

Preface

Over time the impossible becomes understandable and partially possible.

Here begins *new art* created by *capturing pictures of objects that live in the 4th dimension*. The new method is applied to the *hyper-sphere*, which is a higher-dimensional offspring of an ordinary sphere. We see an ordinary sphere, also called a *2-sphere*, each time we look at a full moon or the surface of a basketball, but the hyper-sphere lives in the 4th-dimension beyond human view. A hyper-sphere is also called a *3-sphere*.

The “global structure” of a hyper-sphere requires four dimensions, but human vision is only 3-dimensional. In other words, it is humanly impossible to “see” a hyper-sphere in the 4th dimension.

Nevertheless, the historical value of the hyper-sphere cannot be overstated: Einstein used it in 1917 to describe the shape of our universe, and Dante, circa 1300 AD, used it to describe the Christian afterlife. These descriptions are presented in chapters 2, 3, and 4 using numerous illustrations.

Over time, our understanding of the meaning of *dimension* evolved. By 1970 a *4-web* (think of a spider’s web) from the 4th dimension emerged. In the 4-web case, however, by 2003 it was discovered that its “global structure” could be moved into human view (see left-side graphic above and reference [27]). The 4-web is an example of a construction that was applied in the article “On Imbedding Finite-Dimensional Metric Spaces”, which I published in the *Transactions of the American Mathematical Society* Vol. 211, 1975.

While the “human view” of the 4-web is a relatively recent discovery, the idea of “reasoning well with images of objects” dates back to the late 1800s. For example, within the introduction of Poincaré’s famous *Analysis Situs* (1895), we find

... geometry is the art of reasoning well with badly made figures. Yes, without doubt, but with one condition. The proportions of the figures might be grossly altered but their elements must not be interchanged and must conserve their relative situation.

By moving the 4-web into human view, we “see” its structure: The line segments that comprise the 4-web are organized into *five groups*, each group “just touching” the other four. As Poincaré might say, the five groups were not interchanged and their relative situation was conserved.

Moving objects from one dimension into another begs the question *What do we mean when we say that space has three dimensions?* Such a question concerned Poincaré in 1912, when he stated

... Of all the theorems of analysis situs, the most important is that which we express by saying that space has three dimensions. ...

In this book the concept of *dimension* is tailored for a general audience. To accommodate such an audience, we introduce our friend *Freddy the penguin* whose dimensionally-varying vision is illustrated in numerous situations — Freddy motivates an intuitive understanding of dimension.

The *dimension* concept and the 4-*web* structure are fundamental to the generation of our hyper-sphere art. Roughly speaking, the graphic is generated in two steps: First, the 4-web is used to capture tens-of-thousands of points of a hyper-sphere, like a spider uses his web to capture insects. Second, the captured points are moved into human view using the algorithm for moving the 4-web into human view. The image points that define the graphic, being part of the 4-web graphic, meet Poincaré's requirement — they are *not interchanged and their relative situation is conserved*.

So given the modern, 21-Century mathematical, understanding of the 4-web and how it may be moved into human view, it is possible to create a *partial image of the hyper-sphere* (right-side graphic above) — *the impossible becomes partially possible*.

But what would a new work of art be without a title? In this case, the title reflects the history of the hyper-sphere and viewer impressions. Most viewers see the *face of man*, and such impressions combined with Einstein's and Dante's use of the hyper-sphere were key to the selection of the title "*God's Image?*" (with a question mark).

During the writing and rewriting of drafts of this book several people provided valued contributions: Some were colleagues, some are 3-D graphic specialists, and others were students. My former student Chris Edward Dupilka provided computer algorithms, beautiful graphics, and the first movie on the supplemental Blu-ray Disc; my colleagues Allen Danforth Parks, John Earl Gray, Eugene LeRoy Miller, Jimmy Gardner, Larry Lehman, and, Janet and Richard Zeleznock carefully read drafts and provided suggestions that greatly improved the presentation for a general audience; and the Rhino 3-D software (www.rhino3d.com) specialists Mary Fugier, Jerry Hambly, and Pascal Golay provided Rhino script and macros that supported the development of graphics and videos.

Spotsylvania, VA
June 2011

Stephen L. Lipscomb

Art Meets Mathematics in the Fourth Dimension

Lipscomb, S.

2014, XVII, 184 p. 147 illus., 28 illus. in color.,

Hardcover

ISBN: 978-3-319-06253-2