Chapter 2
The Paradox Theory

In this chapter, it is explained for what purposes an IRA can be used, and how an IRA should be evaluated, according to the Paradox Theory of IRAs (one of the two theories discussed in this book).

2.1 Use

According to this theory, the goal of an IRA is to refute a proposition. An IRA can refute two kinds of propositions: namely, a universally quantified proposition (such as 'for all propositions x, x is justified to an agent S only if S has a reason for x') or an existentially quantified proposition (such as 'for at least proposition x, x is justified to an agent S'). These options will be explained in turn in the following. The main supporters of this theory are Black (1996) and Gratton (1997, 2009).[1]

The argument schema to refute universally quantified propositions may be represented as follows:

Paradox Schema A

(1) For all x in domain K, x is F only if there is a new item y in K and x and y stand in R.
(2) For all x and y in K, x and y stand in R only if y is F.
(3) At least one item in K is F.
(4) An infinity of items in K are F. [from 1–3]
(5) (4) is false: No infinity of items in K are F.
(C) (1) is false: It is not the case that for all x in K, x is F only if x stands in R to a new item y in K. [from 1–5]

[1] Versions of this theory have also been discussed or suggested, if only briefly, by Russell (1903, Sect. 329), Beth (1952), Yalden-Thomson (1964), Gettier (1965), Schlesinger (1983, Chap. 8), Sanford (1975, 1984), Day (1986, 1987), Clark (1988), Post (1993), Jacquette (1996), Nolan (2001), Klein (2003), Orilia (2006), Oppy (2006, Chap. 9), Maurin (2007, 2013), Cling (2008, 2009), Rescher (2010), Wieland (2012, 2013).

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This schema has one hypothesis for Reductio Ad Absurdum (RAA), i.e. line (1); three premises, i.e. lines (2), (3) and (5); and two main inferences, i.e. lines (4) and (C). For details about the inferences, see Sect. 2.4. To obtain instances of this schema, 'K' has to be replaced with a specific domain, and 'F' and 'R' with a predicate which expresses a property of or relation between the items in that domain. For example, this schema can be read on the basis of the following key[2]:

- domain K: persons;
- x is F: x is reliable;
- x and y stand in R: x is guarded by y.

These instructions yield the following instance of Paradox Schema A:

Guardians (Paradox A instance)

(1) For all persons x, x is reliable only if x is guarded by a guardian.
(2) For all persons x and y, x is guarded by a guardian y only if y is reliable.
(3) At least one person is reliable.
(4) An infinity of persons is reliable. [from 1–3]
(5) (4) is false: No infinity of persons is reliable.
(C) (1) is false: It is not the case that for all persons x, x is reliable only if x is guarded by a guardian. [from 1–5]

As a second example, here is the Problem of the Criterion (introduced in Sect. 1.2) constructed as such an instance:

Disputes (Paradox A instance)

(1) For all propositions x, the dispute about x is settled only if it is settled by a criterion.
(2) For all propositions x and y, the dispute about x is settled by y only if the dispute about y is settled.
(3) The dispute about at least one proposition is settled.
(4) The dispute about an infinity of propositions is settled. [from 1–3]
(5) (4) is false: It is not the case that the dispute about an infinity of propositions is settled.
(C) (1) is false: It is not the case that for all propositions x, the dispute about x is settled only if it is settled by a criterion. [from 1–5]

Such negative conclusions can be associated with certain positive outcomes, in these cases that there is at least one reliable person who is not guarded by a guardian, and that there is at least one dispute which is settled yet not by a criterion. Examples can easily be multiplied. For example, Aristotle's case reconstructed in terms of this schema would conclude that there is at least one thing that is good and not desired for the sake of something else.

[2] For an overview of further instances of the letters, see Sect. 2.3 below.

Next, the argument schema to refute existentially quantified propositions:

Paradox Schema B

(1) For all x in domain K, x is F only if there is a new item y in K and x and y stand in R.
(2) For all x and y in K, x and y stand in R only if y is F.
(3) At least one item in K is F.
(4) An infinity of items in K are F. [from 1–3]
(5) (4) is false: No infinity of items in K are F.
(C) (3) is false: No item in K is F. [from 1–5]

The only difference with the previous schema is that this time (3), rather than (1), is the hypothesis for RAA. In this case, the conclusions of the guardians and disputes IRAs would be respectively: no person is reliable; the dispute about no proposition is settled.

These two argument schemas are labelled ‘Paradox Schemas’ because their instances closely resemble paradoxes. Paradoxes (or at any rate many of them) are such that a set of propositions (which are to some extent independently intuitive) together entail a contradiction so that, by RAA, at least one of them must be false (cf. Sainsbury 1987, p. 1; Clark 2002, pp. 151–154). The same applies to instances of the Paradox Schemas: the propositions (1), (2) and (3) jointly entail (4) which forms a contradiction with (5) so that, by RAA, at least one of (1), (2), (3) or (5) must be false.

2.2 Evaluation

When are IRAs that take the form of the Paradox Schemas sound? Or put differently: how can one resist such arguments? To explain this, we need to consider the broader dialectical context of instances of the Paradox Schemas, i.e. a context between opponents who attack one other’s position and defend their own. If we consider two persons ‘S1’ and ‘S2’, then the dialectical situation is as follows:

Line	What	Dialectical context
(1)/(3)	Hypothesis for RAA	S1’s position
(2)	Premise	S2 shows that S1 has to concede this
(1)/(3)	Premise	S2 shows that S1 has to concede this
(4)	Infinite regress	S2 infers this from (1) to (3)
(5)	Premise	S2 shows that S1 has to concede this
(C)	Rejection	S2 infers this from (1) to (5) by RAA

The slash ‘/’ indicates the difference between the two Paradox Schemas.

Consider for example the following IRA[3]:

Reasons (Paradox B instance)

(1) For all propositions x, x is justified to S only if S has a reason y for x.
(2) For all propositions x and y, S has a reason y for x only if y is justified to S.
(3) At least one proposition is justified to S.
(4) S has an infinity of reasons. [from 1–3]
(5) (4) is false: S does not have an infinity of reasons.
(C) (3) is false: No proposition is justified to S. [from 1–5]

In this case, the dialectic is between epistemological views that hold that justification in fact obtains (S1) and the sceptical position that no proposition is justified to anyone (S2). First, S1's position is constructed as the view that at least one proposition is justified to someone. Second, S2 defends that a proposition is justified to someone only if she has a reason for it. Third, S2 defends that someone has a reason only if that reason itself is justified to her. Fourth, S2 infers an infinite regress from the foregoing. Fifth, S2 defends that the regress is absurd or otherwise vicious (so that (5) is true: it is true that one does not have an infinity of reasons). Last, S2 rejects S1's position and concludes that no proposition is justified to anyone.

From this, it can easily be seen what can be done to resist an instance of a Paradox Schema. There are five main options, corresponding to each of the lines. Person S1 could deny that:

- the hypothesis was in fact her position;
- the first premise that helps generating the regress holds;
- the second premise that helps generating the regress holds;
- an infinite regress is entailed even if the foregoing does hold;
- the regress does not exist (or is unacceptable in another way).

In the regress of reasons case, for example, a popular option is the second. Namely (a version of) foundationalism denies that reasons are always required for justification. The last option goes under the name 'infinitism', and denies that the regress of reasons is unacceptable in the first place.

Indeed, not all infinite regresses are thought to be vicious or unacceptable. There are 'good' and 'bad' cases. What is the difference? To explain this, we need the concept of an infinite regress in general. According to the Paradox Theory, all infinite regresses are entailed by the schematic lines (1)–(3) of the Paradox Schemas and consist of steps each of which is a necessary condition for the previous one. Schematically:

[3] As the focus here is on IRAs generally, I will ignore some details regarding the content of this instance, such as the difference between propositional and doxastic justification (cf. Klein 2007), or the difference between justification as a state and justification as an activity (cf. Rescorla 2014).

(a) a is F;
(b) a and b stand in R;
(c) b is F;
(d) b and c stand in R;
etc.

Here, for example, are the regresses of reasons and guardians in Paradox format (where p_{1-n} are names for propositions and persons respectively):

(a) p_1 is justified to S;
(b) S has a reason p_2 for p_1;
(c) p_2 is justified to S;
(d) S has a reason p_3 for p_2;
etc.

(a) p_1 is reliable;
(b) p_1 is guarded by a guardian p_2;
(c) p_2 is reliable;
(d) p_2 is guarded by a guardian p_3;
etc.

In series like these, (b) is a necessary condition for (a), (c) is a necessary condition for (b), and so on. Importantly, not just any series of the form (a), (b), (c), etc. is a regress. For example, a mere series of guardians that guard one another is not a regress. Likewise, a mere series of numbers (1, 2, 3, etc.) is not a regress. Only those series entailed by the schematic lines (1)–(3) of the Paradox Schemas are considered as regresses.

Furthermore, an infinite regress is vicious iff the entailed infinite regress does not exist (or is shown to be unacceptable, as I will explain below). For in that case is one committed to a contradiction (i.e. between lines (4) and (5) of the Paradox Schemas: an infinity of Ks are F and no infinity of Ks are F), and must one reject one of the propositions that generates the infinite regress.

For example, infinite regresses of reasons are vicious only if it has been shown that there are no such regresses, i.e. that the infinity of necessary conditions are not, or cannot be, in place (e.g. that one does not or cannot have an infinity of reasons). For example, one worry would be that it is mentally impossible for human beings to possess so many of reasons. Furthermore, such regresses are non-vicious if it has been shown that they can and do exist. As noted, the view that they are not vicious is called ‘infinitism’.[4] In Sect. 5.3, I will provide one extended example of a regress (generated in a Paradox way) that is arguably harmless.

One general reason why regresses are vicious concerns paradoxes of infinity (for an overview, cf. Oppy 2006, Chap. 3). For example, in the case of Hilbert’s Hotel with an infinite number of rooms, all of which are occupied, the question is

[4] For defences, cf. Klein (1999, 2007), Peijnenburg (2010), Aikin (2011).

how it can be that there is always room for a new guest (namely by moving the guest from room 1 to room 2, the guest from 2 to 3, and so on). Moreover, if infinities are regarded as paradoxical and absurd from the very start, then all regresses (generated in a Paradox way) are vicious. For surely if they are impossible, they do not exist.

In many cases, however, it is generally accepted that the regress is possible, i.e. that it could exist, yet debated whether it is *acceptable*, i.e. whether the benefits of rejecting one of the propositions that generate the regress outweigh the costs of the regress (cf. Lewis 1983, pp. 353–354; Nolan 2001; Cameron 2008). Consider for example the regressive claim that for every x, there is a singleton {x} (i.e. the set of itself). This generates regresses such as: Socrates, {Socrates}, {{Socrates}}, and so on. The question is whether the benefits of rejecting the claim that anything has a singleton outweigh the costs of such regresses (in this case infinitely many sets). The regress of singletons is vicious, then, only if the costs are too high.

2.3 Classic Instances

Here is a list of further filling instructions for the Paradox Schemas (based on the classic cases from Sect. 1.3). In each case, to be sure, further details of the schematic letters could be spelled out. Yet this would go beyond what theories of IRAs have to provide: namely, the form that IRAs (or a group among them) have in common.

Items in K	x is F	x and y stand in R
Sets of large things	the members of x are large	y contains the form Largeness in which the members of x participate
Actions	x is good	y is an end for the sake of which x is performed
Propositions	x is justified to S	y is a reason for x that is available to S
Contingent beings	x exists	x is caused by y
Inductive inferences	x is justified	x is derived from past facts and the assumption y that the future resembles the past
Relations	x is unified with its relata	y unifies x with its relata
Sets of premises	a conclusion follows from x	y contains an additional premise ‘if the members of x are true, then the conclusion must be true’
Sets of relata	the members of x stand in an asymmetric relation	y is a set of properties such that the members of x have these properties
A-series	x is contradiction-free	y is an A-series such that the members of x are past, present and future at different times of y
Rules	x has a fixed use	y fixes the use of x
Actions	x is performed intelligently	y is an action of applying knowledge that x must be performed in such and such way
Languages	x is liar paradox-free	that sentences of x are true or not is only stated in a meta-language y

2.4 Logical Analysis

In this section, I provide the logical details of the Paradox Schemas presented in Sect. 2.1. It shows that they are valid according to classical first-order logic. A few comments are in order. I use the propositional calculus by Nolt et al. (1988, Chap. 4), and the first-order extension by Gamut (1982, pp. 142–147). This means that I employ standard natural deduction abbreviations of the inference rules, a strict distinction between premises (PREM) and hypotheses (HYP), and the hypothetical rules Reductio Ad Absurdum (¬I) and Conditional Proof (→I). All portions of hypothetical reasoning are clearly marked by vertical lines. Some of the predicates and premises need some explanation. These explanations are provided right after the formalisation.

Key:

Kx: x is in domain K
Fx: x has property F
Rxy: x stands in relation R to y

Example:

Kx: x is a proposition
Fx: the dispute about x is settled
Rxy: the dispute about x is settled by y

Paradox Schema A

(1)	$\forall x \forall y((Ky \wedge Rxy) \rightarrow Fy)$	PREM
(2)	$\exists x(Kx \wedge Fx)$	PREM
(3)	$\neg(\exists x(Kx \wedge Fx) \wedge (\forall x((Kx \wedge Fx) \rightarrow \exists y(Ky \wedge Fy \wedge Rxy))))$	PREM
(4)	$\forall x((Kx \wedge Fx) \rightarrow \exists y(Ky \wedge Rxy))$	HYP ¬I
(5)	$Ka \wedge Fa$	HYP →I
(6)	$(Ka \wedge Fa) \rightarrow \exists y(Ky \wedge Ray)$	4; ∀E
(7)	$\exists y(Ky \wedge Ray)$	5, 6; →E
(8)	$Kb \wedge Rab$	HYP →I
(9)	$\forall x((Kb \wedge Rxb) \rightarrow Fb)$	1; ∀E
(10)	$(Kb \wedge Rab) \rightarrow Fb$	9; ∀E
(11)	Fb	8, 10; →E
(12)	$Kb \wedge Fb \wedge Rab$	8, 11; ∧I
(13)	$\exists y(Ky \wedge Fy \wedge Ray)$	12; ∃I
(14)	$Kb \wedge Rab \rightarrow \exists y(Ky \wedge Fy \wedge Ray)$	8-13; →I
(15)	$\exists y(Ky \wedge Fy \wedge Ray)$	7, 14; ∃E
(16)	$(Ka \wedge Fa) \rightarrow \exists y(Ky \wedge Fy \wedge Ray)$	5-15; →I
(17)	$\forall x((Kx \wedge Fx) \rightarrow \exists y(Ky \wedge Fy \wedge Rxy))$	16; ∀I
(18)	$\exists x(Kx \wedge Fx) \wedge (\forall x((Kx \wedge Fx) \rightarrow \exists y(Ky \wedge Fy \wedge Rxy)))$	2, 17; ∧I
(19)	$(18) \wedge \neg(18)$	18, 3; ∧I
(20)	$\neg(\forall x((Kx \wedge Fx) \rightarrow \exists y(Ky \wedge Rxy)))$	4-19; ¬I

Lines (1)–(4) may have variants in terms of one- or many-place predicates and their number (this does not hold for the Failure Schemas). Also, it is easy to see how variant B of the Paradox Schema can be constructed where line (2), rather than (4), is the hypothesis for Reductio Ad Absurdum (¬I).

Line (18) requires some explanation. Literally, it does not yet express that there is an infinity of Ks that are F. The reason is that the existential quantifier does not yet say what it should say, namely that 'y' has to be a new item in the domain. The phrase 'there is a new item y' cannot be expressed by a familiar logical constant, for it does not mean merely 'there is an item y that is distinct from x', but rather 'there is an item y that is distinct from *all other items mentioned earlier in the regress*'. To express this, we could introduce an additional relation '<', distinct from R, whose only job is to order the Ks, and make sure that all items introduced in the regress are new items (so that they form an infinite, non-circular series). To do this, '$x < y$' can be read as 'x occurs earlier in the regress than y' and has to satisfy the following conditions[5]:

- $\forall x \neg x<x$
- $\forall x \forall y \forall z((x<y \wedge y<z) \rightarrow x<z)$
- $\forall x \forall y((x \neq y \wedge Kx \wedge Ky) \rightarrow (x<y \vee y<x))$
- $\forall x \forall y(x<y \rightarrow (Kx \wedge Ky))$

Moreover, this allows us to formulate the contradiction in (19) between 'an infinity of Ks are F' and 'no infinity of Ks are F' in first-order terms:

- $\exists x(Kx \wedge Fx) \wedge \forall x((Kx \wedge Fx) \rightarrow \exists y(x<y \wedge Ky \wedge Fy))$
- $\exists x(Kx \wedge Fx \wedge \forall y((Ky \wedge x<y) \rightarrow \neg Fy))$

For example: The dispute about at least one proposition is settled and the dispute about any proposition is settled only if there is a new proposition about which the dispute is settled; For at least one proposition x, the dispute about x is settled, and for all new propositions y, the dispute about y is not settled.

Another option, suggested by Cling (2009, p. 343), would be to drop the idea of 'infinity', and replace 'there is an infinity of Ks that are F' with 'there is an endless regress of Ks that are F' (where the latter, but not the former, includes finite, circular regresses). If we change this throughout the argument we would not need to block loops, and yet we would still obtain a contradiction in (19) so that we can apply ¬I. This solution will work in all cases where infinity is not really an issue (i.e. where the unacceptability of a regress does not derive from its infinity).

Finally: a very similar logical analysis can be provided for Paradox Schema B (i.e. which differs mainly regarding HYP ¬I).

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5 These ensure that '<' is irreflexive and transitive, and that all and only Ks stand in '<'. Thanks to Christian Straßer for suggesting this solution.

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