Preface

Magische Symmetrie—Die Ästhetik in der modernen Physik

I like sailing. On many of my sailing trips I join my former colleague and friend Frieder[1], who is an electrical engineer by education. You can imagine that especially if the wind has slowed down we have plenty of time to chat about on-board short-circuits (which is Frieder's task) and the influence of relativity theories on our navigational GPS (which is supposed to be my task, being a theoretical physicist by education). Both non-working batteries and well-working GPS naturally lead to conversations about the laws of nature–how physicists formulate and understand them, and how engineers are utilizing them for our daily life. And I can tell you: these are worlds apart. When I tried to explain to Frieder why and how I'm giving lectures on "Symmetries in Physics" he was completely confused, because symmetry for him is not a category of pure and applied science, but–if at all–maybe for art. As a matter of fact, it is not easy to convey a feeling to a person outside of physics (and mathematics) how arguments of elegance, beauty, aesthetics, ... can be leitmotifs in such rational disciplines.

Let me give Frieder another chance to get my message through: Isn't it surprising that we can do physics at all (and engineering, in order to suit him)? Only because certain patterns in Nature occur here and there (and today and yesterday, etc.) in a comparable manner, can we dare to establish certain "laws of nature".[2] And strangely enough the laws of physics can be expressed in the language of mathematics. During the course of time, mankind discovered more and more phenomena of nature. But physics did not simply grow in the same way; now and then it was realized that certain phenomena, previously not known to be related, astoundingly do have common roots. As a matter of fact, physics today is simpler than the physics at the beginning of the last century. And one of the reasons for this are symmetries of Nature–or to be more precise: "symmetries of fundamental physics".

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[1] Frieder and I worked for more or less twenty years in an industrial research institute on robotics and artificial intelligence.

[2] Here we have the first disparity between laws of nature and batteries: a battery may have worked yesterday, but it does not work today, and low and behold, if one just waits until tomorrow, it will work again.

After some struggling with myself I decided to use the wording “fundamental physics” in the title of this book. This is a risk in the sense that every scientist more or less claims to do fundamental research. And, as explained also in this book, the notion of “fundamental” received a change in attitude with the very idea of effective field theories. Apart from that, I had to indicate that this is not a book on symmetries in “physics”, because for instance symmetries in solid state and atomic physics are left out completely. Instead of “fundamental physics” I could have chosen the more apt wording “elementary particle physics and relativity theories”. Actually my lectures at the Freie Universität Berlin, from which this book originates, were once announced this way, but I thought it too clumsy to be used as the title of a book.

The theme “symmetries in elementary particle physics and relativity theories” is also dealt with in the nice book by A. Zee[3] [578], which very emphatically brings the symmetry arguments of today’s fundamental physics to the layman–in the sense that no mathematical formula is used. The present book is neither meant for layman nor for experts, but for readers with some background in theoretical physics. Nothing of the material is really new. Nearly everything can be found in textbooks or review articles. However, I know of no other attempt to treat fundamental physics entirely under the aspect of symmetries. You may take this monograph as a “mathematically enriched” version of Zee’s book. And although it is meant for physicists, it may also be of interest to philosophers and mathematicians (even though it may not completely satisfy their standards). Hopefully also Frieder can benefit from browsing through the text.

The very idea of this book is to explore established physics and introduce modern particle and relativity physics following the golden thread of symmetries. I asked myself how much of physics can be deduced from symmetries. To be sure, the symmetry arguments need to be fed and supported by experimental results. But it seems that the interaction between the theoretician and the experimentalist can be channelled by symmetries. And therefore in this book I start from very well established notions of analytical mechanics and follow the symmetry thread to highly topical issues such as, for example, the entropy of black holes.

Only at the beginning of the last century with the advent of the relativity theories and of quantum physics it was realized that symmetries and the related notion of invariance are a kind of principle of nature. For instance E. P. Wigner, one of the main symmetry players, coined the phrase “… if we knew all the laws of nature, or the ultimate Law of nature, the invariance properties of these laws would not furnish us new information.” Today two types of symmetry play a prominent role, namely space-time symmetries–as visible in the relativity theories and in relativistic field theories–and symmetries in abstract mathematical spaces, so called internal symmetries. Symmetries imply conservation laws, they allow us to classify particles and field variants, and they make sure that infinities otherwise arising in a quantum field theory cancel each other. In particle physics it became

[3] The exceedingly well-chosen German title is the motto at the beginning of this preface.

apparent that the basic interactions are based on local internal symmetries (or gauge symmetries). And in a sense to be described, also general relativity is a gauge theory. Since the symmetry transformations constitute a group in the mathematical sense, group theory plays a prominent rule in fundamental physics. Predominantly we are dealing with continuous groups, named after Sophus Lie, their algebras and–as a consequence of quantum-mechanical principles–the irreducible unitary representations of these groups. Some of the symmetries seem to be exact within the present experimental boundaries (like the symmetry with respect to Lorentz transformations), others are more or less broken. Examples are the spontaneous breaking of a symmetry in the electroweak theory, giving rise to the Higgs boson, or the violation of space reflection and of time reversal in weak interactions. Given the success of symmetry arguments for constructing the standard model in particle physics, it is no surprise that further symmetries (as for instance supersymmetries) are envisaged. Indeed, at present, the drawers of particle and gravity theorists can hardly hold the huge amount of speculations. All of these are symmetry inspired, but need either to be falsified or supported by experimental results such as are expected from the Large Hadron Collider, the most powerful microscope available today–and probably for years to come.

It might sound strange and it does not shed light on the imagination of an author, if he must concede in his preface that the headings of the chapters in his book do not cover what they pretend to contain. You may read each heading “XXX” as having the real meaning of “Symmetries and XXX”. In order to make this clear, each chapter starts with a short preamble of what “XXX” adds[4] to our understanding of symmetries.

Instead of treating the topic directly and top-down from a modern perspective, I preferred to follow the historical course in unraveling symmetries in physics.

- Therefore I start with “Classical Mechanics” (Chap. 2), its conservation laws and the Galilei group as its symmetry group. Although hardly mentioned in textbooks and lectures, the so-called first Noether theorem is hidden behind the scene.
- Chapter 3 on “Electrodynamics and Special Relativity” is the motivation for discussing the Poincaré group and the two Noether theorems. This became a rather long chapter: Together with the Poincaré symmetry we can consider not only its limit to Galilei symmetry, but also understand the Poincaré symmetry as a limit of de Sitter symmetry, and as a specific restriction of conformal symmetries. The latter show up for instance as approximate symmetries in the high-energy limit of quantum chromodynamics, or as essential symmetries of string models. The pivotal work by E. Noether on variational symmetries remained outside the focus of physicists for more than half a century (and was re-derived by them several times). In the context of present-day theoretical

[4] … or added–in an historical point of view.

research it experienced an extension from finite and from closed algebras to infinite-dimensional and to open algebras.

- A book on symmetries in fundamental physics must reflect that fundamental physics is quantized (at least particle physics, but also gravity of black holes or near the big bang). The notion of symmetry in "Quantum Mechanics" (Chap. 4) gives rise to discuss both Wigner's theorem about symmetry generators and unitary (respectively anti-unitary) operators, and the ensuing important relation between symmetries and ray representations.
- In "Relativistic Field Theory" (Chap. 5) one requires for the defining properties of particles the irreducible unitary representations of the Poincaré group, and the representation of the Lorentz group for classifying fields. Notice that this chapter is headed "Relativistic Field Theory" and not "Quantum Field Theory". Certainly, today's theories of particle physics are quantum field theories. However, going into details here would take us far beyond the context of this book. Nevertheless, I felt obliged to have a subsection on effective theories including the idea of the renormalization group. For one thing, the notion of running couplings is needed to fully understand the standard model of particle physics and its possible extensions, and for another thing, going from one effective field theory to a neighboring one is related to the introduction or deletion of symmetry-related terms in the Lagrangians of the theories. There is a further section on spontaneous symmetry breaking, arising if the symmetry of the action is not a symmetry of the vacuum state. This phenomenon is essential in the formulation of the electroweak sector of the standard model. Another section deals with the discrete symmetries related to charge conjugation, space reflection and time reversal.
- After having established in Chap. 5 the symmetry (and the dimensional renormalization) principle for constructing actions for field theories the technical apparatus for understanding the gauge-symmetry-based aspects of "Particle Physics" (Chap. 6) is at our disposal. Chapter 6 also contains material on 'anomalies'. In a QFT-based presentation of symmetries, anomalies could be worth a separate chapter, since these represent situations where symmetries in the classical laws are spoiled or even completely destroyed by radiative quantum corrections.
- The next chapter (Chap. 7) deals with symmetries in "General Relativity and Gravitation". You doubtless know that general relativity (GR) deals with 'curved' spacetime, and therefore to understand gravitodynamics there is a section on Riemann geometry and more general geometries. The symmetry group of GR is related to general coordinate transformations which constitute an infinite-dimensional Lie group. It is an open issue in which sense general relativity (respectively possible extensions) is a gauge theory. There are common features but from its basics, GR is different from Yang-Mills type theories. This is specifically true considering the point of energy-momentum conservation. Theories of gravity also opened the understanding of boundary terms in action

functionals and their relation to topological invariants. Although without doubt GR is the best theory of gravity, symmetries allow many ways to extend Einstein's theory. Among these are alternative geometries (manifolds with torsion) and/or dynamics (actions quadratic in curvature and torsion).

- The chapter on "Unified Field Theories" (Chap. 8) extends the scope of currently observed gauge symmetries (grand unified theories), reflects about general relativity extended to higher dimensions (Kaluza–Klein models), to a symmetry comprising bosons and fermions (supersymmetry and supergravity). I also added some material on superstrings since these–although not amenable to the world as we know it–seem to be a testing ground for all kind of down-to-earth, exotic, and still unknown symmetries.
- In the "Conclusion" (Chap. 9) I present the symmetries of the 'World' action in a condensed form. Further I sketch how far symmetries serve for unifying physics, and I also reflect about some philosophically inspired issues around the topic of symmetries, like their possible origin, or their role in a basket of 'principles'.

As for the appendices: In order to fully understand symmetries some knowledge in "Group Theory" (Appendix A), especially on representation of groups and Lie algebras is indispensable. Anti-commuting entities enter physics in the context of "Spinors, $\mathbb{Z}_2$-gradings, and Supersymmetry" (Appendix B). Since all theories in fundamental physics are invariant with respect to local symmetry transformations, they necessarily are singular systems, and as such give rise to constraints, dealt with in Appendix C on "Symmetries and Constrained Dynamics". The theorems of E. Noether play a key role in the symmetries dealt with in this book. These assume a different form in classical mechanics, quantum mechanics, and in relativistic field theories. Since the latter are conceptually most often formulated in the language of path integrals, an appropriate Appendix D "Symmetries in Path Integral and BRST Quantization" is added. In a top-down approach to the theme of symmetry, the path integral and specifically the Faddeev-Popov recipe for quantizing Yang-Mills and generally covariant theories would probably constitute the first chapter; immediately followed by the BRST symmetries and there role in field quantization. The basic structures of fundamental physics (especially with regard to symmetries) can best be understood in the mathematical language of differential manifolds and bundles. It is in the language of modern "Differential Geometry" (Appendix E) where one can see for example the commonalities and the distinctions of Yang-Mills theories and general relativity. Especially in terms of differential forms (Appendix F "Symmetries in Terms of Differential Forms"), symmetries and their consequences can be described in a very elegant and compact way; as a matter of fact invariance with respect to general coordinate transformations is guaranteed by the calculus per se. In some contexts (e.g. for Poincaré gauge theories of gravity and in their generalization to supermanifolds) they are far superior to an index notation.

As may be already clear, this book is written by a physicist and it addresses itself to physicists in the first instance.[5] The text originates from lectures, and my audience was made up of advanced undergraduate students, graduate students, and curious colleagues from science departments. From the students I only required knowledge in analytical mechanics, Maxwell's form of electrodynamics, and quantum mechanics. But I expected no prior knowledge of the standard model or of general relativity. The lectures were not part of a curriculum with some final examinations on the topic, but were considered by the students as a "tip of cream" on top of their required lectures. I think, that the theme of the lecture serves to satisfy the curiosity which drives young people to study physics anyhow.

By far not all of the material in this book was covered in my 32 h of lectures. So I did not have the time to talk about symmetries in unified field theories and all those other topics that are marked with an asterisk in the Contents. I also did not cover completely what is here the Appendix A on group theory. However, since this material is absolutely essential, I gave it as a handout to the students. Together with this appendix and the part on spinors in Appendix B, the main text is completely self-contained. All other appendices are additional material going deeper either into field theory or into a modern differential geometry notation. For a first reading these appendices are not needed.

As to the literature I not only refer to articles and books which are on the same level as the lectures, but also more advanced material, including topical articles. And, given my relation to the Max Planck Institute for the History of Science, I aim to be as correct as possible in citing historical sources. Everybody has a chance to follow the discussion on symmetries, their scope and their applicability in more detail. Everyone is invited to start on his/her level of knowledge, pick up the golden symmetry thread, and hopefully will be led to areas of new knowledge.

[5] If you prefer a book on the topic written by a mathematician you may consult [467], a book (in German) which regrettably is out of print.

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