Preface

The first objective of this work is to present, to some extent, a deep introduction to the basic concepts on real and functional analysis.

In principle, the text is written for applied mathematicians and postgraduate students in applied mathematics, with interest in applications of functional analysis, calculus of variations, and optimization to problems in physics and engineering.

However, engineers, physicists, and other professionals in related areas may find the text very interesting by the possibility of background development towards graduate-level mathematics applicable in their respective work fields.

We have proven almost all results presented. The proofs are rigorous, but we believe are almost all very clear and relatively easy to read, even at the most complex text parts.

The material presented in Parts I and II concerns standard real and functional analysis. Hence in these two parts the results in general are not new, with the exception of some sections on domains of class $\hat{C}_1$ and relating Sobolev spaces and some sections about Lagrange multiplier results and the basic theorem about relaxation for the scalar case, where we show a different proof concerning the original one in the book *Convex Analysis and Variational Problems* (indeed such a book is the theoretical base of the present work) by Ekeland and Témam's.

About the basic part, specifically Chaps. 1–3 correspond to standard functional analysis. In Chaps. 4–6 we present basic and advanced concepts in measure and integration which will be relevant in subsequent results (in fact perhaps a little more than the minimum necessary). Moreover, Chaps. 7 and 8 correspond to a basic exposition on Sobolev spaces and again, the fundamental results presented are relevant for subsequent developments. In Chaps. 9–11 we introduce some basic and more advanced concepts on calculus of variations, convex analysis, and optimization.

Finally, the applications presented in Chaps. 12–23 correspond to the work of the present author along the last years, and almost all results including the applications of duality for micro-magnetism, composites in elasticity, and conductivity and phase transitions are extensions and natural developments of prior ones presented in the author's Ph.D. thesis at Virginia Tech, USA, and the previous book *Topics on Functional Analysis, Calculus of Variations and Duality* published by Academic

Publications. The present book overlaps to some extent with the previous one just on a part concerning standard mathematics. The applications in the present one are almost all new developments.

Anyway, a key feature of the present work is that while all problems studied here are nonlinear with corresponding non-convex variational formulation, it has been almost always possible to develop convex (in fact concave) dual variational formulations, which in general are more amenable to numerical computations.

The section on relaxation for the vectorial case, as its title suggests, presents duality principles that are valid even for vectorial problems. It is worth noting that such results were used in this text to develop concave dual variational formulations in situations such as for conductivity in composites and vectorial examples in phase transitions. In Chap. 15 we present the generalized method of lines, a numerical procedure in which the solution of the partial differential equation in question is written on lines as functions of boundary conditions and boundary shape. In Chap. 22 we develop some examples concerning the Navier–Stokes system.

Summary of Main Results

The main results of this work are summarized as follows.

Duality Applied to Elasticity

Chapter 12 develops duality for a model in finite elasticity. The dual formulations obtained allow the matrix of stresses to be nonpositive or nonnegative definite. This is, in some sense, an extension of earlier results (which establish the complementary energy as a perfect global optimization duality principle only if the stress tensor is positive definite at the equilibrium point). The results are based on standard tools of convex analysis and the concept of the Legendre transform.

Duality Applied to a Plate Model

Chapter 13 develops dual variational formulations for the two-dimensional equations of the nonlinear elastic Kirchhoff–Love plate model. We obtain a convex dual variational formulation which allows nonpositive definite membrane forces. In the third section, similar to the triality criterion introduced in [36], we obtain sufficient conditions of optimality for the present case. Again the results are based on the fundamental tools of convex analysis and the Legendre transform, which can easily be analytically expressed for the model in question.

Duality Applied to Ginzburg–Landau-Type Equations

Chapters 14–16 are concerned with existence theory and the development of dual variational formulations for Ginzburg–Landau-type equations. Since the primal formulations are non-convex, we use specific results for distance between two convex functions to obtain the dual approaches. Note that we obtain a convex dual formulation for the simpler real case. For such a formulation optimality conditions are also established.

Duality Applied to Multi-well Variational Problems

The main focus of Chaps. 17 and 18 is the development of dual variational formulations for multi-well optimization problems in phase transitions, conductivity, and elasticity. The primal formulation may not have minimizers in the classical sense. In this case, the solution through the dual formulation is a weak limit of minimizing sequences for the original problem.

Duality for a Model in Quantum Mechanics

In Chap. 19 we develop a duality principle and computation for a class of nonlinear eigenvalue problems found in quantum mechanics models. We present numerical results for one- and two-dimensional problems. We highlight that this chapter is coauthored by myself and my colleague Professor Anderson Ferreira.

Duality Applied to the Optimal Design in Elasticity

The first part of Chap. 20 develops a dual variational formulation for the optimal design of a plate of variable thickness. The design variable, namely the plate thickness, is supposed to minimize the plate deformation work due to a given external load. The second part is concerned with the optimal design for a two-phase problem in elasticity. In this case, we are looking for the mixture of two constituents that minimizes the structural internal work. In both applications the dual formulations were obtained through basic tools of convex analysis. Finally, we highlight that this chapter is coauthored by myself and my colleague Professor Alexandre Molter.

Duality Applied to Micro-magnetism

The main focus of Chap. 21 is the development of dual variational formulations for functionals related to ferromagnetism models. We develop duality principles for the so-called hard and full (semi-linear) uniaxial cases. It is important to emphasize that the dual formulations here presented are convex and are useful to compute the average behavior of minimizing sequences, specially as the primal formulation has no minimizers in the classical sense. Once more the results are obtained through standard tools of convex analysis.

Duality Applied to Fluid Mechanics

In Chap. 22 we develop approximate solutions for the incompressible Navier–Stokes system through the generalized method of lines. We also obtain a linear system whose solution solves the steady-state incompressible Euler equations.

Duality Applied to the Optimal Control and Optimal Design of a Beam Model

Chapter 23 develops duality for the optimal control and design of a beam model. We emphasize the dual formulation is useful to obtain numerical results. Finally, numerical examples of optimal design are provided, concerning the maximization of buckling load and fundamental frequency, respectively.

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