Preface

> The following empirical rule is sound advice to number-crunchers and arithmurgists: *Parabola/Sine Rule*. If a solution is well-approximated by a parabola or a sine function or otherwise is very, very smooth, then it is a poor example for comparing and evaluating numerical methods because even a dreadful, awful numerical method will work tolerantly well for such nice function.
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> **—J. P. Boyd, 2011**

Spectral methods, *collocation*, *Galerkin* and *tau*, offer useful alternatives to finite differences or finite elements type methods in solving boundary value problems attached to differential equations. Advantages of such methods include the production of global solutions which are rapid convergent and, in some cases, the avoidance of Gibbs phenomenon at domain boundaries.

This work is not oriented on formal reasoning which means the well known sequence of axioms, theorems, proofs, corollaries, etc. Instead, it is mainly oriented to the *constructive and practical aspects* of spectral methods. Consequently, we rigorously examine the most important qualities as well as drawbacks of spectral methods in the context of numerical methods devoted to solve non-standard eigenvalue problems. Some nonlinear singularly perturbed boundary value problems along with eigenproblems obtained by their linearization around constant solutions are also considered.

By non-standard eigenvalue problems, we mean singular Sturm–Liouville problems, high order (larger than two) singular and nonsingular generalized eigenvalue problems, eigenproblems involving boundary conditions dependent on the eigenparameter and multiparameter eigenvalue problems. We consider them challenging and thus suitable in evaluating spectral methods, according to above Boyd's advice. Thus, one of the main aims of this work is to review our contributions in tuning spectral methods in order to obtain reliable eigenvalues and eigenmodes, at least for a specified region of the spectrum.

For problems formulated on finite domains, we have used mainly families of Chebyshev polynomials in order to build up test and trial spaces for all three methods. In working with spectral tau and Galerkin methods, we have succeeded to construct both type of these fundamental spaces such that the discretization matrices inherit the properties of differential operators and additionally are sparse (even banded) with fairly good conditioning properties.

With respect to the Chebyshev collocation, its spectral accuracy has enabled us, among other results, to formulate an important conjecture corresponding to the first eigenvalue of the singularly perturbed Viola's problem.

For problems defined on the half-line, we have considered only collocation methods based on Laguerre functions. With such basis functions, we avoid the domain truncation or mapping and enforce exactly any type of boundary conditions at infinity.

Frequently, tau scheme or collocation coupled with some factorization of a high-order differential operator cast a problem into singular algebraic generalized eigenvalue problems. They are singular in the sense that the first matrix in the pencil has a larger rank than the second one. In this situation, some spurious eigenvalues (at infinity) are inevitable when QR/QZ algorithms are used. Moreover, for high order eigenvalue problems, i.e., sixth and eighth orders, and for orders of thousands for the cut-off parameters, the above-mentioned collocation algorithms could be expensive with respect to the CPU consumed time and storage. Thus, in order to improve the accuracy in computing a specified part of the finite spectrum and to shorten the elapsed time we adapted some subspace methods to solve eigenproblems such as Jacobi–Davidson methods. These methods are target-oriented and systematically avoid spurious eigenvalues.

Two classes of applications from mechanics of continua are envisaged. In the first one, the linear stability of elastic systems along with some linear hydrodynamic stability problems are analyzed. Both type of problems lead, in some instances, to eigenvalue problems containing eigenparameter-dependent boundary conditions. In the second class, we gather a lot of second and fourth order genuinely nonlinear two-point boundary value problems formulated on finite or infinite intervals. Some of them exhibit singularities in origin and at infinity and are originated in fluid mechanics, foundation engineering, etc.

The work can be used as a self contained supplementary textbook for various review courses. One has all ingredients, i.e., differentiation matrices, in both physical and phase spaces, and complete procedures in order to implement various types of boundary conditions. Consequently, linear and nonlinear two-point boundary value problems, possibly singularly perturbed, as well as eigenvalue problems of various orders can be solved.

I have to acknowledge first one special colleague who worked with enthusiasm besides me in spectral methods early in the turbulent years '90. He is Sorin Iuliu Pop (TU Eindhoven and alumnus of UBB Cluj-Napoca). I am lucky to have found Sorin at a stage of his career when he still had so much time to spend with others. I am especially grateful to Bor Plestenjak (University of Ljubljana) for providing suggestive colorful pictures concerning Mathieu's eigenmodes. To him, Joost Rommes (NXP Semiconductors, Eindhoven) and Michiel Hochstenbach (TU Eindhoven) must go my thanks for many illuminating discussions on the implementation of JD methods.

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