, such that $1 + \lambda = \prod_{k=1}^{K} [1 + \lambda w_k]$. In the case $\lambda = 0$ the multiplicative function reduces to an additive one.

Under the more general setting, the multilinear value function can be considered:

$$\begin{aligned} V(\mathbf{x}) = & \sum_{k=1}^{K} w_k v_k(x_k) + \sum_{k=1}^{K} \sum_{\ell > k} w_{k\ell} v_k(x_k) v_\ell(x_\ell) \\ & + \sum_{k=1}^{K} \sum_{\ell > k} \sum_{z > \ell} w_{k\ell z} v_k(x_k) v_\ell(x_\ell) v_z(x_z) + \cdots \\ & + w_{123\ldots} v_1(x_1) v_2(x_2) v_3(x_3) \cdots \end{aligned}$$

This general model has $2^K - 1$ scaling constants as opposed to K trade-offs involved in the additive and multiplicative forms, and includes these two simpler models as special cases. However, the additional complexity of the multilinear model makes it difficult to use in cases with $K \geq 4$. Nevertheless, Keeney and Raiffa [139] note that even when their underlying assumptions do not hold, additive and multiplicative are reasonable approximations to the general case.

2.3.3 Outranking Relations

The founding principles of outranking techniques can be traced to social choice theory [8]. An operational framework in the context of decision aiding, was first introduced by Roy [208] with the ELECTRE methods (ELimination Et Choix Traduisant la REalité).

In contrast to the functional models employed in the context of MAVT, outranking models are expressed in relational form through which the validity of affirmations such as "alternative i is at least as good as (or preferred over) alternative j" can be analyzed. Exploiting such pairwise comparisons through appropriate procedures leads to the final evaluation results (i.e., choice of the best ways of action, ranking or classification of finite set of alternatives from the best to the worst ones).

For instance, in the context of the ELECTRE methods [89] the evaluation process is based on pairwise comparisons used to assess the strength of the outranking relation "alternative i is at least as good as alternative j" ($\mathbf{x}_i \, S \, \mathbf{x}_j$). The comparisons are performed at two stages. The first involves the concordance test, in which the strength of the indications supporting the outranking relation is assessed. This can be done through the following concordance index:

$$C(\mathbf{x}_i, \mathbf{x}_j) = \sum_{k=1}^{K} w_k c_k(x_{ik}, x_{jk})$$

where x_{ik} and x_{jk} are the data for the two alternatives on criterion k, w_k is the weight (relative importance) of criterion k, and $c_k(x_{ik}, x_{jk})$ is the criterion's partial concordance index, defined such that:

$$c_k(x_{ik}, x_{jk}) = \begin{cases} 0 & \text{if } x_{ik} < x_{jk} - p_k \\ \frac{x_{ik} - x_{jk} + p_k}{p_k - q_k} & \text{if } x_{jk} - p_k \leq x_{ik} \leq x_{jk} - q_k \\ 1 & \text{if } x_{ik} > x_{jk} - q_k \end{cases}$$

where p_k and q_k are the user-defined preference and indifference thresholds for criterion k ($p_k \geq q_k \geq 0$). The case $C(\mathbf{x}_i, \mathbf{x}_j) = 1$ indicates that the outranking relation is clearly verified by all performance criteria, whereas the case $C(\mathbf{x}_i, \mathbf{x}_j) = 0$ indicates that there is no evidence to support the hypothesis that alternative i outranks alternative j.

At the second stage, the strength of the indications against the outranking relation is assessed through the calculation of a discordance index for each criterion:

$$d_k(x_{ik}, x_{jk}) = \begin{cases} 0 & \text{if } x_{ik} > x_{jk} - p_k \\ \frac{x_{ik} - x_{jk} + p_k}{p_k - v_k} & \text{if } x_{jk} - v_k \leq x_{ik} \leq x_{jk} - v_k \\ 1 & \text{if } x_{ik} < x_{jk} - v_k \end{cases}$$

The discordance indices examine the existence of veto conditions, in which the performance of alternative i may be too low in one or more criteria (i.e., $d_k(x_{ik}, x_{jk}) \approx 1$ for some k) and consequently it cannot be concluded that it outranks alternative j, irrespective of its performance on the rest of the evaluation factors. The veto threshold $v_k \geq p_k$ defines the minimum difference $x_{jk} - x_{ik}$ above which veto applies according to the performances of alternatives i and j on criterion k.

The combination of the two stages can be performed in different ways. For example, in the ELECTRE III method the following credibility index is used:

$$\sigma(\mathbf{x}_i, \mathbf{x}_j) = C(\mathbf{x}_i, \mathbf{x}_j) \prod_{k \in \mathscr{F}} \frac{1 - d_k(x_{ik}, x_{jk})}{1 - C(\mathbf{x}_i, \mathbf{x}_j)}$$

where $\mathscr{F}$ is the set of performance criteria such that $d_k(x_{ik}, x_{jk}) > C(\mathbf{x}_i, \mathbf{x}_j)$. Credibility indices close to one indicate that the outranking relation $\mathbf{x}_i \, S \, \mathbf{x}_j$ is almost surely true, whereas $\sigma(\mathbf{x}_i, \mathbf{x}_j) \approx 0$ indicates that the relation cannot be verified. On the basis of the results of such pairwise tests, different procedures can be used to chose the best alternatives, or to rank and classify them into categories.

Some particular special features of outranking models, include the consideration of non-compensatory and intransitive preferences. Non-compensation enriches the traditional preference and indifference relations, through the modeling of incomparability. Incomparability arises in situations where alternatives with special characteristics are considered (e.g., excellent performance on some criteria, but very poor performance on others). In such cases it may be difficult to derive straightforward conclusions on the overall performance of the alternatives. On the other hand, handling intransitive preference structures enables the modeling of situations where for example $\mathbf{x} \succ \mathbf{y}$ and $\mathbf{y} \succ \mathbf{z}$ does not imply $\mathbf{x} \succ \mathbf{z}$.

These features of outranking methods are particularly well-suited for financial decision making. For instance, the non-compensatory character of outranking models fits well the emphasis that finance practitioners and policy makers put on minimizing downside risk. With non-compensation, particularly risky features of the available options are identified and their trade-offs with other performance criteria are eliminated. On the other hand, cases where intransitivity and incomparability arise should be examined more closely with additional analysis possibly focusing on qualitative factors. For instance, Doumpos and Zopounidis [74] found that, in the context of corporate credit scoring, such cases are most likely to arise for firms whose financial characteristics are not enough to formulate an accurate recommendation.

Apart from the ELECTRE methods, the PROMETHEE [33] methods have also been widely used for building and exploiting outranking/preference relations in decision aiding. An example of using such an evaluation technique in the context of banking management is given in Sect. 3.3.1. An overview of other outranking techniques can be found in [169].

2.3.4 Preference Disaggregation Analysis

The development of the decision models in MCDA is based on direct and indirect procedures. Direct procedures require the decision maker to specify the parameters of the model (e.g., the criteria trade-offs) through interactive, structured communication sessions in cooperation with the decision analyst. In some cases this might be a feasible process, mainly when the decision involves strategic choices of non-repetitive character. In other cases, however, where real-time decision making is required, such direct procedures are not applicable. Furthermore, the cognitive difficulties associated with direct elicitation procedures, are also an important factor. Indirect preference disaggregation methods are very helpful in this context [128]. Preference disaggregation analysis (PDA) uses regression-like techniques to infer a decision model from a set of decision examples on some reference alternatives, so that the model is as consistent as possible with the actual evaluation of the alternatives by the decision maker.

The key assumption in PDA is that the decision maker is unable or unwilling to provide direct information about his/her system of preferences, other than a sample of decisions that he/she has taken in the past or would take in a given future situation. Given this set of sample decisions (reference set), the analyst should provide the decision maker with a starting basis upon which he/she can elaborate on the specific details of his/her preferential system. In this context, inferring a model that is consistent with the given sample decisions can be of great help to the decision aiding process.

The reference set is the main input in a PDA process; it may consist of past decisions, a subset of the alternatives under consideration, or a set of fictitious alternatives which can be easily judged by the decision maker [128]. Depending on the decision problematic, the evaluation of the reference alternatives may be expressed by defining an order structure (total, weak, partial, etc.) or by classifying them into appropriate classes.

Formally, let $Y(X')$ denote the decision maker's evaluation of a set X' consisting of M reference alternatives described over K criteria. Such an evaluation may involve a complete or partial ranking of the alternatives or their classification in predefined categories and it is assumed to be based (implicitly) on a decision model $F(\mathbf{x}; \boldsymbol{\alpha})$ defined by some parameters $\boldsymbol{\alpha}$, which represent the actual preferential system of the decision maker. The objective of PDA is to infer the "optimal" parameters $\widehat{\boldsymbol{\alpha}}^*$ that approximate, as accurately as possible, the actual preferential system of the decision maker as represented in the unknown set of parameters $\boldsymbol{\alpha}$, i.e.:

$$\widehat{\boldsymbol{\alpha}}^* = \arg \min_{\widehat{\boldsymbol{\alpha}} \in \mathscr{A}} \|\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}\| \tag{2.4}$$

where $\mathscr{A}$ is a set of feasible values for the parameters $\widehat{\boldsymbol{\alpha}}$. With the obtained parameters, the evaluations performed with the corresponding decision model $F(\mathbf{x}; \widehat{\boldsymbol{\alpha}}^*)$ will be consistent with the evaluations actually performed by the decision maker for any set of alternatives.

Problem (2.4), however, cannot be solved explicitly because $\boldsymbol{\alpha}$ is unknown. Instead, an empirical estimation approach is employed using the decision maker's evaluation of the reference alternatives to proxy $\boldsymbol{\alpha}$. Thus, the general form of the optimization problem is expressed as follows:

$$\widehat{\boldsymbol{\alpha}}^* = \arg \min_{\widehat{\boldsymbol{\alpha}} \in \mathscr{A}} L[Y(X'), \widehat{Y}(X')] \tag{2.5}$$

where $\widehat{Y}(X')$ denotes the recommendations of the model for the alternatives in X' and $L(\cdot)$ is a function that measures the differences between $Y(X')$ and $\widehat{Y}(X')$.

For instance, consider an ordinal regression setting, where a decision maker wants to construct a model for ranking some alternatives from the best to the worst ones. The decision maker has evaluated the six alternatives of Table 2.1 under three criteria and provided a ranking (last column) from the best ($y_i = 1$) to the worst one ($y_i = 6$).

The decision maker has decided to construct a decision model of the form $V(\mathbf{x}) = w_1x_1 + w_2x_2 + w_3x_3$, such that $w_1, w_2, w_3 \geq 0$ and $w_1 + w_2 + w_3 = 1$, which is as consistent as possible with the provided evaluations. In order to be consistent with the information in the given reference set, the model should satisfy the inequality

Table 2.1 A reference set for constructing a ordinal regression model

Alternatives	Criteria			Ranking
	x_1	x_2	x_3	(y)
$\mathbf{x}_1$	7	1	8	1
$\mathbf{x}_2$	4	5	8	2
$\mathbf{x}_3$	10	4	2	3
$\mathbf{x}_4$	2	4	1	4
$\mathbf{x}_5$	4	1	1	5
$\mathbf{x}_6$	1	2	5	6

$V(\mathbf{x}_i) > V(\mathbf{x}_j)$, for all pairs of alternatives such that $y_i < y_j$ (i.e., alternative i is preferred over alternative j, $\mathbf{x}_i \succ \mathbf{x}_j$). Such a model can be constructed through the solution of the following linear program:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{6} \sum_{j \neq i} \varepsilon_{ij} \\
\text{s.t.} \quad & \sum_{k=1}^{3} w_k (x_{ik} - x_{jk}) + \varepsilon_{ij} \geq \delta \quad \forall \mathbf{x}_i \succ \mathbf{x}_j \\
& w_1 + w_2 + w_3 = 1 \\
& w_k, \varepsilon_{ij} \geq 0 \qquad\qquad \forall i, j, k
\end{aligned}
$$

where $\varepsilon_{ij} = \max\{0, V(\mathbf{x}_j) - V(\mathbf{x}_i)\}$ is the absolute error for the pair of alternatives $\mathbf{x}_i \succ \mathbf{x}_j$ and δ is a small positive constant.

The foundations of PDA have been set during the 1950s with the introduction of non-parametric regression techniques using goal-programming formulations [254] and their later extension to ordinal regression models [226]. Jacquet-Lagrèze and Siskos [127] first defined the PDA framework in the context of decision aiding through the introduction of the UTA method, which is based on an additive utility modeling approach. However, other decision models can also be employed, including non-linear utility functions [37], rule-based models [105], outranking models [67, 182], Choquet integrals [104], and kernel models [191].

A comprehensive bibliography on preference disaggregation methods can be found in Jacquet-Lagrèze and Siskos [128], whereas some recent trends are discussed in [220].

Springer

http://www.springer.com/978-3-319-05863-4

Multicriteria Analysis in Finance
Doumpos, M.; Zopounidis, C.
2014, VIII, 109 p. 15 illus., Softcover
ISBN: 978-3-319-05863-4